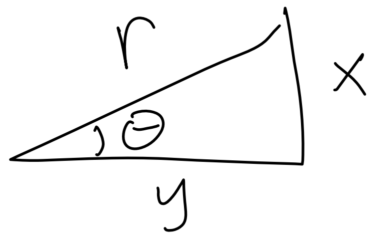
Lec. No. (1)

Revision

$$\sin\theta = \frac{x}{r}$$



$$\cos\theta = \frac{y}{r}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{y}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{y}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{x}$$

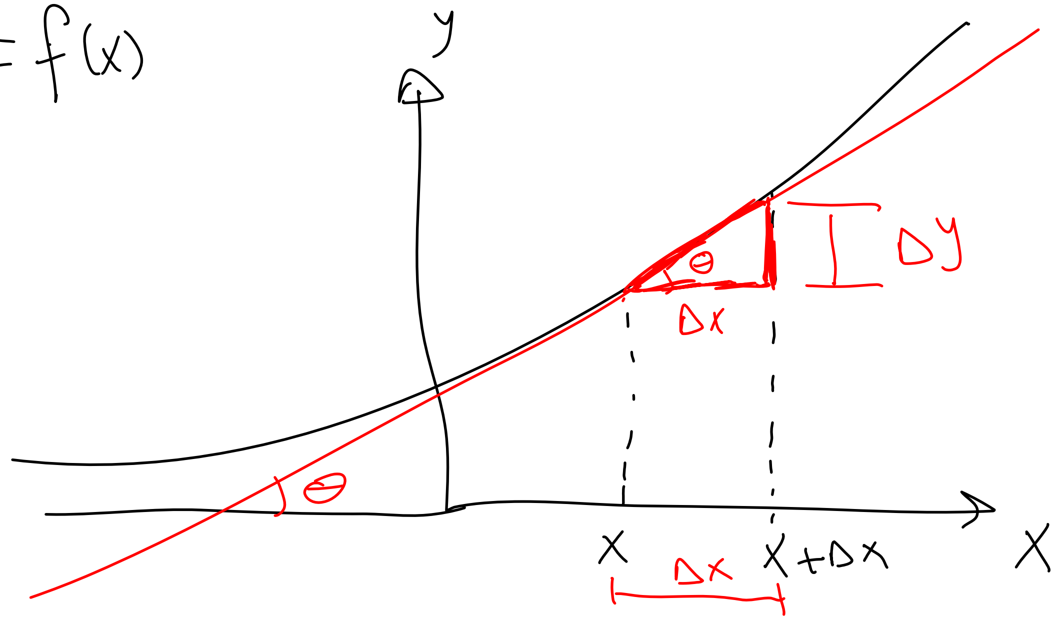
$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} = \frac{y}{x}$$

Differentiation

What is meant by Differentiation?

It's the slope of the tangent for the curve $y = f(x)$ at a certain Point.

Given $y = f(x)$

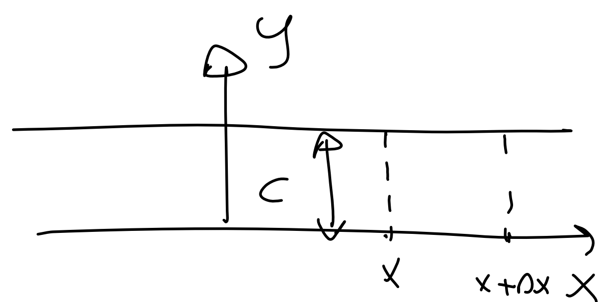


$$y' = \frac{dy}{dx} = m = \tan\theta = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$y' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

All basic rules of differentiation come from this rule.

Say $y = C$



$$y' = \lim_{\Delta x \to 0} \frac{C - C}{\Delta x} = 0$$

Say $y = f(x) = x$

$$y' = \lim_{\Delta x \to 0} \frac{(\cancel{x} + \Delta x) - \cancel{x}}{\Delta x} = 1$$

Say $y = f(x) = x^2$

$$y' = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 - \cancel{x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x (2x + \Delta x)}{\Delta x} = 2x$$

Basic Rules of differentiation

$\boxed{1}$ $y = c \Longrightarrow y' = 0$

Ex. $y = 3 \Longrightarrow y' = 0$

$\boxed{2}$ $y = x^n \Longrightarrow y' = n x^{n-1}$

Ex. $y = x^5 \Longrightarrow y' = 5x^4$

$\boxed{3}$ $y = c f(x) \Longrightarrow y' = c \cdot f'(x)$

Ex. $y = 5x^3 \Longrightarrow y' = 5 \cdot 3x^2 = 15x^2$

$\boxed{4}$ $y = f(x) \pm g(x)$

$y' = f'(x) \pm g'(x)$

Ex. $y = 5x^4 - 3x^2 + 5x - 7$

$y' = 20x^3 - 6x + 5$

$\boxed{5}$ $y = U \cdot V \Longrightarrow y' = U \cdot V' + V \cdot U'$

[Product rule]

Ex. $y = (x^3 + 3x^5)(2x^2 + 7)$

$y' = (x^3 + 3x^5) \cdot [4x] + (2x^2 + 7) \cdot [3x^2 + 15x^4]$

$\boxed{6}$ $y = \dfrac{U}{V} \Longrightarrow y' = \dfrac{VU' - UV'}{V^2}$

[Quotient Rule]

Ex. $y = \dfrac{(x^3 + 5x)}{(x^7 + 3x^2)}$

$$y' = \frac{(x^7 + 3x^2)(3x^2 + 5) - (x^3 + 5x)(7x^6 + 6x)}{(x^7 + 3x^2)^2}$$

$\boxed{7}$ $y = u^n \Rightarrow y' = n u^{n-1} \cdot u'$

Ex. $y = (3x^6 + 5x^2 + 3x + 7)^6$

$y' = 6(3x^6 + 5x^2 + 3x + 7)^5 \cdot [18x^5 + 10x + 3]$

$\boxed{8}$ $y = \sqrt{x} \Rightarrow y = x^{\frac{1}{2}} \Rightarrow y' = \frac{1}{2} x^{-\frac{1}{2}}$

$y' = \frac{1}{2} \frac{1}{\sqrt{x}} \Rightarrow \boxed{y' = \frac{1}{2\sqrt{x}}}$

Ex. $y = 5\sqrt{x} \Rightarrow y' = 5 \cdot \frac{1}{2\sqrt{x}} = \frac{5}{2\sqrt{x}}$

Ex. $y = 7\sqrt{x} \Rightarrow y' = \frac{7}{2\sqrt{x}}$

Ex. $y = \sqrt[3]{x} \Rightarrow y = x^{\frac{1}{3}} \Rightarrow y' = \frac{1}{3} x^{-\frac{2}{3}}$

Ex. $y = \sqrt[4]{x} \Rightarrow y = x^{\frac{1}{4}} \Rightarrow y' = \frac{1}{4} x^{-\frac{3}{4}}$

$\boxed{9}$ $y = \sqrt{u} \Rightarrow y' = \frac{1}{2\sqrt{u}} \cdot u'$

Ex. $y = \sqrt{x^3 + 2x^2} \Rightarrow y' = \frac{1}{2\sqrt{x^3+2x^2}} [3x^2 + 4x]$

Ex. $y = \frac{1}{x} \Rightarrow y = x^{-1} \Rightarrow y' = -x^{-2}$

$\therefore y' = -\frac{1}{x^2}$

Ex. $y = \frac{5}{x} \Rightarrow y' = \frac{-5}{x^2}$

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No. 1

$f(x) = x^3 - 2x + 1 \Rightarrow f' = 3x^2 - 2$

No. 3 $f(x) = 3t^3 - 2\sqrt{t} \Rightarrow f' = 9t^2 - \frac{1}{\sqrt{t}}$

No. 5 $f(w) = \frac{3}{w} - 8w + 1 \Rightarrow f' = \frac{-3}{w^2} - 8$

No. 7

$$h(x) = \frac{10}{\sqrt[3]{x}} - 2x + \pi$$

$$h(x) = 10x^{-\frac{1}{3}} - 2x + \pi$$

$$h' = -\frac{10}{3}x^{-\frac{4}{3}} - 2$$

Higher order Derivatives

Say $\quad y = x^4 - 5x^2 + 3x - 7$

1^{st} derivative $\quad y' = \frac{dy}{dx} = 4x^3 - 10x + 3$

2^{nd} ,, $\quad y'' = \frac{d^2y}{dx^2} = 12x^2 - 10$

3^{rd} ,, $\quad y''' = \frac{d^3y}{dx^3} = 24x$