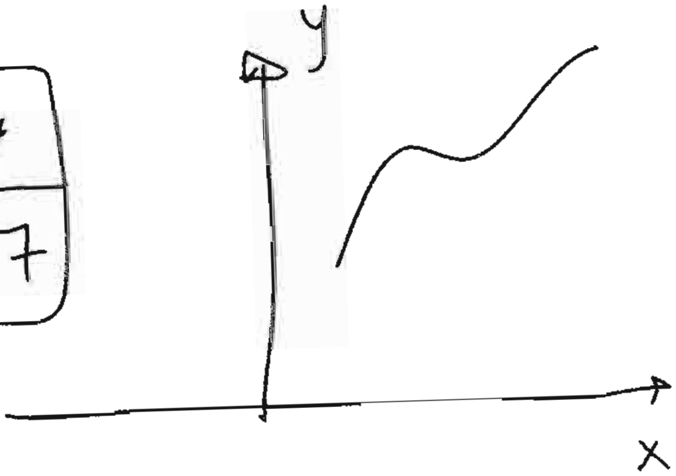
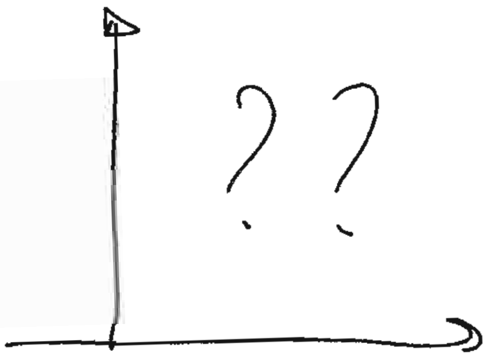
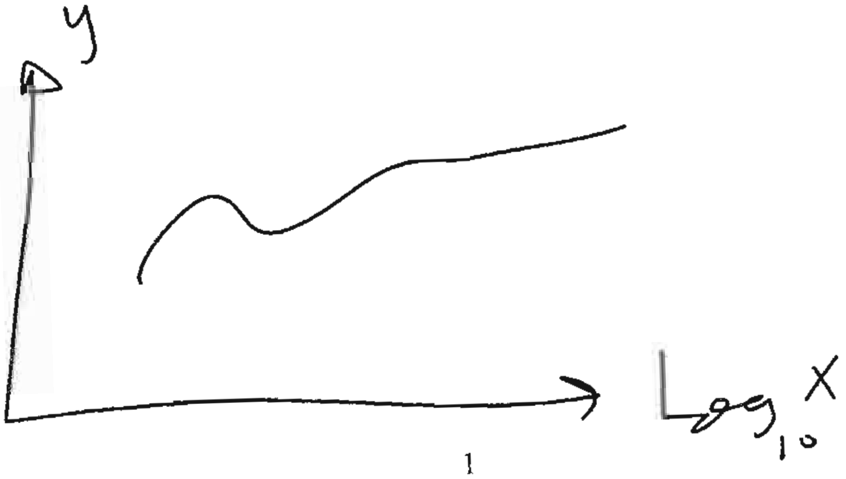
Lec. No. (4)
Logarithmic functions and their derivatives

X	1	2	3	4
y	5	10	12	17



Log x	1	2	3	4	5
X	10	100	1000	10000	100000
y	✓	✓	✓	✓	✓





You know

① $\log_{10} 10 = 1$

② $\log 100 = \log_{10} 10^2 = 2$

③ $\log 1 = \log_{10} 10^0 = 0$

④ $\log_2 8 = \log_2 2^3 = 3$

⑤ $\log_a a^b = b$

⑥ $\log xy = \log x + \log y$

⑦ $\log x^y = y \log x$

⑧ $\log \frac{x}{y} = \log x - \log y$

Irrational number e

$$\lim_{n\to\infty} \left[1+\frac{1}{n}\right]^n = e$$

$n=1$ $\quad \lim_{n\to 1} \left[1+\frac{1}{n}\right]^n = 2$

$n=2$ $\quad \lim_{n\to 2} \left[1+\frac{1}{2}\right]^2 = 2.25$

$$\lim = 2.718281828\ldots$$
$$= \boxed{e}$$

$$\log_e \square = \ln \square$$

Rules

1. $\ln 1 = \log_e e^0 = \boxed{0}$

2. $\ln x^y = y \ln x$

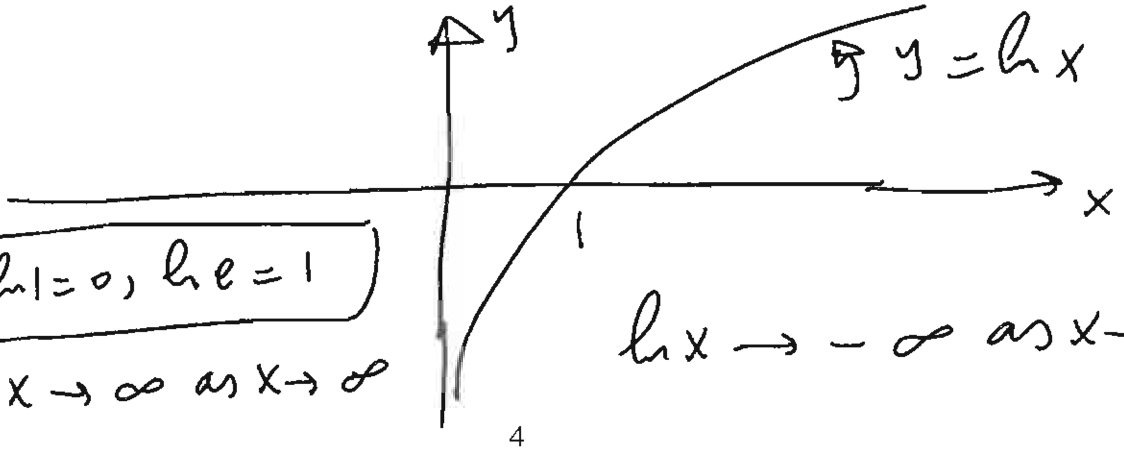
3. $\ln xy = \ln x + \ln y$

4. $\ln \frac{x}{y} = \ln x - \ln y$

5. $\ln e = \log_e e = \boxed{1}$

6. $\ln e^{\Box} = \Box \ln e = \Box$

7. $y = \ln x \Rightarrow \boxed{x = e^y}$



$\boxed{\ln 1 = 0,\ \ln e = 1}$

$\ln x \to \infty$ as $x \to \infty$

$\ln x \to -\infty$ as $x \to 0$

$\boxed{8}$ If $y = \ln u \Rightarrow y' = \frac{1}{u} \cdot u'$

$\boxed{9}$ If $y = \log_a u \Rightarrow y' = \frac{1}{u} \cdot u' \cdot \frac{1}{\ln a}$

<u>Text book</u> <u>Ex. 2.7 Page 181</u>

<u>No. 13</u> $f = \ln 2x \Rightarrow f' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

<u>No. 15</u> $f = \ln(t^3 + 3t)$

$$f' = \frac{1}{(t^3+3t)} \cdot [3t^2 + 3]$$

<u>No. 17</u> $g = \ln(\cos x) \Rightarrow g' = \frac{1}{\cos x} \cdot (-\sin x)$

$$= -\tan x$$

No.19 (a) $f = \sin(\ln x^2)$

$f' = \cos(\ln x^2) \cdot \frac{1}{x^2} \cdot 2x$

(b) $g = \ln(\sin t^2)$

$g' = \frac{1}{\sin t^2} \cdot \cos t^2 \cdot 2t = \boxed{2t \cot t^2}$

sheet (4)

(a) (1) $y = x^3 \ln x$

$y' = x^3 \cdot \frac{1}{x} + \ln x \cdot 3x^2$

(2) $y = \ln\left[x^{-4}(x^5-2)^6\right]$

$y = \ln x^{-4} + \ln(x^5-2)^6$

$y = -4\ln x + 6\ln(x^5-2)$

$y' = -4 \cdot \frac{1}{x} + 6 \cdot \frac{1}{x^5-2} \cdot 5x^4$

(b) If $y = \ln(\sec x + \tan x)$

show $y'' = \sec x \tan x$

<u>Proof</u>

$$y' = \frac{1}{(\sec x + \tan x)} \cdot \left[\sec x \tan x + \sec^2 x\right]$$

$$y' = \frac{1}{\cancel{(\sec x + \tan x)}} \cdot \sec x \cancel{\left[\tan x + \sec x\right]}$$

$$y' = \sec x \Longrightarrow y'' = \sec x \tan x$$

o.k

<u>Ex.</u> If $y = x^2 \Longrightarrow y' = 2x$

<u>Ex.</u> If $y = x^x$

$$y' \neq x \cdot x^{x-1}$$ [since x is a vari

$$y = x^x$$

By taking logarithm for both sides

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

Diff. w.r.t x

$$\frac{1}{y} \cdot y' = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$y' = y\left[1 + \ln x\right] = x^x\left[1 + \ln x\right]$$

Ex.

$$y = x^{\tan x}$$

$$\ln y = (\tan x) \ln x$$

$$\frac{1}{y} \cdot y' = \tan x \cdot \frac{1}{x} + \ln x \cdot \sec^2 x$$

$$y' = x^{\tan x}\left[\frac{\tan x}{x} + \ln x \cdot \sec^2 x\right]$$

a) (3) $y^x = x^y$

$$x \ln y = y \ln x$$

$$x \cdot \frac{1}{y} \cdot y' + \ln y \cdot 1 = y \cdot \frac{1}{x} + \ln x \cdot y'$$

$$y' \left[\frac{x}{y} - \ln x \right] = \frac{y}{x} - \ln y$$

$$y' = \frac{\quad}{\quad}$$

(4) $y = \ln \left[\frac{(x^3)(1-x^2)^4}{\sin x \, (x-1)^5} \right]^{\frac{7}{2}}$

$$y = \frac{7}{2} \left[\left[\ln x^3 + \ln (1-x^2)^4 \right] - \left[\ln \sin x + \ln (x-1)^5 \right] \right]$$

$$y = \frac{7}{2} \left[3 \ln x + 4 \ln (1-x^2) - \ln \sin x - 5 \ln (x-1) \right]$$

$$y' = \frac{7}{2} \left[3 \cdot \frac{1}{x} + 4 \cdot \frac{1}{(1-x^2)} \cdot (-2x) - \frac{1}{\sin x} \cdot \cos x - 5 \cdot \frac{1}{x-1} \cdot 1 \right]$$

$\boxed{5}$ $y = \sin x^2 - 3x\cos x - x^x$

let $z = x^x \Longrightarrow \ln z = x \ln x$

$$\frac{1}{z} \cdot z' = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$z' = x^x \left[1 + \ln x \right]$$

$$y' = \cos x^2 \cdot 2x - 3\left[-x\sin x + \cos x \cdot 1 \right] -$$

$\boxed{6}$ $(\sin x)^{\cos y} = (\sin y)^{\cos x}$

$$(\cos y)\ln \sin x = (\cos x)(\ln \sin y)$$

$$\cos y \cdot \frac{1}{\sin x} \cdot \cos x + \ln \sin x \left[-\sin y \right] \cdot y'$$

$$= \cos x \cdot \frac{1}{\sin y} \cdot \cos y \cdot y' + (\ln \sin y)(-\sin x)$$

$$y' \left[-\sin y \ln \sin x - \cos x \cot y\right]$$

$$= -\sin x \ln \sin y - \cos y \cot x$$

$$y' = \frac{}{}$$

Ex-

$$y = \sqrt[3]{\frac{(1-x^2)^4 \cos^{-1} x}{x^{\tan x} \sin x^3}}$$

$$\ln y = \frac{1}{3}\left[4 \ln(1-x^2) + \ln \cos^{-1} x - (\tan x) \ln x - \ln \sin x^3\right]$$

$$\frac{1}{y} \cdot y' = \frac{1}{3}\left[4 \cdot \frac{1}{1-x^2}(-2x) + \frac{1}{\cos^{-1} x} \cdot \frac{-1}{\sqrt{1-x^2}} - \tan x \cdot \frac{1}{x} - \ln x \cdot \sec^2 x - \frac{1}{\sin x^3} \cdot \cos x^3 \cdot 3x^2\right]$$

$$y' = \frac{1}{3}\left(\qquad\right)\left[\frac{-8}{1-x^2} - \frac{1}{\sqrt{1-x^2}\cos^{-1} x} - \frac{\tan x}{x} - \ln x \cdot \sec^2 x - 3x^2 \cot x^3\right]$$

Ex.

$$\sqrt[4]{y} = \frac{x^4 \cot^{-1} x}{(1+x)^4 \sqrt[5]{x^2+1}}$$

$$\boxed{x^{\frac{1}{4}} = y \Rightarrow x = y^4}$$

$$y = \left(\qquad \right)^4$$

$$\ln y = 4\left[4\ln x + \ln \cot^{-1} x - 4\ln(1+x) - \frac{1}{5}\ln(x^2+1)\right]$$

$$\frac{1}{y}\, y' = 4\left[4\cdot\frac{1}{x} + \frac{1}{\cot^{-1}x}\cdot\frac{-1}{1+x^2} - 4\cdot\frac{1}{1+x}\cdot 1 - \frac{1}{5}\cdot\frac{1}{x^2+1}\cdot 2x\right]$$

$$y' = 4\left(\qquad \right)\left[\frac{4}{x} - \frac{1}{(1+x^2)\cot^{-1}x} - \frac{4}{1+x} - \frac{2x}{5(x^2+1)}\right]$$