Lecture 1
Integration I

Anti-derivatives

Anti-derivatives and Integration

- So far we have worked with ***known*** (or given) ***functions*** that ***provided information*** about the ***total amount*** of a quantity.

- By working with these functions we can obtain information about the rate of change of these quantities.

- The latter allow us to answer important questions about the intervals of increase or decrease as well as the extreme values of the functions.

Anti-derivatives and Integration

- It is ***not always possible to find ready-made functions that provide information*** about the ***total amount*** of a quantity,

- But it is ***often possible*** to collect enough data ***to come up with a function that gives the rate of change*** of a quantity.

Anti-derivatives and Integration

- We know that the ***rate of change*** is given by the ***derivative***,

- And so it is natural to try ***reversing the process***, i.e. ***using derivatives to find original functions.***

- This ***reverse process*** is called ***anti-differentiation*** or ***integration.***

In Summary

- Differentiation of f(x) gives f`(x)

- Integration of f`(x) gives f(x)

Anti-derivatives and Integration

- The ***antiderivative*** of a function is defined as follows,
- *If F`(x) = f(x)*
- *then the function F is called*
- *an* ***antiderivative*** *of f.*

Example

- If $F(x) = x^3$,
- then $F`(x) = f(x) = 3x^2$,
- and so F is an antiderivative of $f(x) = 3x^2$.

Example

- Find ***an antiderivative*** of $f(x) = 6x^5$.
- We recall that $(x^n)` = nx^{n-1}$
- and get $F(x) = x^6$.

The Integration Constant

- We observe that the function $F(x) = x^3$ is not the only function whose derivative is $f(x) = 3x^2$.

- Set $G(x) = x^3 + 5$. It is obvious that $G`(x) = 3x^2$.

- In general, every function G of the form $G(x) = x^3+C$, where C is a real number, is an antiderivative of $f(x) = 3x^2$.

The Integration Constant

- This is the case in general,

- If F and G are anti-derivatives of f on some interval, then there is a constant C such

- that $G(x) = F(x) + C$

- *The real number C is called* ***integration constant***

Anti-derivatives and Integrals

- The set of all ***anti-derivatives*** of *f* is called the ***indefinite integral*** of *f*. This is denoted by

$$\int f(x)dx$$

- Thus

$$\int f(x)dx = F(x) + C$$

- *where F is an antiderivative of f*

Integration Toolbox

- In the previous section we saw how intimately differentiation and indefinite integrals are related.

- It is therefore natural to expect that techniques for finding integrals can be obtained from the rules of differentiation which we had learned before.

Integration Toolbox

- We begin by recalling some of the techniques we used to find derivatives.

1. $(kf(x))' = kf'(x)$;
2. $(f(x) \pm g(x))' = f'(x) \pm g'(x)$;
3. $(x^n)' = nx^{n-1}$;
4. $(\ln|x|)' = \frac{1}{x},\ x \neq 0$;
5. $(e^x)' = e^x$.

Integration Toolbox 1

- Rewriting the rules of differentiation in the integral form we obtain the first part of our Integration Toolbox.

Integration Toolbox 1

1. $\int kf(x)\,dx = k\int f(x)\,dx;$

2. $\int (f(x) \pm g(x))dx = \int f(x)\,dx \pm \int g(x)\,dx;$

Integration Toolbox 1

3. $\int x^n \, dx = \frac{x^{n+1}}{n+1}$, $n \neq -1$;

4. $\int x^{-1} \, dx = \ln|x| + C, \quad x \neq 0$;

5. $\int e^x \, dx = e^x + C$.

Example

- Find

$$\int (3x^4 - \frac{2}{x} + 7e^x)dx$$

Solution

- We make consecutive use of Rules 1 and 2,
- and then apply Rules 3, 4 and 5 to the first, second and third terms, respectively.

$$\int (3x^4 - \frac{2}{x} + 7e^x)dx = 3\int x^4\,dx - 2\int x^{-1}\,dx + 7\int e^x\,dx$$
$$= \frac{3x^5}{5} - 2\ln|x| + 7e^x + C.$$

Example

- Find

$$\int \frac{x^4 - \sqrt{x}}{x^2}\,dx$$

Solution

- We first rewrite the function to be integrated in the simplest form,
- and then using rules 2 and 3, we get

$$\int \frac{x^4 - \sqrt{x}}{x^2} dx = \int \frac{x^4}{x^2} - \frac{x^{\frac{1}{2}}}{x^2} dx = \int (x^2 - x^{-\frac{3}{2}}) dx$$

$$= \frac{x^3}{3} + 2x^{-\frac{1}{2}} + C = \frac{x^3}{3} + \frac{2}{\sqrt{x}} + C.$$

Find the following integrals

$$\int 3e^x dx$$

$$\int \frac{2}{x}\, dx$$

$$\int z^3 dz$$

$$\int 5 dx$$

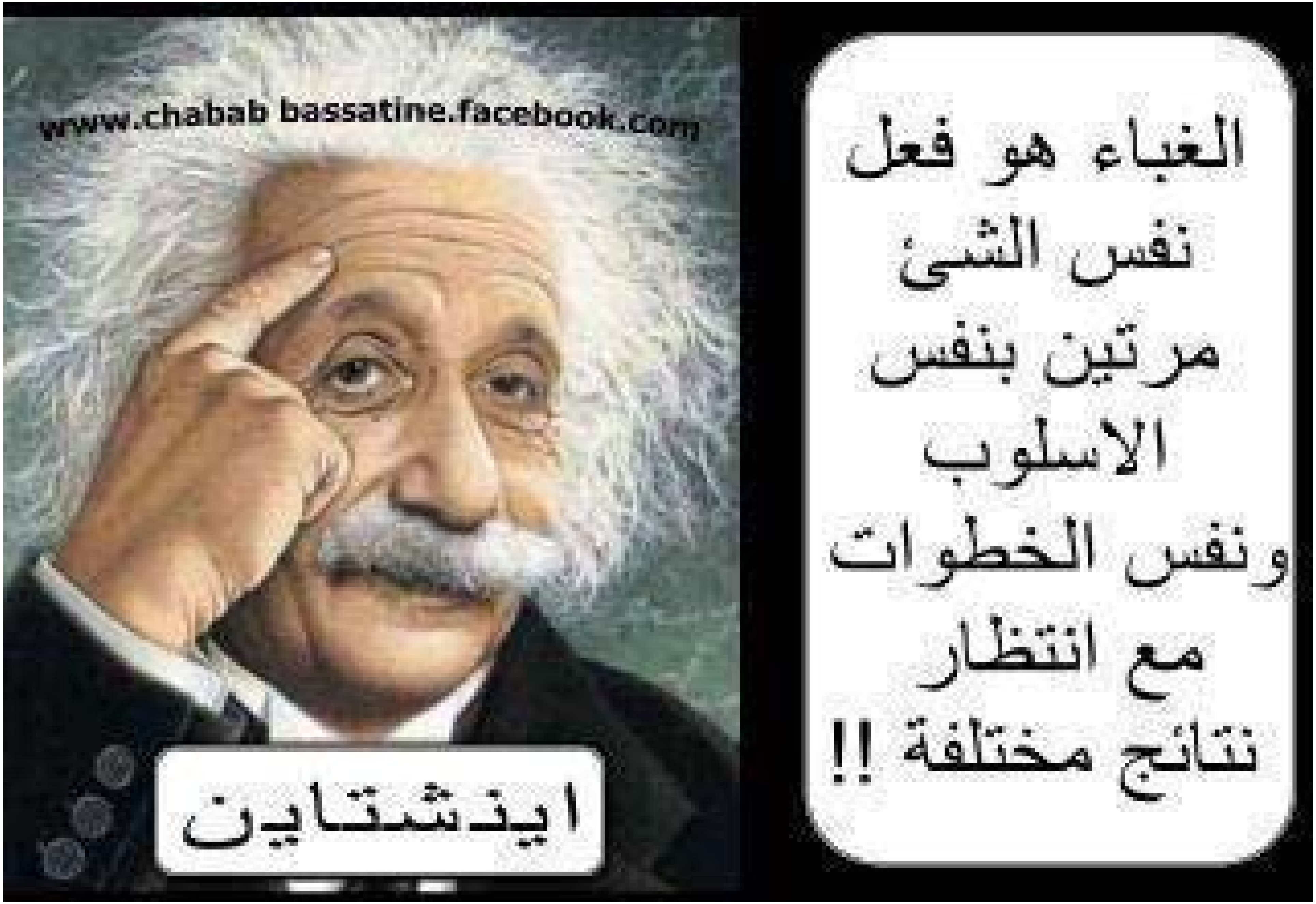
$$\int e^2 dx$$

$$\int y^2 dx$$

$$\int y^2 dx\,, \quad where \;\; y = x + 1$$

$$\int 2e^x - \sqrt{x} + \frac{7}{x} dx$$

$$\int 2e^x - \sqrt{x} + \frac{7}{x^2} dx$$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