# Lecture 2
# Basic Integration Rules

- Fundamental Theorem of Calculus
- Integration by Substitution
- Completing the squares

# Integration Toolbox 2

- We shall face difficulties if we try finding the following integral, using only integration toolbox1

$$\int \frac{2x}{\sqrt{x^2+7}}\,dx$$

# Integration Toolbox 2

- None of the tools from our toolbox matches the integral above.
- This means that new tools need to be found.
- We recall that the chain rule provided us with three convenient formulae for finding derivatives of composite functions of particular types.

# Integration Toolbox 2

- The formulae are called the general power rule, the general logarithmic rule and the general exponential rule.

$$\text{I} \quad ((f(x))^{n+1})' = (n+1)(f(x))^n f'(x);$$

$$\text{II} \quad (\ln|f(x)|)' = (f(x))^{-1} f'(x), \;\; f(x) \neq 0;$$

$$\text{III} \quad (e^{f(x)})' = e^{f(x)} f'(x);$$

# Integration Toolbox 2

- The last three rules give rise to their integral counterparts

I $\int (f(x))^n f'(x)\,dx = \dfrac{(f(x))^{n+1}}{n+1} + C,\ n \neq -1$ **General Power Rule**;

II $\int (f(x))^{-1} f'(x)\,dx = \ln|f(x)| + C, \quad f(x) \neq 0$ **General Logarithmic Rule**;

III $\int e^{f(x)} f'(x)\,dx = e^{f(x)} + C$ **General Exponential Rule**

# Example

- Find

$$\int \frac{2x}{\sqrt{x^2+7}}\,dx$$

# Solution

- We notice that $(x^2 + 7)` = 2x$ and rewrite the integral as follows

$$\int \frac{2x}{\sqrt{x^2+7}}\,dx = \int (x^2+7)'(x^2+7)^{-\frac{1}{2}}\,dx.$$

- It is now obvious that we can integrate using the general power rule.

$$\int (x^2+7)'(x^2+7)^{-\frac{1}{2}}\,dx = \frac{(x^2+7)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x^2+7} + C.$$

# Example

- Find

$$\int \frac{2x}{x^2 + 7} dx$$

# Solution

- We notice that $(x^2 + 7)` = 2x$ and rewrite the integral as follows

$$\int 2x(x^2+7)^{-1}\,dx = \int (x^2+7)'(x^2+7)^{-1}\,dx.$$

- It is now clear that the general logarithmic rule can be applied.

$$\int (x^2+7)'(x^2+7)^{-1}\,dx = \ln|x^2+7| + C.$$

# Example

- Find $$\int xe^{-x^2}\,dx$$

# Solution

- We notice that *(-x² )` = -2x* and rewrite the integral as follows

$$\int xe^{-x^2}\,dx = \int -\frac{1}{2}\cdot(-2x)e^{-x^2}\,dx = -\frac{1}{2}\int(-x^2)'e^{-x^2}\,dx.$$

- It is now clear that the general exponential rule can be applied.

$$-\frac{1}{2}\int(-x^2)'e^{-x^2}\,dx = -\frac{1}{2}e^{-x^2} + C.$$

# Examples

- Find

$$\int \left(15y\sqrt{y} + 2\sqrt{y}\right) dy$$

$$\int \left(\frac{9}{x} - 3e^{-0.4x}\right) dx$$

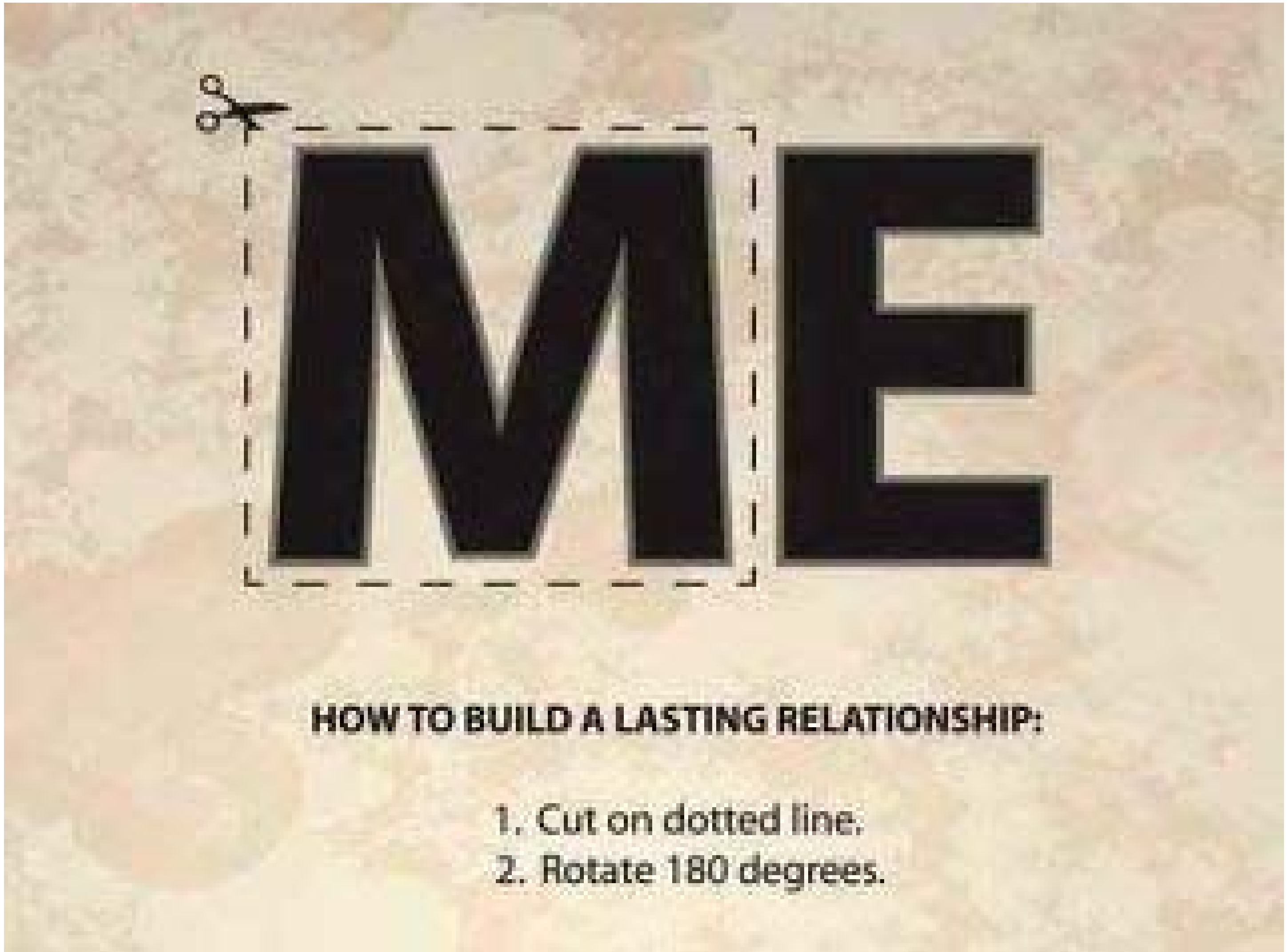
$$\int \frac{\sqrt{2+\ln x}}{x} dx$$

$$\int \frac{e^{2x}}{e^{2x}+5} dx$$

$$\int_1^2 \left(\frac{-1}{B} + 3e^{0.2B}\right) dB$$

$$\int_1^2 \frac{3}{x(1+\ln x)} dx$$

# النذالة


ME
HOW TO BUILD A LASTING RELATIONSHIP:
1. Cut on dotted line.
2. Rotate 180 degrees.

# Indefinite and Definite Integrals

- If we know only the derivative *f`(x)*
- the integration process is trying to find the function *f(x)* itself by a reverse process to the differentiation.
- To be more precise, this process is the ***indefinite integration***

# Definite Integrals

- Now, Let *f* be a function which is continuous on the closed interval **[a, b],** the ***definite integral of f from a to b*** is defined to be the limit

$$\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

- Where

$$\Delta x = \frac{b-a}{n}$$

# Definite Integrals

- The idea of the definite integration will be more clear with the following example of finding the area under the curve $y = f(x)$ from $x = a$ to $x = b$.
- We divide the area to rectangles each of the same width and the height will be the value of $f(x)$ at the middle of this rectangle.

# Definite Integrals

- The total area under the curve is just the summation of all the areas of the rectangles.
- (each area = $f(x_i)\Delta x$ , $x_i$ is usulally the middle value of $x$ in each rectangle)

# Definite Integrals

- The area found by this way is an approximation for the exact area,
- which will be found only when $n \to \infty$
- at which $\Delta x \to dx$

- and in this case the summation is transformed to the integration

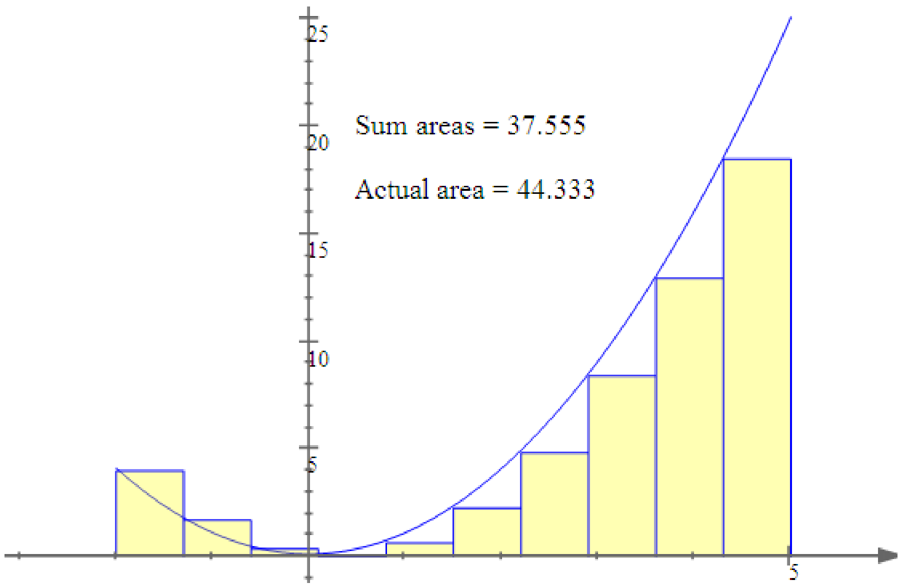
# Illustrative Example

- The following three graphs illustrate the area under the function $y = x^2$

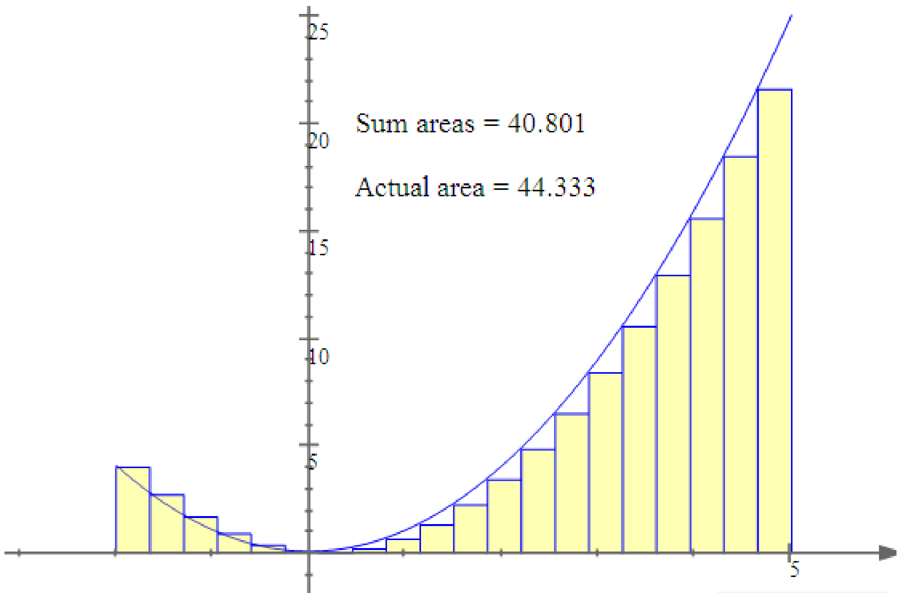
- from $x = -2$ to $x = 5$

- ***calculated for different values of n =10, 20, and 50***.

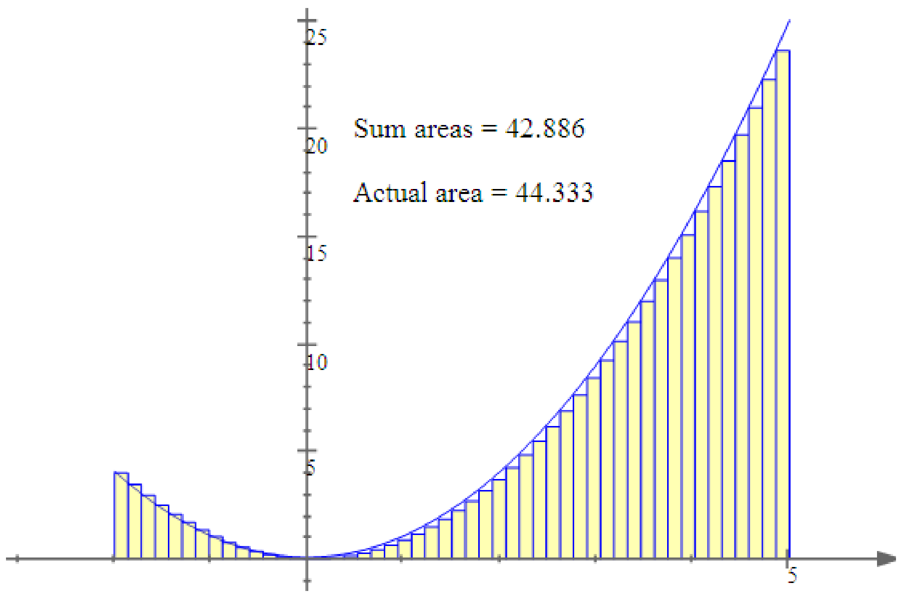
# n = 10



# n = 20



# n = 50



# As n increases, Summation gets Closer to the Exact Value

- It is clear that as we ***increase n*** the approximation area of the ***summation gets closer*** the exact value of the area.

- You can try it yourselves at

http://www.squarecirclez.com/blog/riemann-sums/4715

# Definite Integral of a Function is the Area Under the Function Curve

- Let f be a continuous function on [*a* , *b*].
- Then the Area under the graph of *f*
- on [*a* , *b*]
- is the definite integral from *a* to *b* of *f* and denoted by

$$\int_a^b f(x)dx$$

# Fundamental Theorem of Calculus

- Let f be a continuous function on [*a* , *b*].
  Then

$$\int_a^b f(x)dx = F(b) - F(a)$$

- where *F* is any antiderivative of *f*

# Note

- We point out that the definite integral of a function is a number,

- whereas the indefinite integral is a collection of functions

- that are similar in the independent variable ($x$) and different only by a constant.

# Example

- Find the area under the graph of
- $f(x) = x^2 + 1$
- on [1 , 3]

# Solution

$$Area = \int_{1}^{3} (x^2 + 1)dx = (\frac{x^3}{3} + x)\Big|_1^3 = (\frac{3^3}{3} + 3) - (\frac{1^3}{3} + 1) = \frac{32}{3}$$

# فهمتوا ؟!