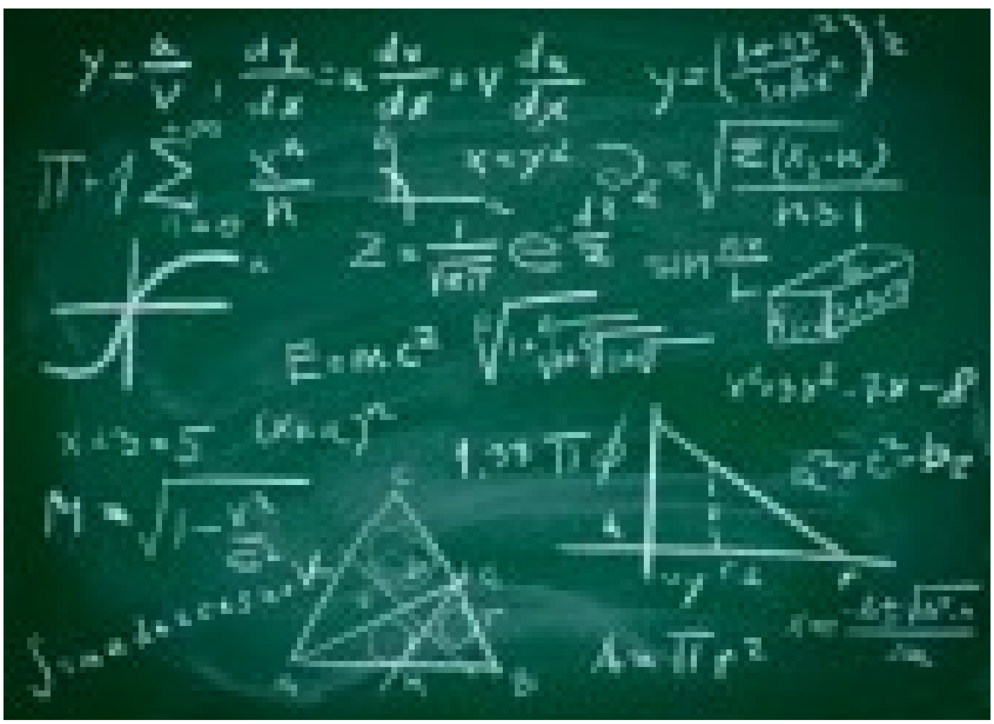
Arab Academy for Science, Technology and Maritime Transport
College of Engineering and Technology
Department of Basic and Applied Science
(Smart Village Campus)

Sheet
Mathematics 1
BA123



Prepared by:
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Arab Academy for Science & Technology and Maritime Transport
College of Engineering & Technology
Department of Basic and Applied Science
Smart Village Campus

BA123 Mathematics (1) Fall 2013-2014

Course Outline

Instructor:	Dr. Hossam Shawky
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Office:	
Off. Hrs:	Thursday (8:30-10:10) and also by appointment
GTA:	Dr. Hossam Shawky
E-mail:	hossams@aast.edu
Office:	
Off. Hrs:	Thursday (8:30-10:10) and also by appointment
Prerequisite:	Non
Course Aim	• Introduce students to differentiation, trigonometric, inverse trigonometric, logarithmic, exponential and hyperbolic functions, as well as parametric, implicit and partial differentiation. • Provide students with a general overview of limits, Taylor’s and Maclaurin’s expansions, curve sketching and conic sections.
Course Objectives	Upon Completion of this course, students should be able to: • Apply the basic rules of differentiation. • Find the derivatives of the trigonometric, inverse trigonometric, logarithmic, exponential and hyperbolic functions, as well as parametric, implicit and partial differentiation. • Define indeterminate forms, L'Hopital's rule and Maclaurin’s expansion • Understand the basics of curve sketching and conic sections.
Text Book	Calculus, Early Transcendental Functions, Robert T.Smith & Roland B.Minton Fourth Edition-Mc-Graw Hill
Reference	Calculus, Sherman K. Stein & Anthony Barcellos Fifth Edition-Mc-Graw Hill
Course Outcomes	An ability to apply knowledge of mathematics, science, and engineering
Grading Policy	**Assignments and attendance:** 10 Marks **Week 3:** Quiz (5Marks) Tutorial **Week 4:** Quiz (5Marks) Lecture **Week 5:** Quiz (5Marks) Tutorial **Week 7:** Exam (15Marks) Lecture **Week 9:** Quiz (5Marks) Tutorial **Week 10:** Quiz (5Marks) Lecture **Week 12:** Exam (10Marks) Lecture **Week 16:** Final Exam (40Marks)

<table>
<tr><th colspan="2">Week of</th><th colspan="2">E V E N T</th></tr>
<tr><td rowspan="3">1</td><td rowspan="3">Sept.22nd</td><td>Lecture</td><td>Rate of change and basic rules of differentiation
Problems : Ex 2.1 Page 134 #1, 3
Ex 2.2 Page 143 # 1, 11
Ex 2.3 Page 151 # 1 - 21 (odd), 25
Ex 2.4 Page 158 # 1 - 12(odd)
Ex 2.5 Page 165 # 1-11 (odd)</td></tr>
<tr><td>Tutorial</td><td>Problems : Ex 2.1 Page 134 #5, 7
Ex 2.2 Page 143 # 3, 5
Ex 2.3 Page 151 # 1 - 21 (even), 26
Ex 2.4 Page 158 # 1 - 12 (even)
Ex 2.5 Page 165 # 1-11 (even)</td></tr>
<tr><td>H.W</td><td>Problems : Ex 2.1 Page 134 #2,4,6
Ex 2.2 Page 143 # 2,4,6,8
Ex 2.3 Page 151 # 22-24
Ex 2.4 Page 158 # 13-16
Ex 2.5 Page 165 # 12-16
Sheet (1): All problems</td></tr>
<tr><td rowspan="3">2</td><td rowspan="3">Sept.29th</td><td>Lecture</td><td>Trigonometric functions and their derivatives
Problems : Ex 2.6 Page 173 #1 - 18 (odd) +Sheet (2)</td></tr>
<tr><td>Tutorial</td><td>Problems : Ex 2.6 Page 173 #1 - 18 (even) +Sheet (2)</td></tr>
<tr><td>H.W</td><td>Problems : Ex 2.6 Page 173 #19-22+ +Sheet (2)</td></tr>
<tr><td rowspan="3">3</td><td rowspan="3">Oct.6th</td><td>Lecture</td><td>Inverse trigonometric functions and their derivatives
Problems : Ex2.8 Page 191 #29,31+Sheet (3)</td></tr>
<tr><td>Tutorial</td><td>Problems : Ex2.8 Page 191 #30,32a, 33+Sheet (3) + Quiz No.1</td></tr>
<tr><td>H.W</td><td>Problems : Ex2.8 Page 191 #34+Sheet (3)</td></tr>
<tr><td rowspan="3">4</td><td rowspan="3">Oct.20th</td><td>Lecture</td><td>Logarithmic functions and their derivatives+ Quiz No.2
Problems : Ex 2.7 Page 181 # 13-20 (odd) +Sheet (4)</td></tr>
<tr><td>Tutorial</td><td>Problems : Ex 2.7 Page 181 # 13-20 (even)+Sheet (4)</td></tr>
<tr><td>H.W</td><td>Problems : Ex 2.7 Page 181 # 23+Sheet (4)</td></tr>
<tr><td rowspan="3">5</td><td rowspan="3">Oct.27th</td><td>Lecture</td><td>Exponential functions and their derivatives
Problems : Ex 2.7 Page 181 # 1-12 (odd) +Sheet (5)</td></tr>
<tr><td>Tutorial</td><td>Problems : Ex 2.7 Page 181 # 1-12 (even)+Sheet (5) + Quiz No.3</td></tr>
<tr><td>H.W</td><td>Problems : Ex 2.7 Page 181 # 21,22,24)+Sheet (5)</td></tr>
</table>

6	**Nov.3rd**	**Lecture**	***Derivatives of hyperbolic and inverse hyperbolic functions*** Problems : Ex 2.9 Page 197 # 5-13 (odd)+Sheet (6)
		Tutorial	Problems : Ex 2.9 Page 197 # 5-13 (even)+Sheet (6)
		H.W	Problems : Ex 2.9 Page 197 # 20,22+Sheet (6)
7	**Nov.10th**	**Lecture**	***Implicit differentiation + 7th week exam*** Problems : Ex2.8 Page 191 #1,2,5-12 (odd)+Sheet (7)
		Tutorial	Problems : Ex2.8 Page 191 #3,5-12 (even)+Sheet (7)
		H.W	Problems : Ex2.8 Page 191 #13-16+Sheet (7)
8	**Nov.17th**	**Lecture**	***Parametric differentiation*** Problems : Sheet (7)
		Tutorial	Problems : Sheet (7)
		H.W	Problems : Sheet (7)
9	**Nov.24th**	**Lecture**	***Partial Differentiation*** Problems : Ex12.3 Page 849#1,3,5,11+Sheet (8)
		Tutorial	Problems : Ex12.3 Page 849#2,4,6,12+Sheet (8)+ **Quiz No.4**
		H.W	Problems : Ex12.3 Page 849#13,14+Sheet (8)
10	**Dec.1st**	**Lecture**	***Indeterminate Forms and L'Hopital's Rule+*** **Quiz No.5** Problems : Ex 3.2 Page 230 # 1 - 33(odd) +Sheet (9)
		Tutorial	Problems : Ex 3.2 Page 230 # 1 - 33(even)+Sheet (9)
		H.W	Problems : Ex 3.2 Page 230 # 36 - 43+Sheet (9)
11	**Dec.8th**	**Lecture**	***Maclaurin's expansion*** Problems : Ex 8.7 Page 605 #1,3,5 +Sheet (10)
		Tutorial	Problems : Ex 8.7 Page 605 #2,4,6+Sheet (10)
		H.W	Problems : Ex 8.7 Page 605 #7,8 +Sheet (10)
12	**Dec.15th**	**Lecture**	***Curve sketching: Critical, maximum, minimum and inflection points + 12th week exam*** Problems : Sheet (11)
		Tutorial	Problems : Sheet (11)
		H.W	Problems : Sheet (11)
13	**Dec.22nd**	**Lecture**	***Curve sketching (rational functions) and physical application (velocity and acceleration)*** Problems : Sheet (11)
		Tutorial	Problems : Sheet (11)
		H.W	Problems : Sheet (11)
14	**Dec.29th**	**Lecture**	***Conic Sections : Parabola*** Problems : Ex 9.6 Page 683 #1,13 +Sheet (12)
		Tutorial	Problems : Ex 9.6 Page 683 #2,14+Sheet (12)
			Problems : Ex 9.6 Page 683 #3,4+Sheet (12)
15	**Jan.5th**	**Lecture**	***Revision***
		Tutorial	***Revision***
16	**Jan.12th**	**Final Exam**	

Good Luck

Sheet 1 : Basic Differentiation Rules

Lecture Examples

a. **Find** $\frac{dy}{dx}$ **for each of the following functions**

1) $y = x^4 - 3x^{-2} + 15x + 10$

2) $y = (\sqrt{x} - 1)^7$

3) $y = \frac{1}{(x^6 - 2)^5}$

4) $y = (x^3 - 1)^5 (2 + 3x^{-4})^7$

5) $y = \frac{x^3 - 1}{x^3 + 1}$

6) $y = \left(\frac{x^2 - 3}{x^{-4} + 2} \right)^{4/3}$

b. **Find** $\frac{d^2y}{dx^2}$ **for each of the following functions**

1) $y = x^7 - \frac{2}{x^3} + x^{-5} + 16x + 5$

2) $y = (2 - x^3)^8$

c. If $y = (x + \sqrt{x^2 - 1})^4$, Show that $\frac{dy}{dx} = \frac{4y}{\sqrt{x^2 - 1}}$

Classroom Exercises

d. **Find** $\frac{dy}{dx}$ **for each of the following functions**

1) $y = \frac{3}{x} - \frac{4}{x^2} + 6x^5 + 7$

2) $y = (x^4 - 1)^6$

3) $y = \left(\frac{1 - x^4}{1 + x^4} \right)^{3/2}$

4) $y = \sqrt{x^3 - 1}\, (1 - 3x)^5$

5) $y = \sqrt{\frac{x - 1}{x + 1}}$

6) $y = \left(1 - \frac{1}{\sqrt{x}} \right)^{-4/3}$

e. **Find** $\frac{d^2y}{dx^2}$ **for each of the following functions**

1) $y = (x^3 - 1)^6$

2) $y = \frac{x^2 - 1}{x^2 + 1}$

Homework

f. **Find $\frac{dy}{dx}$ for each of the following functions**

1) $y = \sqrt{x^5 - 4}$

2) $y = x^{-3}(1 + x^4)^5$

3) $y = (x^2 + 4)^6 (1 - 2x)^7$

4) $y = \sqrt{x}(1 - \sqrt{x})^6$

5) $y = (\sqrt{1 + x^2})^5 \sqrt[3]{x^4 - 1}$

6) $y = \left(\frac{x^2 - 4}{x^2 + 2}\right)^{7/2}$

g. **Find $\frac{d^2y}{dx^2}$ for each of the following functions**

1) $y = (x^{3/2} - 1)^4$

2) $y = \frac{x^2 - 1}{\sqrt{x + 1}}$

Sheet 2 : Trigonometric Functions and their Derivatives

y	$y' = \frac{dy}{dx}$
$y = \sin u$	$y' = \cos u . u'$
$y = \cos u$	$y' = -\sin u . u'$
$y = \tan u$	$y' = \sec^2 u . u'$
$y = \cot u$	$y' = -\operatorname{cosec}^2 u . u'$
$y = \sec u$	$y' = \sec u . \tan u . u'$
$y = \operatorname{cosec} u$	$y' = -\operatorname{cosec} u . \cot u . u'$

Lecture Examples

a. **Find** $\frac{dy}{dx}$ **for each of the following functions**

1) $y = \sin x^3$

2) $y = (1 + \cos^3 x)\cot^2 2x$

3) $y = x^3 \cos x^2 - 2\cot x^{-3}$

4) $y = \frac{x \sin 2x}{1 - \cos^2 3x}$

5) $y = \sec^3 \sqrt{4x^2 + 1}$

6) $y = \frac{\sin(x-1)}{x-1}$

b. **Find** $\frac{d^2 y}{dx^2}$ **for each of the following functions**

1) $y = (1 - \cos^2 x)^{-3/2}$

2) $y = x \sec x$

c. If $y = a \sin cx + b \cos cx$ **,** where a,b and c are constants, prove that

$$\frac{d^2 y}{dx^2} = -c^2 y$$

Classroom Exercises

d. Find $\frac{dy}{dx}$ for each of the following functions

1) $y = \tan^2(\cos^3 x^2)$

2) $y = \sqrt{x^2+1}\cos^3\sqrt{x^2-1}$

3) $y = x^2 \sec^3 x - 4\cot^2 x^3$

4) $y = \frac{1-\sin 2x}{1+\cos 2x}$

5) $y = \sqrt{\tan^2 x + x\cos^3 x}$

6) $y = \sqrt{x-1}\sin\sqrt{x-1}$

e. Find $\frac{d^2y}{dx^2}$ for each of the following functions

1) $y = x^4(\cos 2x)$

2) $y = \frac{\sin x}{x}$

Homework

f. Find $\frac{dy}{dx}$ for each of the following functions

1) $y = \sec^3\sqrt{\cos x}$

2) $y = \cot(\sqrt{x}\tan\sqrt{x})$

3) $y = x^{3/2}\cot x^3$

4) $y = \operatorname{cosec}^4\sqrt{x^2-1}$

5) $y = (1-\sin\sqrt{x})^3\cos\sqrt{x}$

6) $y = \sqrt{x}\operatorname{cosec}\sqrt{x}$

g. Find $\frac{d^2y}{dx^2}$ for each of the following functions

1) $y = x\tan x^3$

2) $y = \sin^3 x$

Sheet 3: Inverse Trigonometric Functions and their Derivatives

y	$y' = \frac{dy}{dx}$
$y = \sin^{-1} u$	$y' = \frac{1}{\sqrt{1-u^2}} .u'$
$y = \cos^{-1} u$	$y' = \frac{-1}{\sqrt{1-u^2}} .u'$
$y = \tan^{-1} u$	$y' = \frac{1}{1+u^2} .u'$
$y = \cot^{-1} u$	$y' = \frac{-1}{1+u^2} .u'$
$y = \sec^{-1} u$	$y' = \frac{1}{u\sqrt{u^2-1}} .u'$
$y = \operatorname{cosec}^{-1} u$	$y' = \frac{-1}{u\sqrt{u^2-1}} .u'$

Lecture Examples

a. Find $\frac{dy}{dx}$ for each of the following functions

1) $y = \cos^{-1}\sqrt{x}$

2) $y = (x^2+4)\operatorname{cosec}^{-1} 2x$

3) $y = x^3(1-\sec^{-1} x)$

4) $y = x^3 \sin^{-1}\sqrt{x} - 2\cot^{-1} x^2$

5) $y = \tan^{-1}\left(\frac{x-1}{x+1}\right)$

6) $y = \frac{1-\sin^{-1} x}{\cos^{-1} x}$

b. If $y = \tan(\cos^{-1} x)$, Prove that $y' = \frac{-(y^2+1)}{\sqrt{1-x^2}}$

Classroom Exercises

c. Find $\frac{dy}{dx}$ **for each of the following functions**

1) $y = \sqrt{x}\,\tan^{-1}\sqrt{x}$

2) $y = \cot^{-1}\left(\frac{\cos 3x}{1+\sin 3x}\right)$

3) $y = x^3 \sec^{-1} x^2$

4) $y = \frac{\cos^{-1} x}{1-\sin^{-1} x}$

5) $y = \sqrt{x^2-1}\,\sin^{-1} x - x\cos^{-1} x$

6) $y = \tan^{-1}(\cos x) + \cot^{-1}(\sin x)$

d. If $y = \cos(2\sin^{-1} x)$, prove that $(1-x^2)(y')^2 = 4(1-y^2)$

Homework

e. Find $\frac{dy}{dx}$ **for each of the following functions**

1) $y = \sin^{-1} x^3$

2) $y = \cot^{-1}(\cos 2x)$

3) $y = \cot^{-1}\left(\frac{x-4}{x+4}\right)$

4) $y = \frac{\tan^{-1} x}{1-\cos^{-1}\sqrt{x}}$

5) $y = x^2 \operatorname{cosec}^{-1}\sqrt{x} - 3x\sin^{-1} x$

6) $y = \sqrt[3]{x}\,\sec^{-1}\left(\frac{x}{4}\right)$

f. Prove that $\frac{d}{dx}\left(\tan^{-1}\left(\frac{x-1}{x+1}\right)\right) = \frac{d}{dx}\left(\tan^{-1}(x)\right)$

Sheet 4 : Logarithmic Functions and their Derivatives

Lecture Examples

a. Find $\frac{dy}{dx}$ **for each of the following functions**

1) $y = x^3 \ln x$

2) $y = \ln(x^{-4}(x^5 - 2)^6)$

3) $y^x = x^y$

4) $y = \ln\left[\frac{x^3(1-x^2)^4}{\sin x\,(x-1)^5}\right]^{7/2}$

5) $y = \sin x^2 - 3x\cos x - x^x$

6) $(\sin x)^{\cos y} = (\sin y)^{\cos x}$

b. If $y = \ln(\sec x + \tan x)$, Prove that $y'' = \sec x \tan x$.

Classroom Exercises

c. Find $\frac{dy}{dx}$ **for each of the following functions**

1) $y = (\ln x)^3$

2) $y = \ln\left(\frac{x^3 - 1}{x^2 + 1}\right)$

3) $y = \sqrt[4]{\frac{(1-x)^3 \tan^{-1} x}{x^x \sec x^3}}$

4) $y^{5/2} = x^{\ln x}$

5) $y = \frac{x^x (2 - \sin x)^{x^2}}{x^{\cos x}(1 - 2\ln x)^5}$

6) $y = x\cos\sqrt{x} - x^{\sec x}$

d. If $y = \cos(\ln x) + \sin(\ln x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Homework

e. Find $\frac{dy}{dx}$ for each of the following functions

1) $y = \ln(1 - \ln x)$

2) $y = \ln\left[\frac{(1-x^2)^5(2-\sin^{-1}x)}{(1-\ln x)^2(3-\cos x)}\right]$

3) $y = \frac{(x-1)^3(1-\sin x)^4}{x^x(2-\cos x)^2}$

4) $\sqrt{y} = \frac{x^5 \tan^{-1} x}{(1+x)\sqrt[3]{x}}$

5) $y = x^{\sin x}$

6) $y = (\ln(\sin x))^{\cos x}$

Sheet 5 : Exponential Functions and their Derivatives

Lecture Examples

a. Find $\dfrac{dy}{dx}$ **for each of the following functions**

1) $y = e^{\sin^{-1}x}$

2) $y = e^{\tan^{-1}\sin x}$

3) $y = \cos^3 e^{x^2}$

4) $y = \cos^{-1}(1 - e^{-x})$

5. $y = \operatorname{cosec}^{-1} e^x - x^4 e^{\cot x}$

6. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

b. If $y = \dfrac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$, show that $\dfrac{dy}{dx} = \dfrac{8}{\left(e^{2x} + e^{-2x}\right)^2}$.

c. If $y = \tan^{-1} \ln e^{\tan\sqrt{x}}$, show that $y\dfrac{dy}{dx} = \dfrac{1}{2}$.

Classroom Exercises

d. Find $\dfrac{dy}{dx}$ **for each of the following functions**

1) $y = x^3 e^{x^5 - 3}$

2) $y = e^{\ln \sin^{-1}(\sin x^3)}$

3) $y = \sqrt{e^{\cos^{-1}x}}$

4) $y = \ln\left[\dfrac{e^{x^2}\sin x^3}{(1 - e^x)(2 - x)}\right]^6$

e. If $y = \ln(\cos x)$, **show that** $y'' + e^{-2y} = 0$

f. Find $\dfrac{d^2y}{dx^2}$ **for each of the following functions**

1) $y = e^{\sin x}$

2) $y = \cos e^{3x}$

Homework

g. Find $\frac{dy}{dx}$ **for each of the following functions**

1) $y = e^{x^3}$

2) $y = e^{\cos^{-1}(\sin x)}$

3) $y = \ln\left[\frac{1+e^{2x}}{(1-e^{-2x})^3}\right]$

4) $y = x^5 \sec e^{-x}$

h. If $y = ae^{-2x} + be^{3x}$ where a and b are constants, Show that

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$$

i. **Find** $\frac{d^2y}{dx^2}$ **for each of the following functions**

1) $y = e^{-4x}$

2) $y = \ln(e^{2x} - 4)$

Sheet 6:Derivatives of hyperbolic and inverse hyperbolic functions

Lecture Examples

a. **Find** $\frac{dy}{dx}$ **for each of the following functions**

1) $y = x^4 \cosh^2 x^3$

2) $y = \tanh(x \ln x)$

3) $y = e^{\cosh^{-1} x^2}$

4) $y = (\sin^{-1} \sqrt{x})(1 - \cosh^{-1} x^2)$

5) $y = \sqrt[5]{x^3} \tanh^{-1} x^2$

6) $y = \ln\left[\frac{(x+1)^2 e^{\operatorname{cosech} x}}{\sqrt{x^3 - 1}}\right]$

b. Show that $\cosh^{-1} x = \ln(x \pm \sqrt{x^2 - 1})$

Classroom Exercises

c. **Find** $\frac{dy}{dx}$ **for each of the following functions**

1) $y = \sinh x^3$

2) $y = \tanh^{-1}(\operatorname{sech} 2x)$

3) $y = \sin\left(\cosh^{-1} \sqrt{x^2 + 1}\right)$

4) $y = \sqrt{\cosh^{-1}(e^{-x/2})}$

d. Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$

e. Solve the following equation $e^{\cosh^{-1} x} = 2$

Homework

f. Find $\frac{dy}{dx}$ for each of the following functions

1. $y = x^2 \coth^3 \sqrt{x}$

2. $y = xe^{\sinh^{-1} x}$

3. $y = (1 - \ln \sec x) \cosh^{-1} \sqrt{x}$

4. $y = \ln \sqrt{\tanh 3x}$

5. $y = \sinh^{-1}(\sin 2x)$

6. $y = \tanh^{-1} \sqrt{\sec x}$

g. Show that $\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$

Sheet 7 : Parametric and Implicit Differentiation

Lecture Examples

a. **Find** $\frac{dy}{dx}$ **for each of the following functions**

1) $y = t\ln t$, $y = \ln t/t$

2) $x = e^t \cosh t$, $y = e^t \sinh t$

b. **Find** $\frac{d^2y}{dx^2}$ **for each of the following functions**

1) $x = \operatorname{cosec} t$, $y = \cos 2t$

2) $x = \sqrt{1-t^2}$, $y = \sin^{-1} t$

c. If $x = \cos\frac{t}{1+t}$, $y = \sin\frac{t}{1+t}$, show that $y^3 y'' + 1 = 0$

d. If $x = \frac{t+1}{t-1}$, $y = \left(\frac{t-1}{t+1}\right)^5$, show that $y'' = 30x^{-7}$

e. **Find** $\frac{dy}{dx}$ **for each of the following functions**

1) $x^3 - 3x^2y^4 + 7y^2 = 10$

2) $x + \cos^{-1} y = xy$

3) $\sin^{-1} x + \tan(xy) = 5$

4) $y = e^{-x} + e^y$

Classroom Exercises

f. **Find** $\frac{dy}{dx}$ **for each of the following functions**

1) $x = \frac{3t}{1+t^3}$, $y = \frac{3t}{1+t^3}$

2) $x = \sqrt{1-\sin\theta}$, $y = \sqrt{1+\cos\theta}$

g. **Find $\frac{d^2y}{dx^2}$ for each of the following functions**

(1) $x = \cos\theta + \theta\sin\theta$, $y = \sin\theta - \theta\cos\theta$

(2) $x = \sqrt{t^4 - 1}$, $y = \sec^{-1} t^2$

h. If $x = \tan\frac{t-1}{1+t}$, $y = \sec\frac{t-1}{1+t}$, show that $\frac{d^2y}{dx^2} = y^{-3}$.

i. **Find $\frac{dy}{dx}$ for each of the following functions**

1) $x^{-2}y^5 - 2xy^2 + 7x = 12$ **2)** $x + y^2 = e^{x/y}$

3) $\ln y = x + e^y$ **4)** $y^2 = \sin^3 2x + \cos^3 2y$

5) $x^{1+y} + y^{1+x} = 1$

Homework

j. If $x = \tan t - t$, $y = \tan^3 t$, Find $\frac{d^2y}{dx^2}$.

k. If $x = t + \frac{1}{t}$, $y = t^2 + \frac{1}{t^2}$, Show that $\frac{d^2y}{dx^2} = 2$.

l. If $x = \frac{t-1}{t+1}$, $y = \frac{t+1}{t-1}$, Show that $\frac{d^2y}{dx^2} = 2y^3$.

m. **Find $\frac{d^2y}{dx^2}$ for each of the following functions**

1) $y^4 - 4x^3y^2 + 6x^2 = 7$ **2)** $\tan^{-1} y = x^2 + y^2$

3) $y = e^{(x+y)^3}$ **4)** $\cosh^{-1} \sec y = xy^3$

Sheet 8 : Partial Differentiation

Lecture Examples

a. Find the first partial derivatives for each of the following

1) $z = (x^2 - y)\sin x^3$ **2)** $z = (\sin 2y)^x$

b. If $z = \tan^{-1}\dfrac{y}{x}$ **show that**

1) $x\dfrac{\partial z}{\partial y} - y\dfrac{\partial z}{\partial x} = 1$ **2)** $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x^2} = 0$

c. If $z = f(x^2 + y^2)$ **show that** $x\dfrac{\partial z}{\partial y} - y\dfrac{\partial z}{\partial x} = 0$

Classroom Exercises

d. Find the first partial derivatives for each of the following

1) $z = y^2(x^4 - 1)^5 + 6y^2x$ **2)** $z = x^2 \sin\sqrt{x} + y\cos(xy)$

3) $z = \tan^{-1}\dfrac{y}{x}$ **4)** $z = e^{x/y}\tanh^{-1}(x^2 + y^2)$

e. If $z = \cot^{-1}\dfrac{y}{x}$ show that $\dfrac{\partial^2 z}{\partial y^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

f. If $z = \tan^{-1}\dfrac{x-1}{y-1}$ show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

Homework

g. Find the first partial derivatives for each of the following

1) $z = x^3 - 3x^2y^4 + y^2$ **2)** $z = (x+y)\sin(x-y)$

3) $z = e^{\frac{y}{x}} \ln\frac{x}{y}$ **4)** $z = (1+\sin y)^{1+\cos x}$

h. If $z = \ln(x^2 + y^2)$ show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

i. If $z = \cot^{-1}\frac{x}{y}$, show that

1) $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = -1$ and 2) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

Sheet 9: Indeterminate Forms and L'Hopital's Rule

Lecture Examples

1) $\lim_{x\to\pi/2} \frac{2\cos x}{2x-\pi}$

2) $\lim_{x\to o} \frac{1-\cosh x}{x^2}$

3) $\lim_{x\to o} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$

4) $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$

5) $\lim_{x\to o} \frac{x\cos x+\tan 2x}{x\sec x+\sin 4x}$

6) $\lim_{\phi\to o}(\operatorname{cosec}\phi-\cot\phi)$

7) $\lim_{x\to\infty} (\frac{x+3}{x-1})^x$

8) $\lim_{x\to o} (\cos x)^{1/x^2}$

Classroom Exercises

9) $\lim_{x\to\pi} \frac{1-\sin(x/2)}{\pi-x}$

10) $\lim_{x\to 1} \frac{\cot(\pi x/2)}{1-\sqrt{x}}$

11) $\lim_{x\to o} \frac{\tan x+\sec x-1}{\tan x-\sec x+1}$

12) $\lim_{x\to 0} \frac{1-\sqrt{\cos x}}{x^2}$

13) $\lim_{x\to\pi/2} (\sec x-\tan x)$

14) $\lim_{x\to o} \frac{\sin 4x-x\cos x}{x\sec x-\tan 3x}$

15) $\lim_{x\to o} (\cos x)^{1/x}$

16) $\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x$

Homework

17) $\lim_{x\to 1} \frac{1-x^2}{\sin\pi x}$

18) $\lim_{x\to 1} \frac{\sin(x^3-1)}{x-1}$

19) $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$

20) $\lim_{x\to 0} \frac{\sin 3x}{1-\cos 4x}$

21) $\lim_{x\to o} \frac{\sinh x}{x}$

22) $\lim_{x\to 0} \frac{\tanh x}{x}$

23) $\lim_{x\to 0}(1+\sin x)^{1/x}$

24) $\lim_{x\to\infty} \left(\frac{x}{x+2}\right)^{x-2}$

Sheet 10 : Maclaurin's Expansion

Maclaurin's Expansion:

$$f(x) = f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} \cdots + f^{(n)}(0)\frac{(x)^n}{n!} + \cdots$$

Lecture Examples

a. Find **Maclaurin's Expansion** of each of the following :

(1) $f(x) = \sin 2x$

(2) $f(x) = \ln(2+3x)$, *Find approximate value to* $\ln(2.3)$.

b. Using Maclaurin's Expansion, show that

(1) $e^{-x}\cos x = 1 - x + \frac{1}{3}x^3 - \frac{1}{6}x^4 + \cdots$

(2) $\frac{\cos x}{\sqrt{1+x}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \cdots$

Classroom Exercises

c. Find Maclaurin's Expansion for each of the following

1) $f(x) = \cos 3x$

2) $f(x) = \frac{1}{\sqrt{1+x}}$

3) $f(x) = e^{-3x}$

d. Using Maclaurin's Expansion show that:

1) $e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} + \cdots$

2) $\frac{e^x}{1-x} = 1 + 2x + \frac{5}{2}x^2 + \frac{8}{3}x^3 + \cdots$

Homework

e. Find Maclaurin's Expansion for each of the following

1) $f(x) = \frac{1}{x+1}$ **2)** $f(x) = \sin 3x$

f. Using Maclaurin's Expansion, show that

(1) $\frac{e^{-x}}{1-x} = 1 + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$

(2) $\sinh x + \cosh x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$

Sheet 11 : Differentiation applications

y'
- + ve → the interval is decreasing
- zero → Critical point
- - ve → the interval is decreasing

Critical points → y'
- + ve → the critical point is local minimum
- - ve → the critical point is local maximum

y''
- + ve → the interval is concave up ∪
- zero → Inflection point
- - ve → the interval is concave down ∩

Graphing Rational Functions

1. Find the domain of the rational function.
2. Find the vertical asymptote(s) of the rational function.
3. Find the horizontal asymptote of the rational function.
4. Determine the symmetry of the rational function.
5. Find the intercepts of the rational function.
6. Graph the rational function.

1. The **domain** is the set of all **input values** to which the rule applies. These are called your **independent variables**. These are the values that correspond to the first components of the ordered pairs it is associated with.

2. **Vertical Asymptote**

Let $f(x) = \dfrac{P(x)}{Q(x)}$ be written in lowest terms and P and Q are polynomial functions.

If $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$, then the vertical line x = a is a vertical asymptote.

The line x = a is a vertical asymptote of the graph of f(x) if and only if the denominator Q(a) = 0 and the numerator P(a) $\neq 0$.

You can have zero or many vertical asymptotes. It will be x = whatever number(s) cause the denominator to be zero after you have simplified the function.

3. **Horizontal Asymptote**

Let $f(x) = \dfrac{P(x)}{Q(x)}$ be written in lowest terms and P and Q are polynomial functions and Q(x) $\neq 0$.

If $f(x) \to a$ as $x \to \infty$ or $x \to -\infty$, then the horizontal line y = a is a horizontal asymptote.

If there is a horizontal asymptote, it will fit into one of the two following cases:

Case I

If the degree of P(x) < the degree of Q(x), then there is a horizontal asymptote at y = 0 (x-axis).

Case II

If the degree of P(x) = the degree of Q(x), then there is a horizontal asymptote at

$$y = \frac{\text{leading coefficient of } P(x)}{\text{leading coefficient of } Q(x)}$$

In other words, it would be the ratio between the leading coefficient of the numerator and the leading coefficient of the denominator.

4. **Determine the symmetry**

The graph is **symmetric about the *y*-axis** if the function is **even**.

The graph is **symmetric about the origin** if the function is **odd**.

5. **Find any intercepts that exist.**

The ***x*-intercept** is where the graph crosses the *x*-axis. You can find this by **setting *y* = 0** and solving for *x*.

The ***y*-intercept** is where the graph crosses the *y*-axis. You can find this by **setting *x* = 0** and solving for *y*.

6. **Draw curves through the points, approaching the asymptotes.**

Note that your graph can cross over a horizontal, but it can NEVER cross over a vertical asymptote.

Solved example

1) $y = f(x) = \dfrac{x-1}{x^2}$

Domain:

Our restriction here is that the denominator of a fraction can never be equal to 0. So to find our domain, we want to set the denominator equal to 0 and restrict those values.

let $x^2 = 0$, then $x = 0$, hence the domain will be $(-\infty, 0) \cup (0, \infty)$ i.e. Our domain is all real numbers except zero

Intercepts :

y-intercept $\Rightarrow$ to find the y-intercept, we let $x = 0$ and solve for y $\Rightarrow$ no y-intercept .

x-intercept $\Rightarrow$ to find the x-intercept, we let $y = 0$ and solve for x $\Rightarrow$ $0 = \dfrac{x-1}{x^2}$ $\Rightarrow$ x-1 = 0 $\Rightarrow$ x =1 (x-intercept) **.**

Symmetry :

note : f(-x) = f(x) $\Rightarrow$ symmetry about the y-axis $\Rightarrow$ i.e. even function
f(-x) = -f(x) $\Rightarrow$ symmetry about the origin $\Rightarrow$ i.e. odd function

$f(x) = \dfrac{x-1}{x^2}$, $f(-x) = \dfrac{-x-1}{x^2}$, So f(-x) $\neq$ f(x) and f(-x) $\neq$ -f(x) $\Rightarrow$ no symmetry .

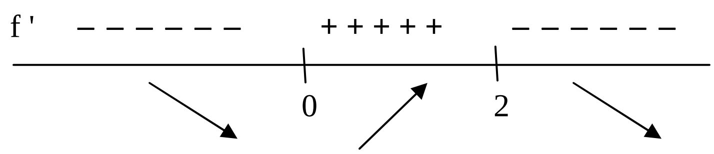
Asymptotes :

1. Vertical Asymptote : $\dfrac{\text{Not } 0}{0}$, $y = \dfrac{x-1}{x^2}$, then $\dfrac{-1}{0}$ $\Rightarrow$ x = 0 is a vertical Asymptote .

2. Horizontal Asymptote: $\lim\limits_{x\to\pm\infty} \dfrac{x-1}{x^2} = 0 \Rightarrow$ y = 0 is a horizontal Asymptote.

Increasing and decreasing intervals :

$$f(x) = \frac{x-1}{x^2} \quad , \quad f'(x) = \frac{2-x}{x^3}$$

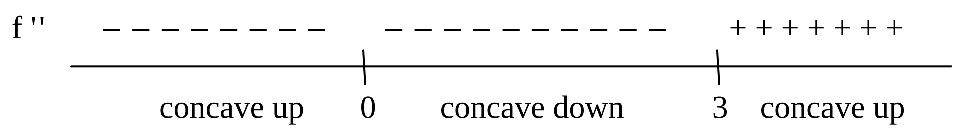


Increasing intervals : (0 , 2) ,   decreasing intervals : ($-\infty$, 0) , (2 , ∞)

Local maximum: at x = 2 **,** **y =** $\frac{1}{4} \Rightarrow$ (2 , $\frac{1}{4}$) is a local maximum

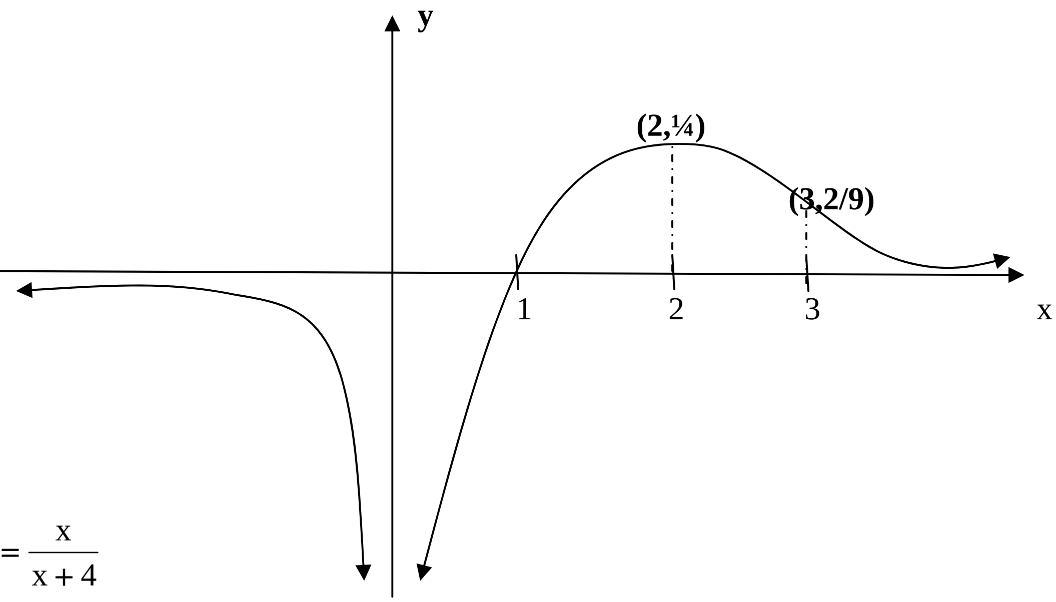
Local minimum: since x = 0 is outside the domain **,** hence no local minimum.

Inflection points : $f'(x) = \dfrac{2-x}{x^3}$, $f''(x) = \dfrac{2(x-3)}{x^4}$



again the function is undefined at x = 0 , hence the inflection point is (3 , $\frac{2}{9}$)

Graph:



2) $y = f(x) = \dfrac{x}{x+4}$

Domain: let $x+4=0$, then $x=-4$, hence the domain will be $(-\infty,-4)\cup(-4,\infty)$

y-intercept : $y=\dfrac{0}{0+4}=0$

x-intercept : $0=\dfrac{x}{x+4} \Rightarrow x=0$

Therefore the function crosses the x-axis and y-axis at the origin (0,0)

Symmetry :

$f(-x)=\dfrac{-x}{-x+4}$. So $f(-x)\neq f(x)$ and $f(-x)\neq -f(x) \Rightarrow$ no symmetry .

Asymptotes :

1. Vertical Asymptote : $f(-4)=\dfrac{-4}{0} \Rightarrow x=-4$ is a vertical Asymptote.
2. Horizontal Asymptote: $\lim\limits_{x\to\pm\infty}\dfrac{x}{x+4}=\dfrac{\text{leading coeff of } x}{\text{leading coeff of } (x+4)}=\dfrac{1}{1}=1$,

   or

$$\lim_{x\to\pm\infty}\frac{x}{x+4}=\frac{\frac{x}{x}}{\frac{x}{x}+\frac{4}{x}}=\lim_{x\to\pm\infty}\frac{1}{1+\frac{4}{x}}=1$$

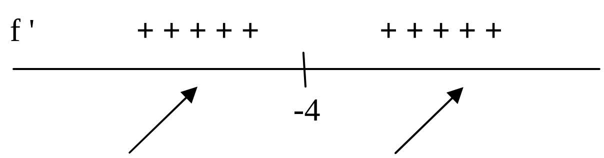
. Hence, y = 1 is a horizontal Asymptote.

Increasing and decreasing intervals :

$f(x)=\dfrac{x}{x+4}$, $f'(x)=\dfrac{4}{(x+4)^2}$, for $f'(x)=0 \Rightarrow f'(x)$ is undefined

i.e. no solution and no critical points. Hence no local max. and local min exist.

Then let $(x+4)^2=0 \Rightarrow x=-4$.



Inflection points : $f'(x)=\dfrac{4}{(x+4)^2}$, $f''(x)=\dfrac{-8}{(x+4)^3}$. Again for $f''(x)=0 \Rightarrow f''(x)$ is undefined . Then let $(x+4)^3=0 \Rightarrow x=-4$.

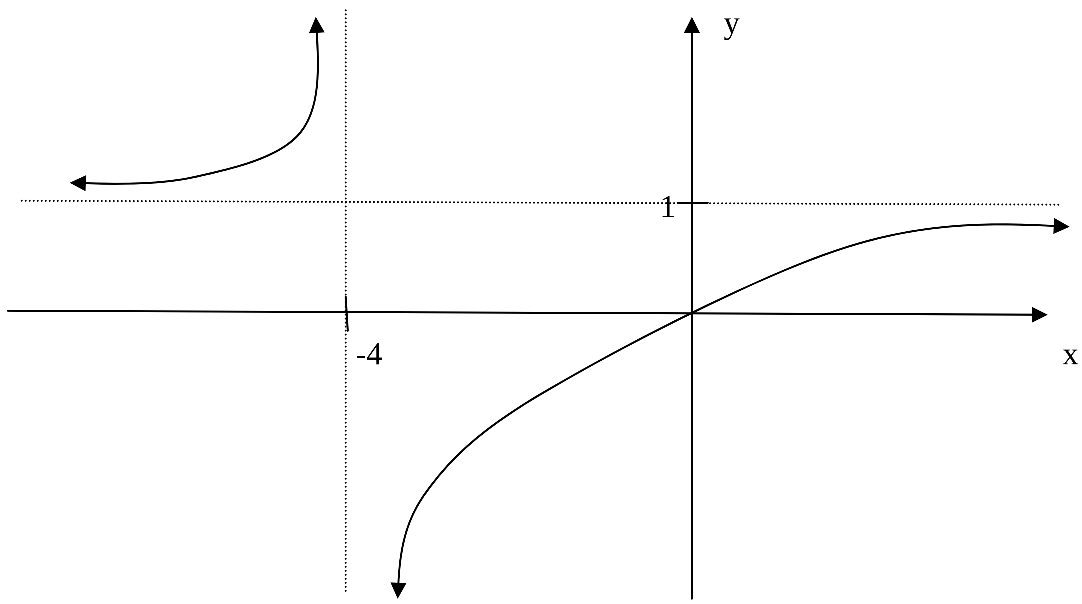
f'' + + + + + + + − − − − − − − − − − −

concave up -4 concave down

In order to check whether the curve crosses the horizontal Asymptote $y = 1$,

let $\frac{x}{x+4} = 1 \Rightarrow x = x + 4 \Rightarrow$ no solution $\Rightarrow$ never crosses line $y = 1$.

Graph:



The equation of motion of any particle is given by:

- The displacement of the particle:

$$s = s(t)$$

- then, its velocity is given by:

$$v = \frac{ds}{dt}$$

- and its acceleration is given by:

$$a = \frac{dv}{dt} = \frac{d^2 s}{d t^2}$$

Lecture Examples

i) In each of the following curves,

1) $y = x^2 - 4x + 3$ 2) $y = -2x^2 + 12x + 7$

find

a. The critical point.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. Sketch the curve.

ii) In each of the following curves,

1) $y = x^3 - 6x^2 + 10$ 2) $y = \left(x^2 - 9\right)^2$

find

a. The critical points.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. The inflection point.
e. The concavity of the curve.
f. Sketch the curve

iii) In each of the following curves,

1) $y = \frac{5x}{x^2 + 1}$ 2) $y = \frac{x}{x - 2}$

1. Find the domain of the rational function.
2. Find the vertical asymptote(s) of the rational function.
3. Find the horizontal asymptote of the rational function.
4. Determine the symmetry of the rational function.
5. Find the intercepts of the rational function.
6. The critical points.
7. The intervals in which the curve is increasing and decreasing.
8. The local maximum and minimum points.
9. The inflection point.
10. The concavity of the curve.
11. Graph the rational function.

iv) Find the velocity and the acceleration for each of the following

1) $s(t) = t^3 - 6t^2 + 7$ 2) $s(t) = t^3 e^{t^4 - 1}$

Classroom Exercises

i) In each of the following curves,

1) $y = 2x^2 - 8x + 10$ 2) $y = -3x^2 - 12x$

find

a. The critical point.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. Sketch the curve.

ii) In each of the following curves,

1) $y = 6x^2 - x^3$ 2) $y = x^3 - 3x^2 - 9x$

find

g. The critical points.
h. The intervals in which the curve is increasing and decreasing.
i. The local maximum and minimum points.
j. The inflection point.
k. The concavity of the curve.
l. Sketch the curve.

iii) In each of the following curves,

1) $y = \dfrac{7x}{x^2 + 3}$ 2) $y = \dfrac{x + 1}{x - 3}$

1. Find the domain of the rational function.
2. Find the vertical asymptote(s) of the rational function.
3. Find the horizontal asymptote of the rational function.
4. Determine the symmetry of the rational function.
5. Find the intercepts of the rational function.
6. The critical points.
7. The intervals in which the curve is increasing and decreasing.
8. The local maximum and minimum points.
9. The inflection point.
10. The concavity of the curve.
11. Graph the rational function.

iv) Find the velocity and the acceleration for each of the following

1) $s(t) = \sin 5t - \cos 5t$ 2) $s(t) = t^6 (1 - \ln t)^4$

Homework

i) In each of the following curves,

1) $y = 12x - 3x^2$ 2) $y = 3x^2 - 6x$

find

a. The critical point.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. Sketch the curve.

ii) In each of the following curves,

1) $y = 2x^3 - 12x^2 + 18x$ 2) $y = x^3 - 9x^2 + 8$

find

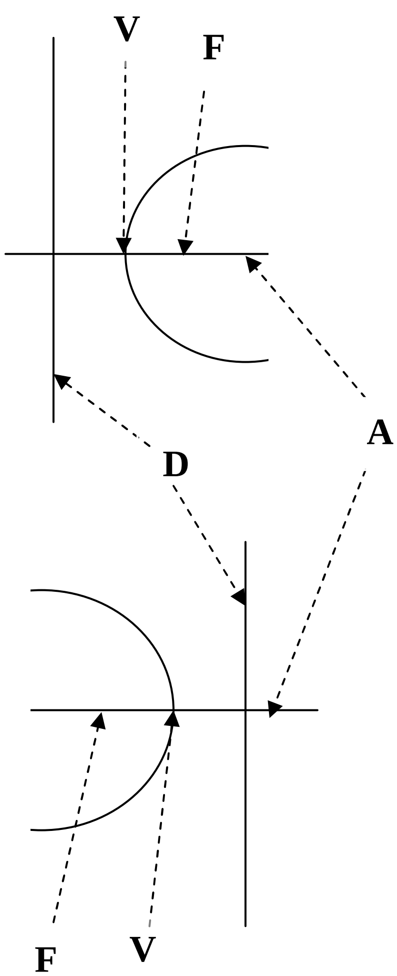
a. The critical points.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. The inflection point.
e. The concavity of the curve.
f. Sketch the curve.

iii) Find the velocity and the acceleration for each of the following

1) $s(t) = t\sinh 3t + \cosh 3t$ 2) $s(t) = \dfrac{1 - e^{2t}}{e^{2t} - e^{2t}}$

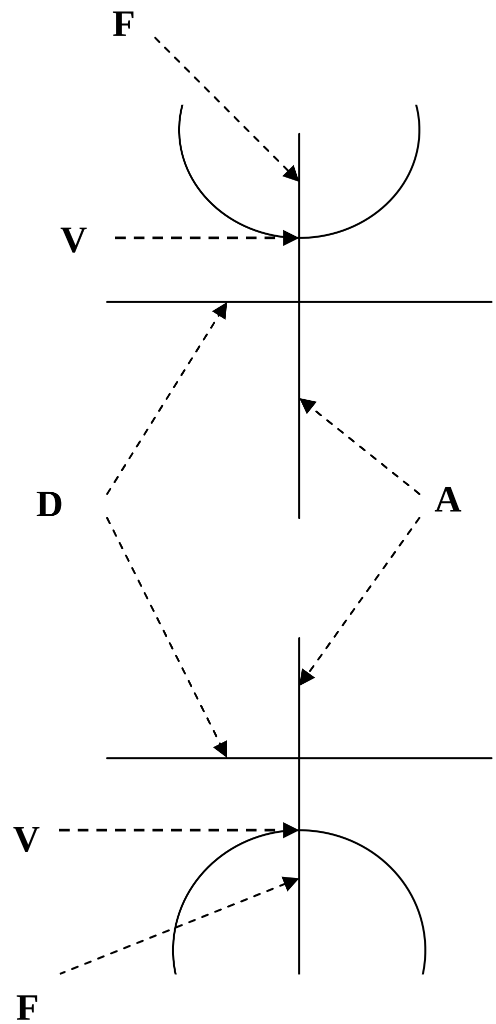
Sheet 12 : Conic Sections

The Parabola



$(y - y_0)^2 = 4c(x - x_0)$

vertex	(x_0, y_0)
focus	$(x_0 + c, y_0)$
axis	$y = y_0$
directrix	$x = x_0 - c$



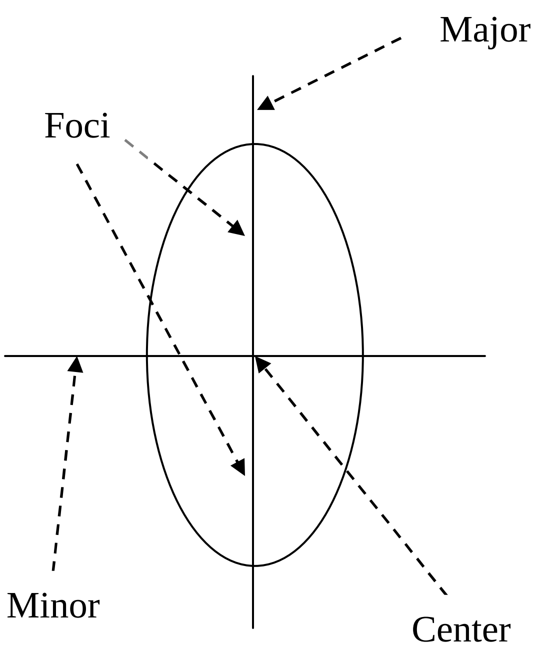
$(x - x_0)^2 = 4c(y - y_0)$

vertex	(x_0, y_0)
focus	$(x_0, y_0 + c)$
axis	$x = x_0$
directrix	$y = y_0 - c$

The Ellipse

General form $\frac{(x-x_0)^2}{b^2}+\frac{(y-y_0)^2}{a^2}=1$

$a > b$



Center (x_0, y_0) **,** $c^2 = a^2 - b^2$**,**

major axis = 2a, minor axis = 2b ,

Vertices $(x_0, y_0 \pm a)$

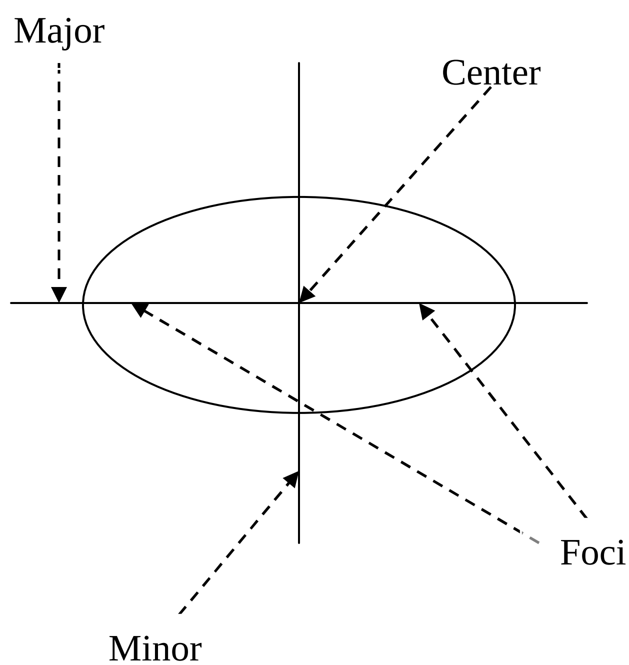
Convertices $(\pm b + x_0, y_0)$

Foci $(x_0, y_0 \pm c)$

Directrix $y = \pm a^2/c$

General form $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$

$a > b$



Center (x_0, y_0) **,** $c^2 = a^2 - b^2$

major axis = 2a , minor axis = 2b ,

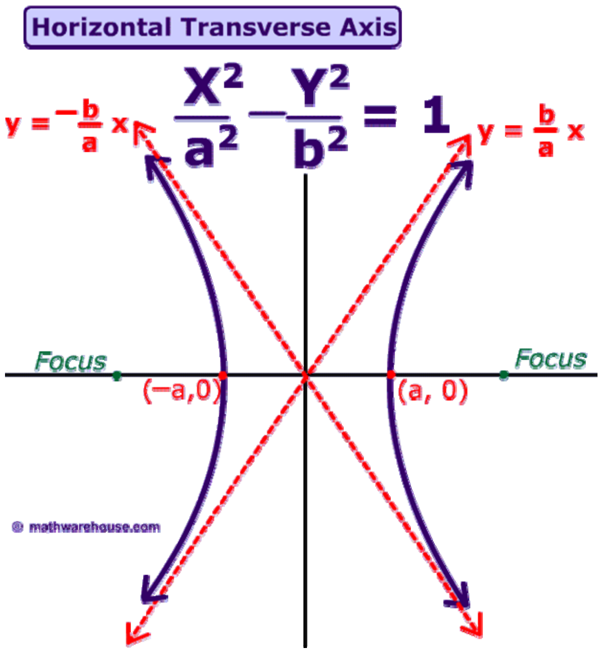
Vertices $(\pm a + x_0, y_0)$

Convertices $(x_0, \pm b + y_0)$

Foci $(\pm c + x_0, y_0)$

Directrix $x = \pm a^2/c$

The Hyperbola



General form $\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$

Center (x_0, y_0) , $c^2 = a^2 + b^2$,

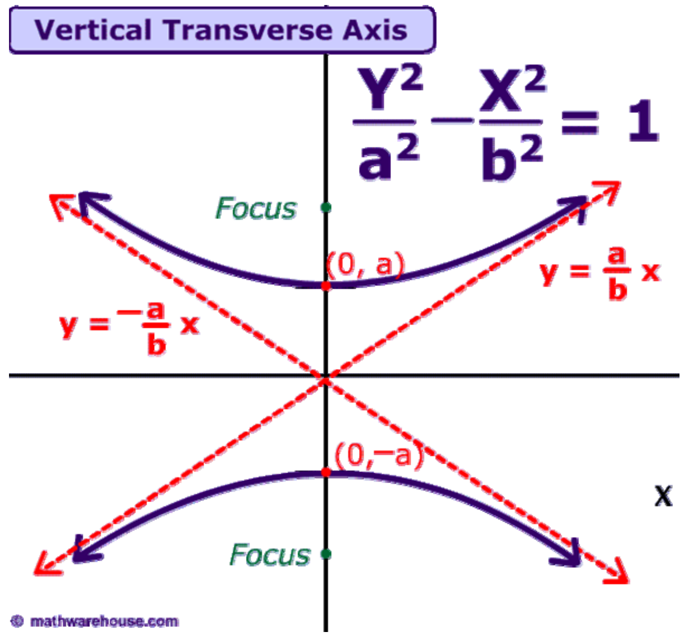
Transverse axis = 2a,
Conjugate axis = 2b ,

Vertices $(\pm a + x_0, y_0)$

Convertices $(x_0, \pm b + y_0)$

Foci $(\pm c + x_0, y_0)$

Directrix $y = \pm a^2/c$



General form $\frac{(y-y_0)^2}{a^2} - \frac{(x-x_0)^2}{b^2} = 1$

Center (x_0, y_0) , $c^2 = a^2 + b^2$

Transverse axis = 2a ,
Conjugate axis = 2b ,

Vertices $(x_0, y_0 \pm a)$

Convertices $(\pm b + x_0, y_0)$

Foci $(x_0, y_0 \pm c)$

Directrix $x = \pm a^2/c$

Lecture Examples

a. Discuss and sketch the following curves:

1) $x^2 + 2x - 4y - 3 = 0$

2) $2y^2 - 4x - 4y - 14 = 0$

3) $4x^2 + 9y^2 + 24x = 0$

4) $2x^2 + 9y^2 + 8x - 72y + 134 = 0$

5) $9x^2 - 16y^2 - 36x - 32y = 124$

6) $16x^2 - 64x - 4y^2 - 8y - 4 = 0$

Classroom Exercises

b. Discuss and sketch the following curves:

1) $x^2 + 10x + 4y + 13 = 0$

2) $y^2 - 4x - 4y + 12 = 0$

3) $x^2 + 4y^2 - 2x - 3 = 0$

4) $25x^2 + 16y^2 + 100x - 32y - 284 = 0$

5) $9x^2 - 4y^2 - 72x + 8y + 176 = 0$

6) $-3x^2 + 12x + 2y^2 - 4y = -8$

Homework

c. Discuss and sketch the following curves:

1) $x^2 - 16y - 6x + 9 = 0$

2) $y^2 + 6x + 8x - 15 = 0$

3) $16x^2 + 4y^2 - 64x + 8y + 4 = 0$

4) $9x^2 + 16y^2 - 36x + 32y = 92$