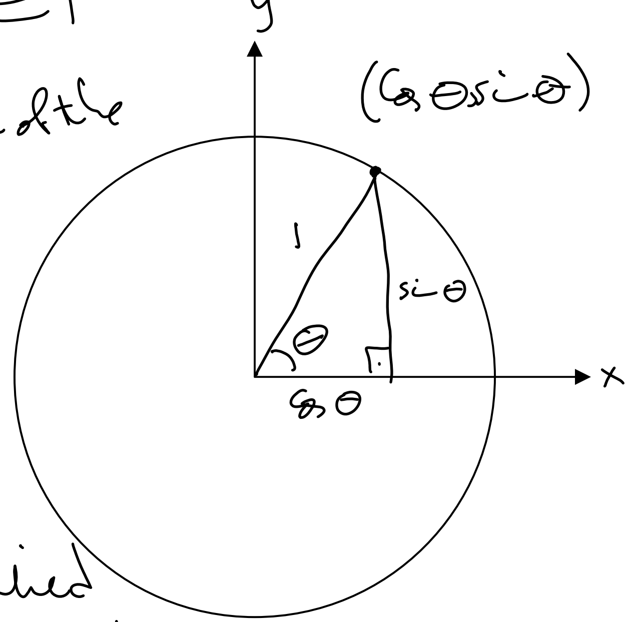
Lec. No. (2)

Trigonometric functions and their derivatives

Define the sine and cosine functions:-

Draw a unit circle $x^2+y^2=1$

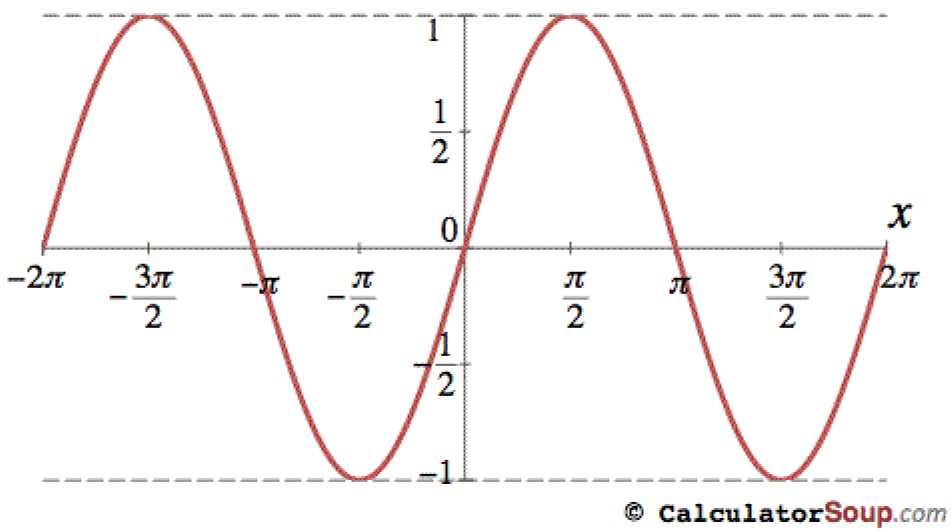


* $\sin\theta$ is the y-coordinate of the point on the circle

* $\cos\theta$ is the x-coordinate of the point

So, $\sin\theta$ and $\cos\theta$ are defined for all values of θ and each has domain $-\infty < \theta < \infty$ with range for each is the interval $[-1, 1]$.

sin x



1
1/2
0
x
−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π
−1/2
−1
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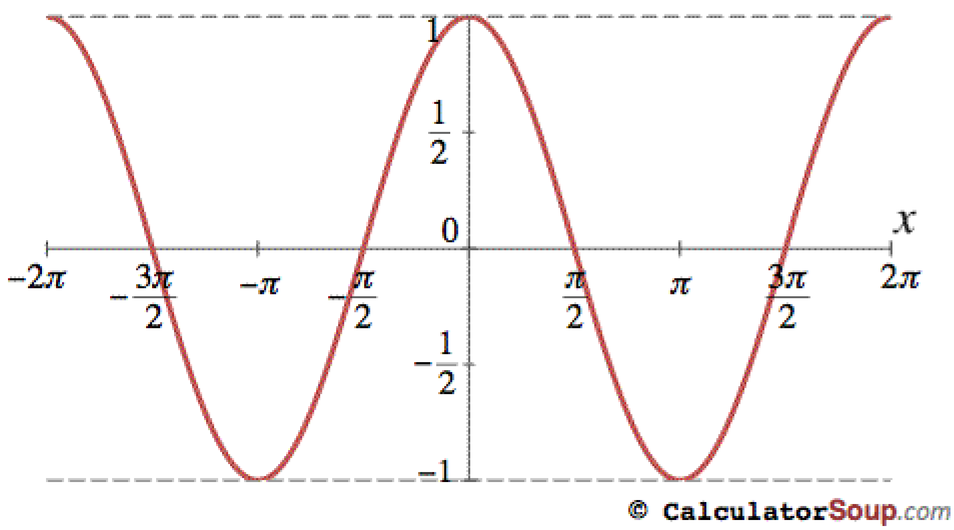
cos x



1
1/2
0
x
−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π
−1/2
−1
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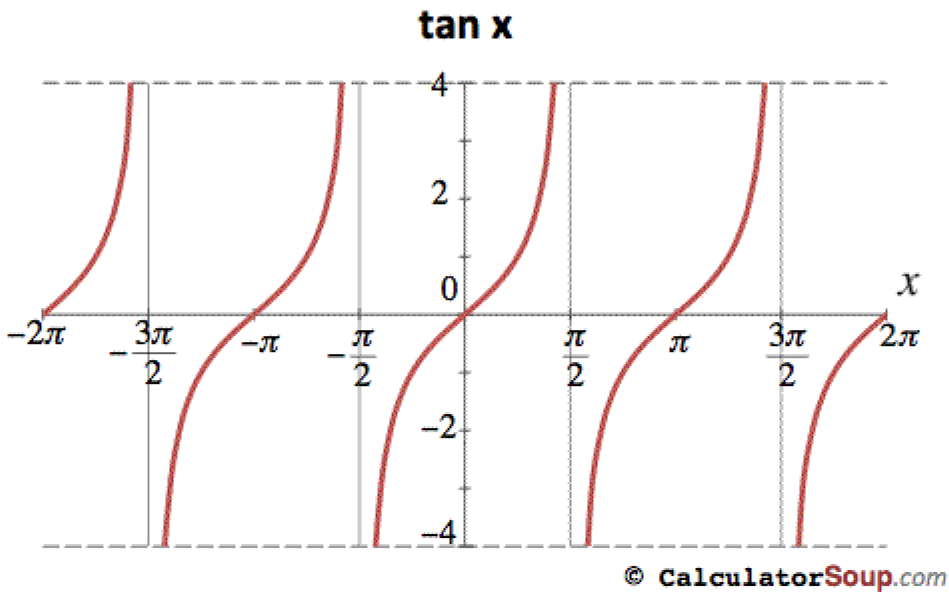
tan x
4
2
0
−2
−4
x
−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π
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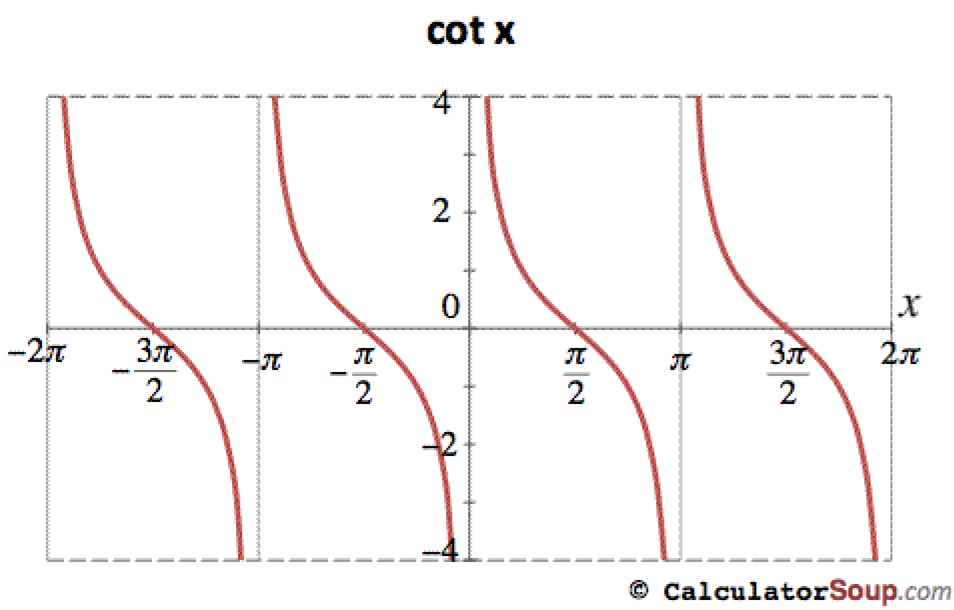
cot x
4
2
0
−2
−4
x
−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π
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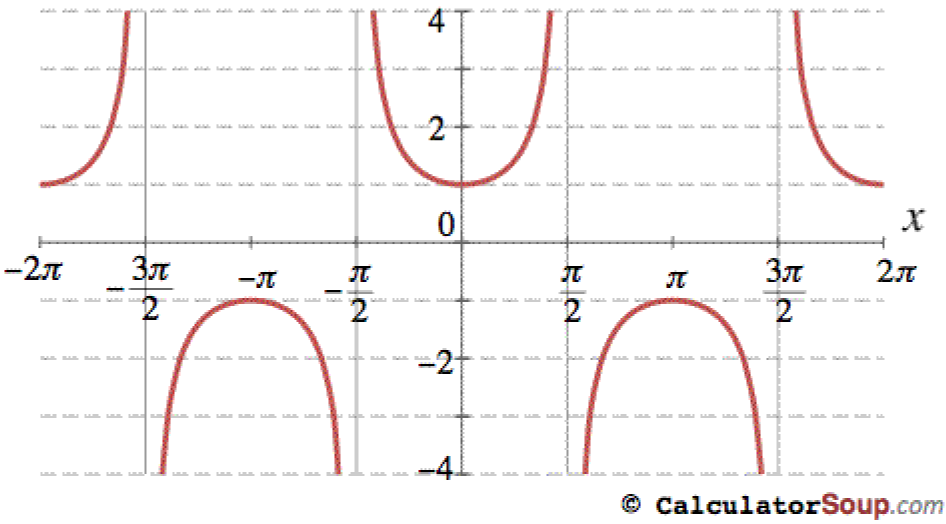
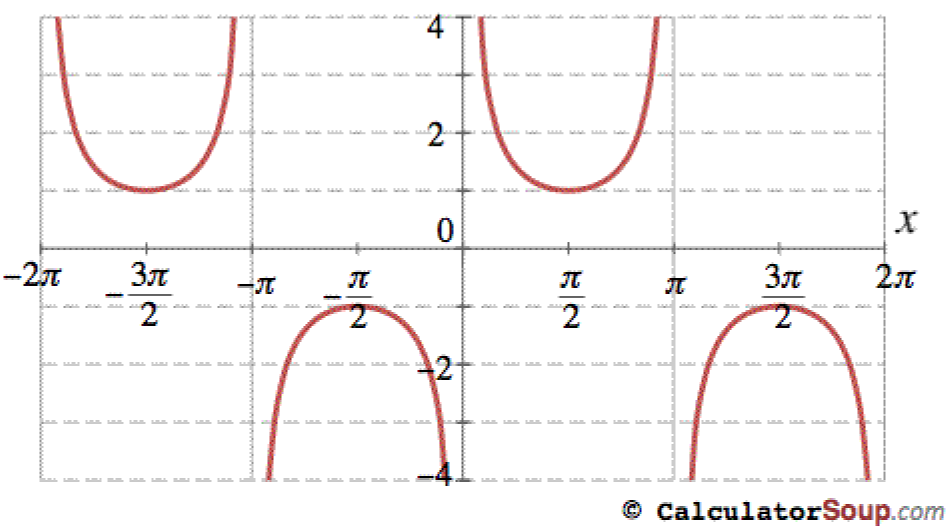
sec x


4
2
0
−2
−4
x
−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π
© CalculatorSoup.com

csc x


4
2
0
−2
−4
x
−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π
© CalculatorSoup.com

Rules:-

1) $\sin^2\theta + \cos^2\theta = 1$

$\div \cos^2\theta$

2) $1 + \tan^2\theta = \sec^2\theta$

$\div \sin^2\theta$

3) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

4) $\sin 2\theta = 2\sin\theta\cos\theta$

5) $\cos^2\theta - \sin^2\theta = \cos 2\theta$

6) $\cos(-\theta) = \cos\theta \Longrightarrow$ since $\cos\theta$ is an even fn.

7) $\sin(-\theta) = -\sin\theta \Longrightarrow$ since $\sin\theta$,, odd ,,

8) $\cos^2\theta = \frac{1}{2}\left[1 + \cos 2\theta\right]$

9) $\sin^2\theta = \frac{1}{2}\left[1 - \cos 2\theta\right]$

Trigonometric Functions and their Derivatives

y	$y' = \dfrac{dy}{dx}$
$y = \sin u$	$y' = \cos u . u'$
$y = \cos u$	$y' = -\sin u . u'$
$y = \tan u$	$y' = \sec^2 u . u'$
$y = \cot u$	$y' = -\operatorname{cosec}^2 u . u'$
$y = \sec u$	$y' = \sec u . \tan u . u'$
$y = \operatorname{cosec} u$	$y' = -\operatorname{cosec} u . \cot u . u'$

Ex. $y = \sin 3x \Rightarrow y' = \cos 3x . 3 = 3\cos 3x$

Ex. $y = \cos x^2 \Rightarrow y' = -\sin x^2 . 2x$

Ex. $y = \tan\sqrt{x} \Rightarrow y' = \sec^2\sqrt{x} . \frac{1}{2\sqrt{x}}$

Ex. $y = \sec 3x \Rightarrow y' = \sec 3x . \tan 3x . 3$

Ex. $y = \operatorname{cosec} x^3 \Rightarrow y' = -\operatorname{cosec} x^3 . \cot x^3 . 3x^2$

Ex. $y = \cot\sqrt{x} \Rightarrow y' = -\operatorname{cosec}^2\sqrt{x} . \frac{1}{2\sqrt{x}}$

Ex. $y = \sin(\cos\sqrt{x})$

$$y' = \cos(\cos\sqrt{x}) \cdot -\sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

sheet No. 2

ⓐ ① $y = \sin x^3 \Rightarrow y' = \cos x^3 \cdot 3x^2$

ⓑ $y = \dfrac{\sin(x-1)}{(x-1)}$

$$y' = \frac{(x-1)\cos(x-1)\cdot 1 - \sin(x-1)\cdot(1)}{(x-1)^2}$$

Note

$$(\sin x)^3 = \begin{cases} \sin x^3 & \times\times \\ \sin^3 x & \checkmark \\ \sin^3 x^3 & \times\times \end{cases}$$

Ex. $y = \sin^3 x \Rightarrow y' = 3\sin^2 x \cdot \cos x$

Say : up - down - left - Right

sheet 2

(a) (2) $y = (1+\cos^3 x)\cot^2 2x$

$$y' = (1+\cos^3 x)\cdot[2\cot 2x \cdot -\csc^2 2x \cdot 2]$$

$$+$$

$$\cot^2 2x \cdot [3\cos^2 x \cdot -\sin x]$$

(3) $y = x^3 \cos x^2 - 2\cot x^{-3}$

$$y' = x^3 \cdot (-\sin x^2)(2x) + \cos x^2 \cdot 3x^2$$

$$- 2\cdot(-\csc^2 x^{-3} \cdot (-3x^{-4}))$$

(4) $y = \dfrac{x\sin 2x}{1-\cos^2 3x}$

$$y = \frac{x\sin 2x}{\sin^2 3x}$$

$$y' = \frac{\sin^2 3x \cdot [x\cdot \cos 2x \cdot 2 + \sin 2x \cdot 1] - (x\sin 2x)[2\sin 3x \cdot \cos 3x \cdot 3]}{\sin^{\cancel{8}\,4} 3x}$$

$\boxed{5}$ $y = \sec^3 \sqrt{4x^2+1}$

$y' = 3\sec^2(\sqrt{4x^2+1}) \cdot \sec\sqrt{4x^2+1} \cdot \tan\sqrt{4x^2+1} \cdot \frac{1}{2\sqrt{4x^2+1}} \cdot 8x$

$\boxed{6}$ (1) $y = (1-\cos^2 x)^{-\frac{3}{2}}$

$y = (\sin^2 x)^{-\frac{3}{2}} = (\sin x)^{-3}$

$y' = -3(\sin x)^{-4} \cdot \cos x$

$y'' = -3(\sin x)^{-4} \cdot (-\sin x) + \cos x \cdot [12(\sin x)^{-5}] \cos x$

(2) $y = x\sec x$

$y' = x \cdot \sec x \tan x + \sec x \cdot 1$

$y'' = x \cdot [\sec x \cdot \sec^2 x + \tan x \cdot \sec x \tan x] + \sec x \tan x \cdot 1$

$+ \sec x \tan x$

$\boxed{C}$ $y = a \sin cx + b \cos cx$

show $y'' = -c^2 y$

$y' = ac \cos cx - bc \sin cx$

$y'' = -ac^2 \sin cx - bc^2 \cos cx$

$= -c^2 \underbrace{[a \sin cx + b \cos cx]}_{y}$

$\therefore y'' = -c^2 y$ O.K

Text book page 173 Ex. 2.6

No. 1 $f = 4 \sin 3x - x \Rightarrow f' = 12 \cos 3x - 1$

No. 3 $f = \tan^3 2t - \csc^4 3t$

$f' = 3 \tan^2 2t \cdot \sec^2 2t \cdot 2 - 4 \csc^3 3t \cdot (-\csc 3t \cot 3t) \cdot 3$

No. 5 $f = x \cos 5x^2 \Rightarrow f' = x \cdot -\sin 5x^2 \cdot 10x + \cos 5x^2 \cdot 1$

No. 7 $f = \frac{\sin x^2}{x^2}$

$$f' = \frac{x^2 \cdot \cos x^2 \cdot 2x - \sin x^2 \cdot 2x}{x^4}$$

No. 9

$$f = \sin 3t \sec 3t$$

$$f = \frac{\sin 3t}{\cos 3t} = \tan 3t$$

$$f' = \sec^2 3t \cdot 3$$

No. 11 $f = \frac{1}{\sin 4w}$

$$f = \csc 4w \Rightarrow f' = -\csc 4w \cot 4w \cdot 4$$

No. 13 $f = 2\sin 2x \cos 2x$

$$f = \sin 4x \Rightarrow f' = 4\cos 4x$$

No. 15

$$f = \tan\sqrt{x^2+1}$$

$$f' = \sec^2\sqrt{x^2+1} \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x$$

No. 17

$$f = \sin^3\left(\cos\sqrt{x^3+2x^2}\right)$$

$$f' = 3\sin^2\left(\cos\sqrt{x^3+2x^2}\right) \cdot \cos\left(\cos\sqrt{x^3+2x^2}\right) \cdot -\sin\sqrt{x^3+2x^2} \cdot \frac{1}{2\sqrt{x^3+2x^2}} \cdot (3x^2+4x)$$