



# COLLEGE OF ENGINEERING & TECHNOLOGY

Department : Electrical & computer Control Engineering

Lecturer : Prof. Dr. Medhat El Singaby

Course : Discrete-Time control systems

Course Code : EE 511

Marks : 40

Date : 20/1/2015

Time : 2 hours

## Final Exam

Answer the following questions:

### Question no.1

A-1

a) Find  $e(0)$  and  $e(10)$  for

$$E(z) = \frac{1}{(z-1)(z-0.3)}$$

By using the partial fraction technique, check the value of  $e(0)$  by the initial value theorem.

b) Solve the given difference equation for  $y(K)$  using

- The sequential technique till  $K = 4$
- The  $z$  - transform.

Will the final value theorem give the correct values of  $y(K)$  as  $K \rightarrow \infty$  ?

$$Y(K+2) - (3/4) Y(K+1) + (1/8) Y(K) = e(K)$$

Where;  $e(K) = 1$  for  $K = 0, 1, 2, \dots$  (unit step)

$$Y(0) = y(1) = 0$$

(10 marks)

### Question no.2

B-2

a) For the unity feedback discrete-data system shown in fig.1 ;

Determine the steady-state error when the system is subjected to

- A unit step input
- A unit ramp input

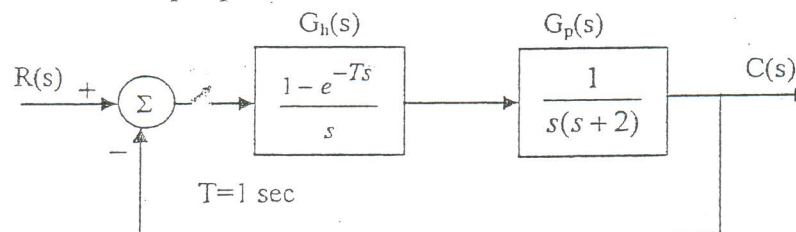


Fig.1

b) Determine, using Routh criteria the stability of the discrete-time system given by its characteristic equation:

$$z^3 + 5z^2 + 3z + 2 = 0$$

(10 marks)

Members of Course Examination Committee:	Signature:	Date:
Lecturer: <i>Medhat El Singaby</i>	<i>Medhat El Singaby</i>	<i>5/1/2015</i>
Course Coordinator: <i>Ahmed El Shewy</i>	<i>Ahmed El Shewy</i>	<i>5/1/2015</i>
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APPENDIX A. TABLE OF LAPLACE TRANSFORMS, z-TRANSFORMS AND MODIFIED z-TRANSFORMS

722 Appendix A Table of Laplace Transforms, z-Transforms and Modified z-Transforms

Laplace Transform F(s)	Time Function f(t) t > 0	z-Transform F(z)	Modified z-Transform F(z,m)
1	$\delta(t)$	1	0
$e^{-kTs}$	$\delta(t - kT)$	$z^{-k}$	$z^{-k-1+m}$
$\frac{1}{s}$	$u_s(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{-1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1} + \frac{T}{(z-1)^2}$
$\frac{2}{s^3}$	$t^2$	$\frac{T^2 z(z+1)}{(z-1)^3}$	$T^2 \frac{m^2 z^2 + (2m - 2m^2 + 1)z + (m-1)^2}{(z-1)^3}$
$\frac{(k-1)!}{s^k}$	$t^{k-1}$	$\lim_{a \rightarrow 0} (-1)^{k-1} \frac{\partial^{k-1}}{\partial a^{k-1}} \left[ \frac{z}{z - e^{-aT}} \right]$	$\lim_{a \rightarrow 0} (-1)^{k-1} \frac{\partial^{k-1}}{\partial a^{k-1}} \left[ \frac{e^{-amT}}{z - e^{-aT}} \right]$
$\frac{1}{s+a}$	$e^{-at}$	$\frac{z}{z - e^{-aT}}$	$\frac{e^{-amT}}{z - e^{-aT}}$
$\frac{1}{(s+a)^2}$	$te^{-at}$	$\frac{Tze^{-aT}}{(z - e^{-aT})^2}$	$\frac{Te^{-amT} [e^{-aT} + m(z - e^{-aT})]}{(z - e^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k e^{-at}$	$(-1)^k \frac{\partial^k}{\partial a^k} \frac{z}{z - e^{-aT}}$	$(-1)^k \frac{\partial^k}{\partial a^k} \left[ \frac{e^{-amT}}{z - e^{-aT}} \right]$
$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$\frac{z(1 - e^{-aT})}{(z-1)(z - e^{-aT})}$	$\frac{(1 - e^{-amT})z + (e^{-amT} - e^{-aT})}{(z-1)(z - e^{-aT})}$

Appendix A Table of Laplace Transforms, z-Transforms and Modified z-Transforms 723

Laplace Transform F(s)	Time Function f(t) t > 0	z-Transform F(z)	Modified z-Transform F(z,m)
$\frac{1}{(s+a)(s+b)}$	$\frac{1}{(b-a)} (e^{-at} - e^{-bt})$	$\frac{1}{(b-a)} \left[ \frac{z}{z - e^{-aT}} - \frac{z}{z - e^{-bT}} \right]$	$\frac{1}{(b-a)} \left[ \frac{e^{-smT}}{z - e^{-aT}} - \frac{e^{-bmT}}{z - e^{-bT}} \right]$
$\frac{a}{s^2(s+a)}$	$t - \frac{1}{a}(1 - e^{-at})$	$\frac{Tz}{(z-1)^2} - \frac{(1 - e^{-aT})z}{a(z-1)(z - e^{-aT})}$	$\frac{T}{(z-1)^2} + \frac{amT - 1}{a(z-1)} + \frac{e^{-amT}}{a(z - e^{-aT})}$
$\frac{1}{(s+a)^2}$	$te^{-at}$	$\frac{Tze^{-aT}}{(z - e^{-aT})^2}$	$\frac{Te^{-amT} [e^{-aT} + m(z - e^{-aT})]}{(z - e^{-aT})^2}$
$\frac{a}{s^2(s+a)}$	$\frac{1}{2} (t^2 - \frac{2}{a}t + \frac{2}{a^2}u_s(t)) - \frac{2}{a^2}e^{-at}$	$\frac{T^2 z}{(z-1)^3} + \frac{(aT-2)Tz}{2a(z-1)^2} + \frac{z}{a^2(z-1)} - \frac{z}{a^2(z - e^{-aT})}$	$\frac{T^2}{(z-1)^3} + \frac{T^2(m+1/2)a - T}{a(z-1)^2} + \frac{(amT)^2/2 - amT + 1}{a^2(z-1)} - \frac{e^{-amT}}{a^2(z - e^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$u_s(t) - (1+at)e^{-at}$	$\frac{z}{z-1} - \frac{z}{z - e^{-aT}} - \frac{aTe^{-aT}z}{(z - e^{-aT})^2}$	$\frac{1}{z-1} - \left[ \frac{1 + amT}{z - e^{-aT}} + \frac{aTe^{-aT}}{(z - e^{-aT})^2} \right] e^{-amT}$
$\frac{a^2}{s^2(s+a)^2}$	$t - \frac{2}{a}u_s(t) + (t + \frac{2}{a})e^{-at}$	$\frac{1}{a} \left[ \frac{(aT+2)z - 2z^2}{(z-1)^2} + \frac{2z}{z - e^{-aT}} + \frac{aTe^{-aT}z}{(z - e^{-aT})^2} \right]$	$\frac{1}{a} \left\{ \frac{aT}{(z-1)^2} + \frac{amT - 2}{z-1} + \left[ \frac{aTe^{-aT}}{(z - e^{-aT})^2} - \frac{amT - 2}{z - e^{-aT}} \right] e^{-amT} \right\}$
$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	$z \sin \omega T$	$\frac{\sin m\omega T + \sin(1-m)\omega T}{z^2 - 2z \cos \omega T + 1}$