A symmetry-based efficient computation of 2D image moments

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***Abstract* — Moments of digital images are widely used to extract invariant image features. Efficient computation of geometric moments is extremely important where all kinds of image moments are expressed as a combination of these geometric moments. In this paper, a new symmetry-based efficient method is proposed for efficient computation of geometric moments. Three types of symmetry are applied to reduce the computational complexity of 2D geometric moments by 87%. A comparison with other existing methods is performed where the numerical experiments ensure the efficiency of the proposed method.**

***Index Terms* — Geometric moments, symmetry property, gray level images, fast computation, orthogonal moments.**

I. INTRODUCTION

Moments and moment invariants of digital images are widely used in image processing, pattern recognition and computer vision. Aircraft identification, scene matching, shape analysis, image normalization, character recognition, accurate position detection, color texture recognition, image retrieval and various other image processing tasks are examples for the implementation of geometric moments. The recent excellent book of Flusser and his coauthors [1] gives an overview of the subject.

Computational process of geometric moments and moment invariants encounter two challenging problems. The first problem is the accuracy, where approximate computation of geometric moments results in a set of inaccurate moments. The degraded accuracy negatively affects the performance of the geometric moment invariants. The second problem is concerned with highly computational requirements especially for large images.

The direct method depends on using zeros-order approximation. This method is time consuming and produces a significant error. Several methods are proposed to overcome these challenging problems.

Spiliotis and Mertzios [2] proposed a novel method which employs binary image representation by non-overlapping rectangular homogeneous blocks, and then the image moments are calculated as the sum of the moments of all blocks. Flusser [3] refined this method. Sossa et. al. [4] proposed a new algorithm based on a morphologic decomposition of the binary image into a set of closed disks. Kuo. et. al. [5], extended the algorithm of Spiliotis and Mertzios to approximately compute the lower order moments for a gray level image using the block representation. Recently, Papakostas et. al [6] extended the method of Spiliotis and Mertzios to compute geometric moments for gray level images.

Liao and Pawlak [7] used another approach. They proposed a formula for computing the 2D geometric moments of a digital image. They numerically integrate the monomial functions over digital image pixels by using Simpson's integration rule. Hosny [8] modified this method where he evaluates the double integration analytically which completely removes the numerical approximation error. In addition to this contribution, Hosny [8] proposed an algorithm for fast computation of the exact geometric moments. Chong et. al. [9] follows the same approach of Hosny and employed the symmetry property to reduce the computational complexity of the exact 2D geometric moments. Recently, Hosny [10] applied a new idea of symmetry to efficiently compute exact 2D geometric moments and employed these exact moments to compute a set of accurate radial moments. Complexity analysis shows that, the recent symmetry-based method of Hosny [10] is superior to its corresponding method of Chong [9].

This paper proposes a new method for efficient computation of accurate geometric moments for both binary and gray level images. Three types of symmetry property are applied to reduce %87 where the set of 2D geometric moments are computed exactly by using a mathematical integration of the monomials over image pixels.

The rest of the paper is organized as follows: In section II, an overview of geometric moments is given. The proposed method is described in section III. Results of numerical experiments are presented in section IV. Conclusion and concluding remarks are presented in section V.

II. OVERVIEW OF GEOMETRIC MOMENTS

Two-dimensional geometric moments are defined as the projection of the image intensity function $f(x, y)$ onto the nominal $x^p\ y^q$. The 2D geometric moments of order (p + q) are defined as:

$$M_{pq}=\int_{-1}^{1}\int_{-1}^{1} x^p\ y^q\ f\left(x,y\right)dx\,dy \qquad (1)$$

For digital images of size $N\times N$ equation (1) usually approximated by replacing integrations by summations as follows:

$$\tilde{M}_{pq}=\sum_{i=1}^{N}\sum_{j=1}^{N} x_i^{\ p}\ y_j^{\ q}\ f\left(x_i,y_j\right)\Delta x\Delta y \qquad (2)$$

Where Δx and Δy are the pixel width in x-and y-direction respectively. Equation (2) is so-called direct method for geometric moment's computations, which is the approximated version using zeroth-order approximation (ZOA).

As indicated by Liao and Pawlak [7], equation (2) is inaccurate approximation of equation (1). To improve the accuracy, they proposed to use the approximated form:

$$M_{pq}=\sum_{i=1}^{N}\sum_{j=1}^{N} h_{pq}\left(x_i,y_j\right)f\left(x_i,y_j\right) \qquad (3)$$

Where

$$h_{pq}\left(x_i,y_j\right)=\int_{x_i-\frac{\Delta x_i}{2}}^{x_i+\frac{\Delta x_i}{2}}\int_{y_j-\frac{\Delta y_j}{2}}^{y_j+\frac{\Delta y_j}{2}} x^p\ y^q\,dx\,dy \qquad (4)$$

Liao and Pawlak proposed an alternative extended Simpson's rule to evaluate the double integral defined by equation (4), then used to calculate the geometric moments defined by equation (3).

III. Proposed Method

The proposed method is presented in this section. Image and object mapping is discussed in the first subsection. The symmetry property for the 2D geometric moments is discussed in the second subsection where the concept of augmented image intensity functions is discussed. The fourth subsection is devoted to discuss the efficient computation of exact 2D geometric moments.

A. Image Mapping

In the literature of digital image processing, the origin of a 2D digital image of size $N\times N$ is located on the upper left of the image and its indices, i and j increase from left to right and from top to bottom, respectively, i.e. $i,j=1,2,........,N$ as shown in Fig.1a. A kind of transformation is applied to the input image where the transformed image is defined in the square $\left[-1,1\right]\times\left[-1,1\right]$ as shown in Fig.1b. This transformation could be done by using the following equations:

$$x_i=\frac{-N+2i-1}{N},\ y_j=-\frac{-N+2j-1}{N} \qquad (5)$$

With $i,j=1,2,3,............N$. The mapped image is represented by $N\times N$ array of pixels where centers of these pixels are the points $\left(x_i,y_j\right)$. The image intensity function is defined for this set of discrete points $\left(x_i,y_j\right)\in\left[-1,1\right]\times\left[-1,1\right]$ as shown in Fig.1b. The sampling intervals in the x-and y-directions are $\Delta x_i=x_{i+1}-x_i$, $\Delta y_j=y_{j+1}-y_j$ respectively. In the literature of digital image processing, the intervals Δx_i and Δy_j are fixed at the constant value equal to $\Delta x_i=\Delta y_j=2/N$. It is clear that, the centre of input image is coinciding with the center of the square $\left[-1,1\right]\times\left[-1,1\right]$.

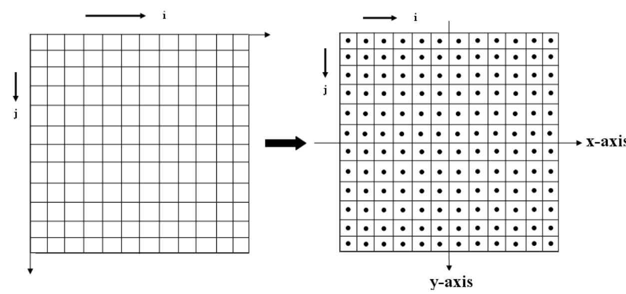

Fig. 1. Image mapping: (a) Original image, (b) Mapped image.

B. Symmetry for 2D Geometric Moments

The x- and y-axis in addition to the two lines $x=y$ and $x=-y$ divided the transformed image into eight octants as shown in Fig. 2. Three types of symmetry could be observed and explored. In the first type; each point P_1 in the first octant with the Cartesian coordinates $\left(x_i,y_j\right)$ has three similar points in other three octants as shown in Fig.3. These points are P_4, P_5 and P_8, where the index associated with the point symbol is referring to the octant number.

The second type of symmetry is concerned with the rest of the eight points where the subscripts i and j are interchanged. The point $P_2\left(x_j,y_i\right)$ in the second octant has three sibling points P_3, P_6 and P_7. It is must be noted that all of these eight points have the same radial distance to the origin point. The third type of symmetry is concerned with the points lies on the symmetrical lines, $x = y$ and $x = -y$. These points are $Q_1(x_i, x_i)$, Q_2, Q_3 and Q_4. All points of three symmetrical types and their Cartesian coordinates are shown in the table (1).

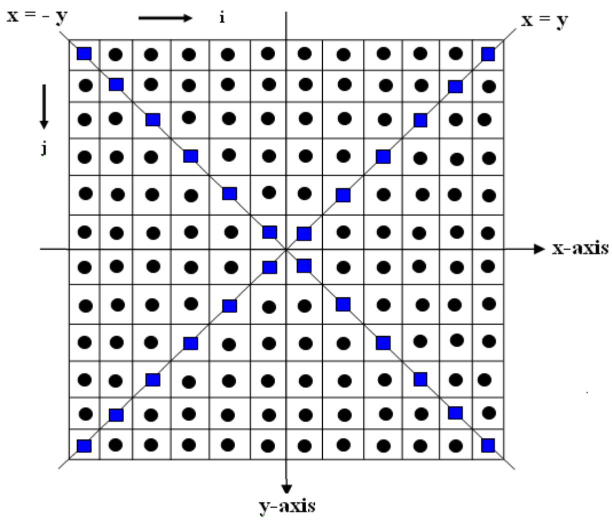

Fig. 2. The x- axis, y-axis, the lines $x = y$ and $x = -y$ divided the transformed image into eight octants

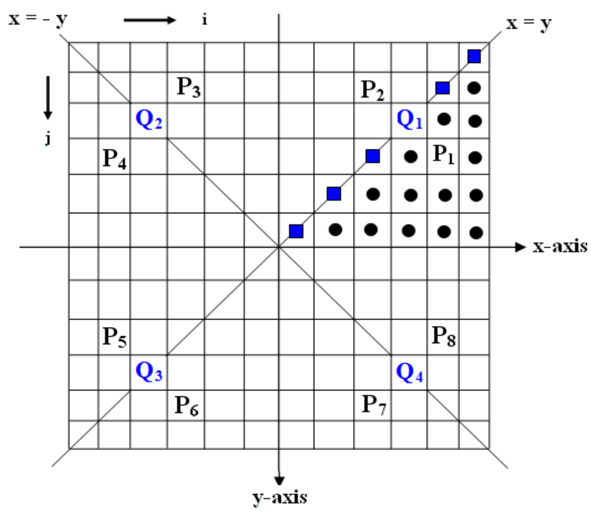

Fig. 3. Three types of symmetry

Since the points P_1, P_2, P_3, P_4, P_5, P_6, P_7 and P_8 has the same radial distance from the coordinate origin then; the numerical value of $x^p y^q$ will be dependent on whatever p and q are even or odd. Based on this symmetry property and the results obtained in table (1), the image intensity function in different octants could be represented by only one augmented function.

This function is a combination of the image intensity functions in the first, fourth, fifth and eighth octants respectively. The augmented function is defined as follows:

Case 1: p =Even, q =Eevn:

$$f_{k1}(x_i, y_j) = f_1 + f_4 + f_5 + f_8 \tag{6.1}$$

Case 2: p =Even, q =Odd:

$$f_{k1}(x_i, y_j) = f_1 + f_4 - f_5 - f_8 \tag{6.2}$$

Case 3: p =Odd, q =Even:

$$f_{k1}(x_i, y_j) = f_1 - f_4 - f_5 + f_8 \tag{6.3}$$

Case 4: p =Odd, q =Odd:

$$f_{k1}(x_i, y_j) = f_1 - f_4 + f_5 - f_8 \tag{6.4}$$

Similar to the first type of symmetry, the second one could be represented by only one function. This function is a combination of the image intensity functions in the second, third, sixth and seventh octants respectively and defined as follows:

Case 1: p =Even, q =Eevn:

$$f_{k2}(x_i, y_j) = f_2 + f_3 + f_6 + f_7 \tag{7.1}$$

Case 2: p =Even, q =Odd:

$$f_{k2}(x_i, y_j) = f_2 - f_3 - f_6 + f_7 \tag{7.2}$$

Case 3: p =Odd, q =Even:

$$f_{k2}(x_i, y_j) = f_2 + f_3 - f_6 - f_7 \tag{7.3}$$

Case 4: p =Odd, q =Odd:

$$f_{k2}(x_i, y_j) = f_2 - f_3 + f_6 - f_7 \tag{7.4}$$

The third kind of symmetry is represented by a function which is a combination of the image intensity functions for points that are lies on the symmetrical lines $x = y$ and $x = -y$ as follows:

Case 1: p =Even, q =Eevn:

$$f_{k3}(x_i, x_i) = Q_1 + Q_2 + Q_3 + Q_4 \tag{8.1}$$

Case 2: p =Even, q =Odd:

$$f_{k3}(x_i, x_i) = Q_1 + Q_2 - Q_3 - Q_4 \tag{8.2}$$

Case 3: p =Odd, q =Even:

$$f_{k3}(x_i, x_i) = Q_1 - Q_2 - Q_3 + Q_4 \tag{8.3}$$

Case 4: p =Odd, q =Odd:

$$f_{k3}(x_i, x_i) = Q_1 - Q_2 + Q_3 - Q_4 \tag{8.4}$$

C. Exact Computation of 2D Geometric moments

The approximation of the integral terms in equation (4) is responsible for the approximation error of geometric moments. These integrals need to be evaluated exactly to

TABLE (1)
THE THREE TYPES OF SYMMETRY POINTS AND THEIR COORDINATES

First type of symmetry	$P_1(x_i, y_j)$	$P_4(x_{N-i+1}, y_j)$	$P_5(x_{N-i+1}, y_{N-j+1})$	$P_8(x_i, y_{N-j+1})$
Second type of symmetry	$P_2(x_j, y_i)$	$P_3(x_{N-j+1}, y_i)$	$P_6(x_{N-j+1}, y_{N-i+1})$	$P_7(x_j, y_{N-i+1})$
Third type of symmetry	$Q_1(x_i, x_i)$	$Q_2(x_{N-i+1}, x_i)$	$Q_3(x_{N-i+1}, x_{N-i+1})$	$Q_4(x_i, x_{N-i+1})$

remove the approximation error. Equation (4) can be written as following:

$$h_{pq}(x_i, y_j) = I_p(i) I_q(j) \tag{9}$$

Where

$$I_p(i) = \int_{x_i - \frac{\Delta x_i}{2}}^{x_i + \frac{\Delta x_i}{2}} x^p dx = \frac{1}{p+1}\left[U_{i+1}{}^{p+1} - U_i{}^{p+1}\right] \tag{10}$$

$$I_q(j) = \int_{y_j - \frac{\Delta y_j}{2}}^{y_j + \frac{\Delta y_j}{2}} y^q dy = \frac{1}{q+1}\left[V_{j+1}{}^{q+1} - V_j{}^{q+1}\right] \tag{11}$$

The upper and lower limits of the integration in equations (10) and (11) have the values:

$$U_{i+1} = x_i + \frac{\Delta x_i}{2} = -1 + i\,\Delta x_i\,, \tag{12.1}$$

$$U_i = x_i - \frac{\Delta x_i}{2} = -1 + (i-1)\Delta x_i\,, \tag{12.2}$$

Similarly,

$$V_{j+1} = y_j + \frac{\Delta y_j}{2} = -1 + j\,\Delta y_j\,, \tag{12.3}$$

$$V_j = y_j - \frac{\Delta y_j}{2} = -1 + (j-1)\Delta y_j \tag{12.4}$$

Substituting equations (10-12) into equation (3) yields a set of exact geometric moments. The computational complexity could be significantly reduced through the computation of the first octant only by applying the three types of symmetry as follows:

$$\begin{aligned}\hat{G}_{pq} = &\sum_{i=2}^{\lfloor N/2 \rfloor} \sum_{j=1}^{i-1} I_p(i) I_q(j) f_{k1}(x_i, y_j) \\ &+ \sum_{i=2}^{\lfloor N/2 \rfloor} \sum_{j=1}^{i-1} I_q(i) I_p(j) f_{k2}(x_i, y_j) \\ &+ \sum_{i=1}^{\lfloor N/2 \rfloor} I_p(i)\, I_q(i) f_{k3}(x_i, x_i)\end{aligned} \tag{13}$$

Where:

$$\left\lfloor \frac{N}{2} \right\rfloor = \begin{cases}(N-1)/2, & N \text{ is odd} \\ N/2, & N \text{ is even}\end{cases} \tag{14}$$

The moment kernel of exact 2D geometric moments is defined by equations (10-11). This kernel is independent of image. Therefore, this kernel can be pre-computed, stored, recalled whenever it is needed to avoid repetitive computation.

IV. NUMERICAL RESULTS

The validity proof of the proposed method is discussed in this section. The performance of the proposed method is compared with the other existing methods. Accuracy and efficiency are essential issues must be addressed and proved. A numerical experiment is conducted. This numerical experiment is devoted to prove the accuracy of the proposed method. A kind of complexity analysis is performed to prove the efficiency of the proposed method.

In the first experiment, low order geometric moments are computed with the approximated ZOA and proposed method. The obtained results are compared with theoretical values of geometric moments. An artificial test image defined by the image intensity function $f(x, y) = 1$ is used in this experiment. The image size is relatively small (4×4), so that, the hand calculations could be done. Theoretical values of geometric moments for this test image are calculated by the following equation:

$$\begin{aligned}M_{pq} &= \int_{-1}^{1}\int_{-1}^{1} x^p\, y^q\; dx\,dy \\ &= \left(\frac{(1)^{p+1} - (-1)^{p+1}}{p+1}\right)\left(\frac{(1)^{q+1} - (-1)^{q+1}}{q+1}\right)\end{aligned} \tag{15}$$

The results are shown in table (2). It is obvious that, the results of the proposed method are identical to the theoretical values while the results of ZOA method are deviated.

Based on the symmetry property, one octant will be used to compute the full set of 2D geometric moments. The computation process required only two points. The first one is fall inside the first octant and has a similar seven points in the other seven octants. The second one is defined according to the third type of symmetry. Consequently, the total number of computed points in the first octant is equal to $\left[1+2+3+.........+\left(N/2-1\right)\right]+N/2$. Therefore, the reduction in the computed points according to the symmetry property is defined as follows:

TABLE (2)
THE NUMERICAL VALUES OF LOW ORDER GEOMETRIC MOMENTS

Theoretical Values				
4.0000	0	1.3333	0	0.8000
0	0	0	0	0
1.3333	0	0.4444	0	0.2667
0	0	0	0	0
0.8000	0	0.2667	0	0.1600
Proposed Exact Values				
4.0000	0	1.3333	0	0.8000
0	0	0	0	0
1.3333	0	0.4444	0	0.2667
0	0	0	0	0
0.8000	0	0.2667	0	0.1600
0	0	0	0	0
Approximated ZOA Values				
4.0000	0	1.2500	0	0.6406
0	0	0	0	0
1.2500	0	0.3906	0	0.2002
0	0	0	0	0
0.6406	0	0.2002	0	0.1026
4.0000	0	1.2500	0	0.6406

$$RP=\left(1-\frac{N\left(N+2\right)}{8}\right)\times 100 \qquad (16)$$

An explicit numerical example is used to explain the efficiency of the proposed method. In table (3), a quick comparison with the other existing methods is presented.

VII. Conclusion

This paper proposes a new method for efficient computation of accurate 2D geometric moments for gray level images. Three types of symmetry property are applied where %87 of the computational demands are removed. The calculated values of geometric moments are very accurate where the integrations are analytically evaluated without any kind of approximation. Computation of the moment invariants is straightforward. The conducted numerical experiments confirm the efficiency of the proposed method.

TABLE (3)
REDUCTION PERCENTAGE (RP) OF THE PROPOSED METHOD

Image size	Direct	Hosny[10]	Proposed	RP
64x64	4096	1024	528	87.1%
128x128	16384	4096	2080	87.3%
256x256	65536	16384	8256	87.4%
512x512	262144	65536	32896	87.5%
1024x1024	1048576	262144	131328	87.5%

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