

Control Systems I

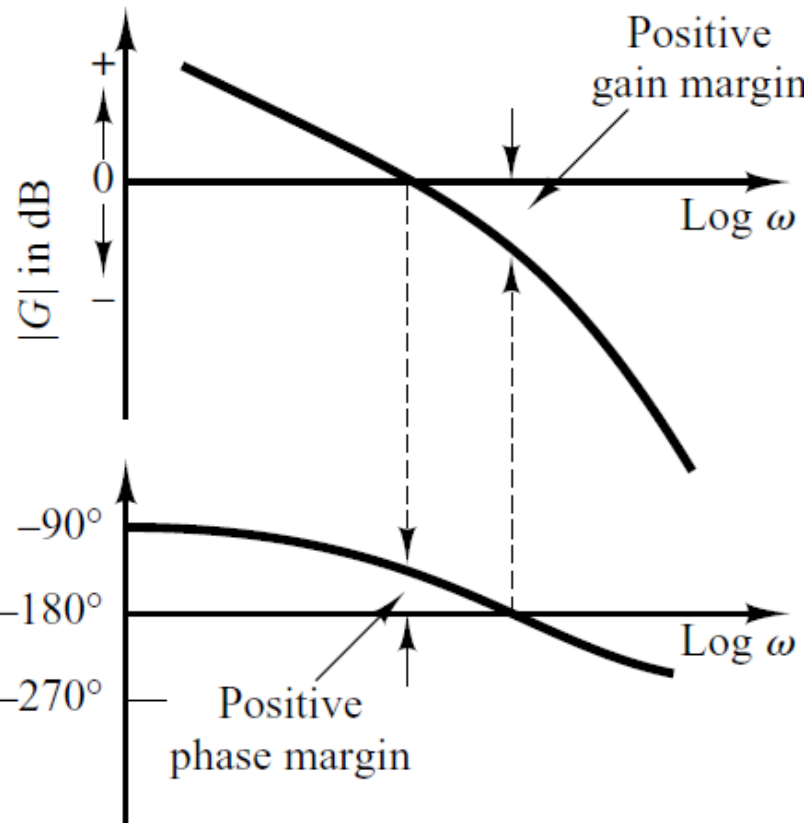
Lecture 8 Compensation-Bode Plot

Emam Fathy

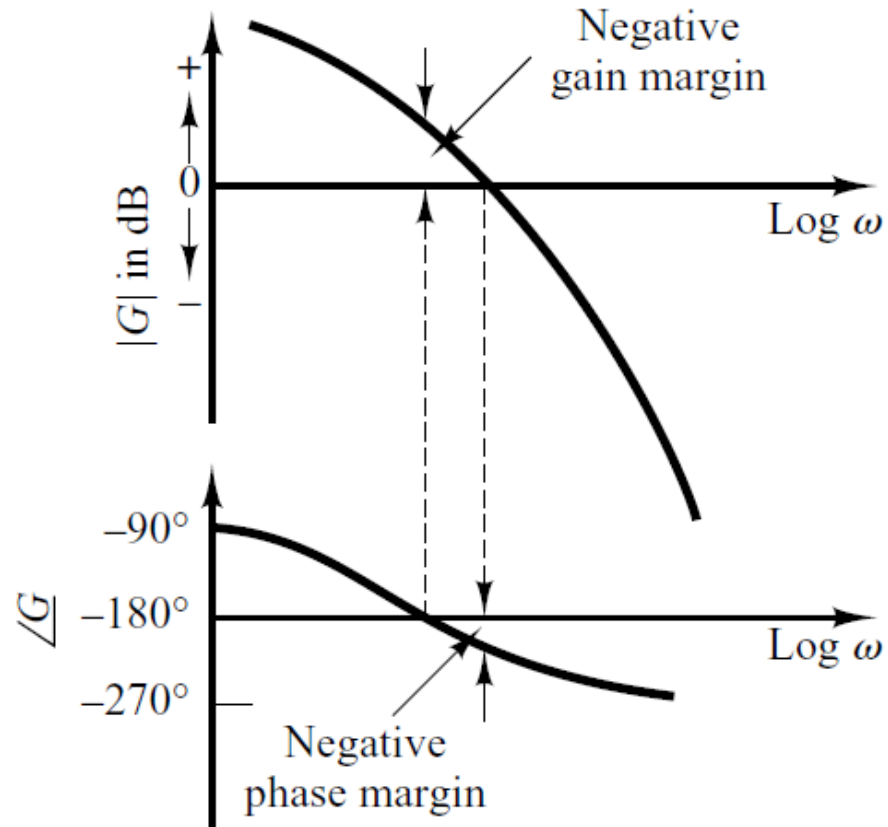
email: emfmz@aast.edu

http://www.aast.edu/cv.php?disp_unit=346&ser=68525

Relative Stability

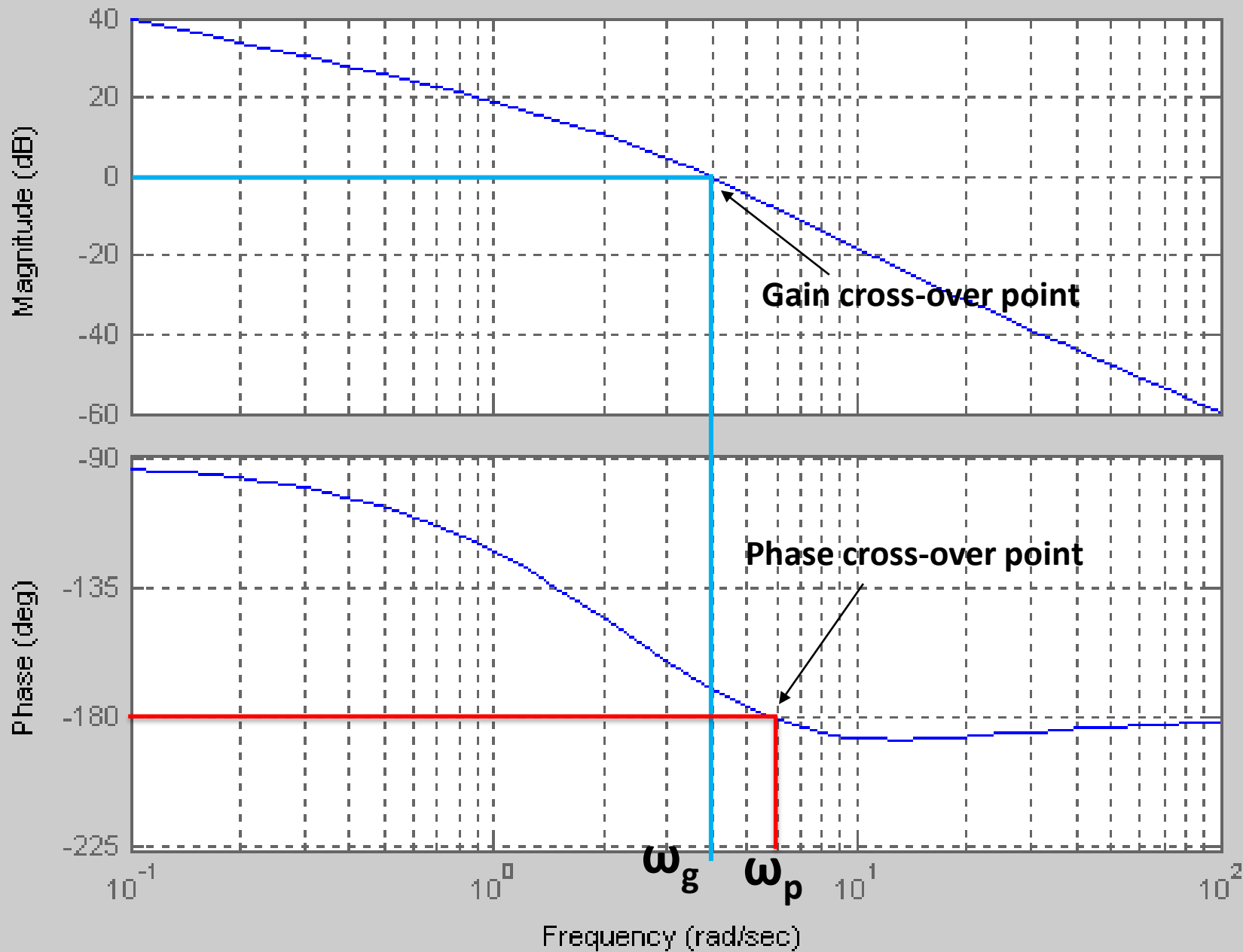


Stable system

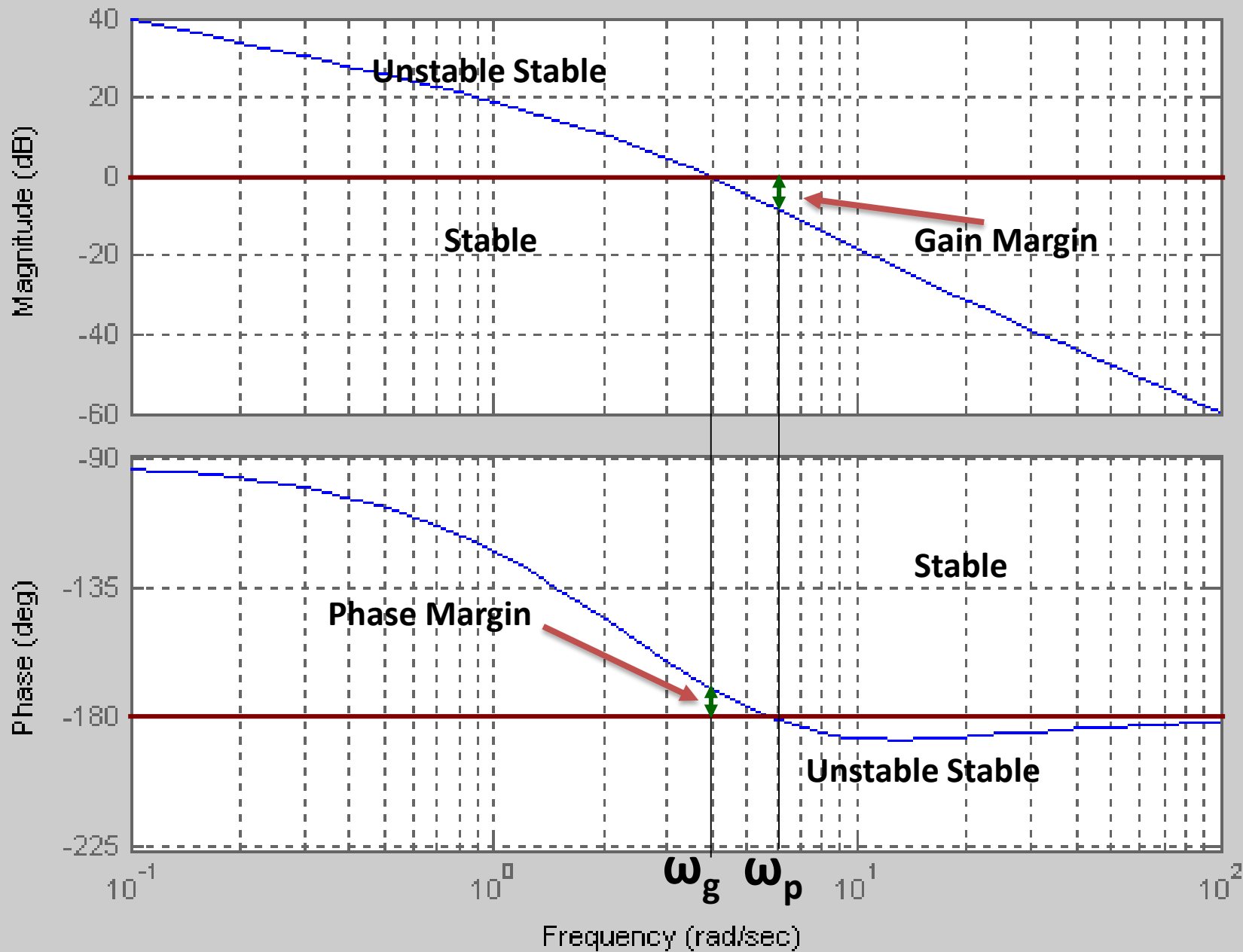


Unstable system

Bode Diagram



Bode Diagram



Compensation design using the bode diagram

The compensator $G_c(s)$ can be written

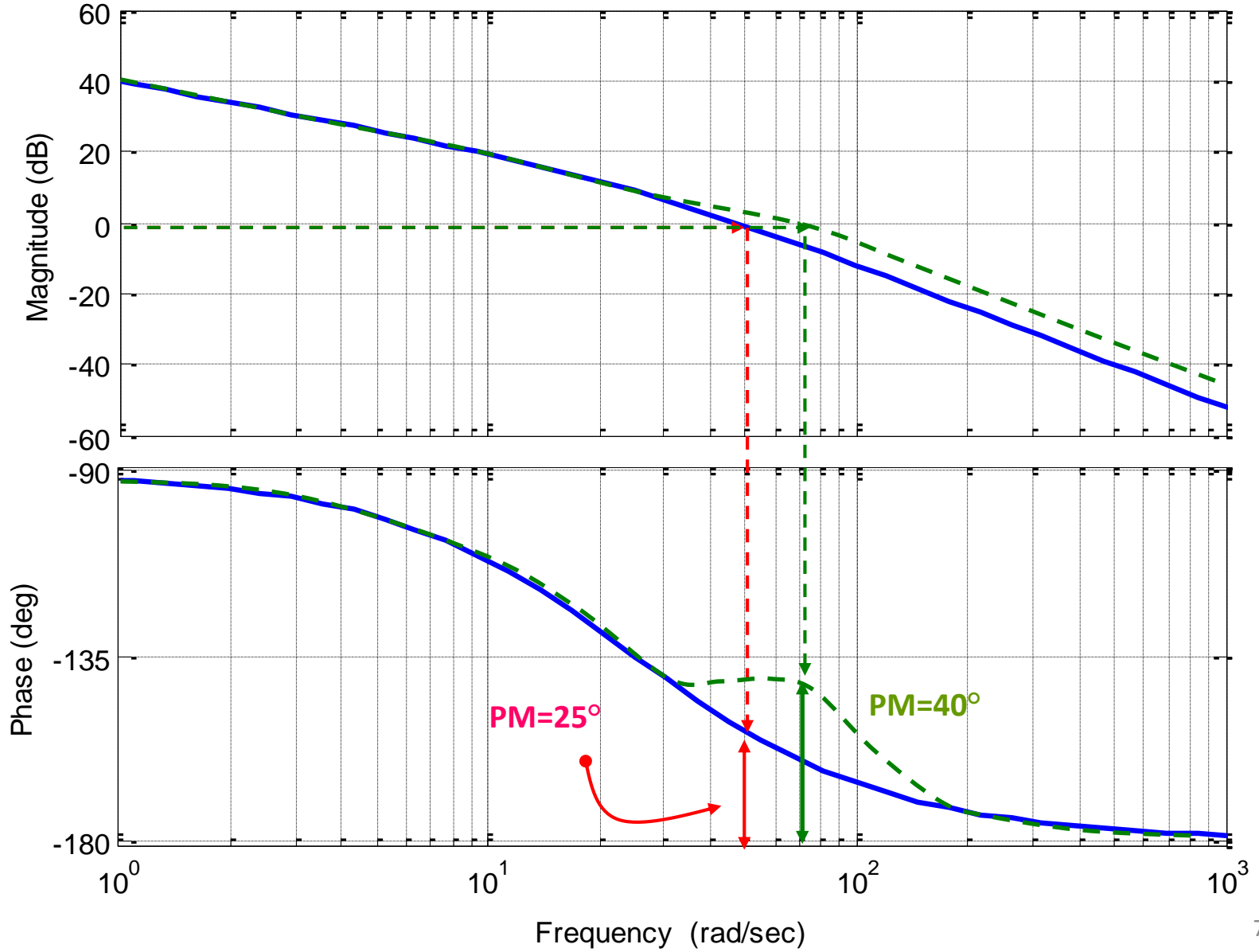
$$G_c(s) = \frac{K(s + z)}{(s + p)}$$

The **phase-lead compensation** when $z < p$.

The **phase-lag compensation** when $z > p$.

phase-lead compensation

Bode Diagram

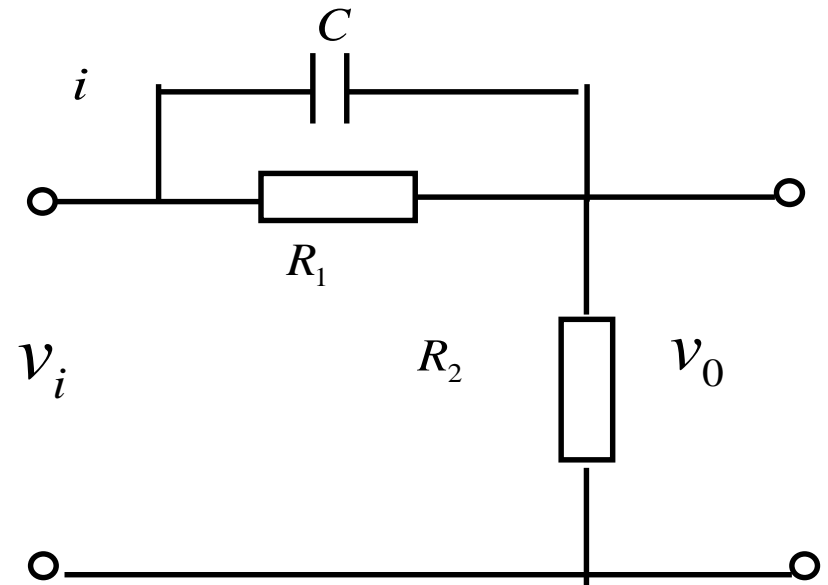


Phase -lead compensation

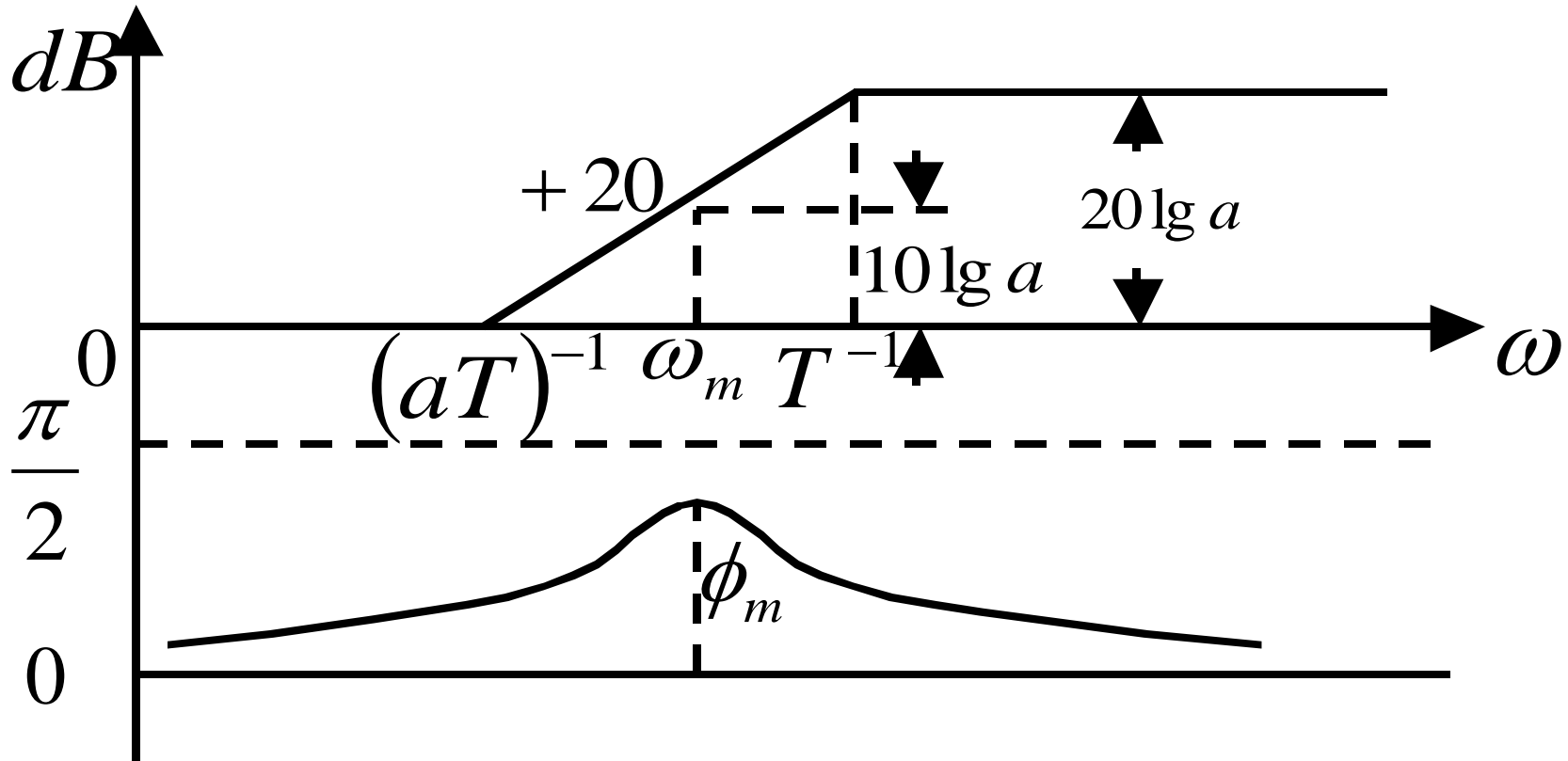
$$G_c(s) = \frac{1 + aTs}{1 + Ts} \quad (a > 1)$$

$$a = \frac{R_1 + R_2}{R_2}$$

$$T = \frac{R_1 R_2}{R_1 + R_2} C$$



Bode diagram of the phase-lead network



$$G_c(s) = \frac{1 + aTs}{1 + Ts} \quad (a > 1)$$

$$\omega_m = \frac{1}{T\sqrt{a}}$$

$$20 \lg |G_c(j\omega_m)| = -10 \lg a$$

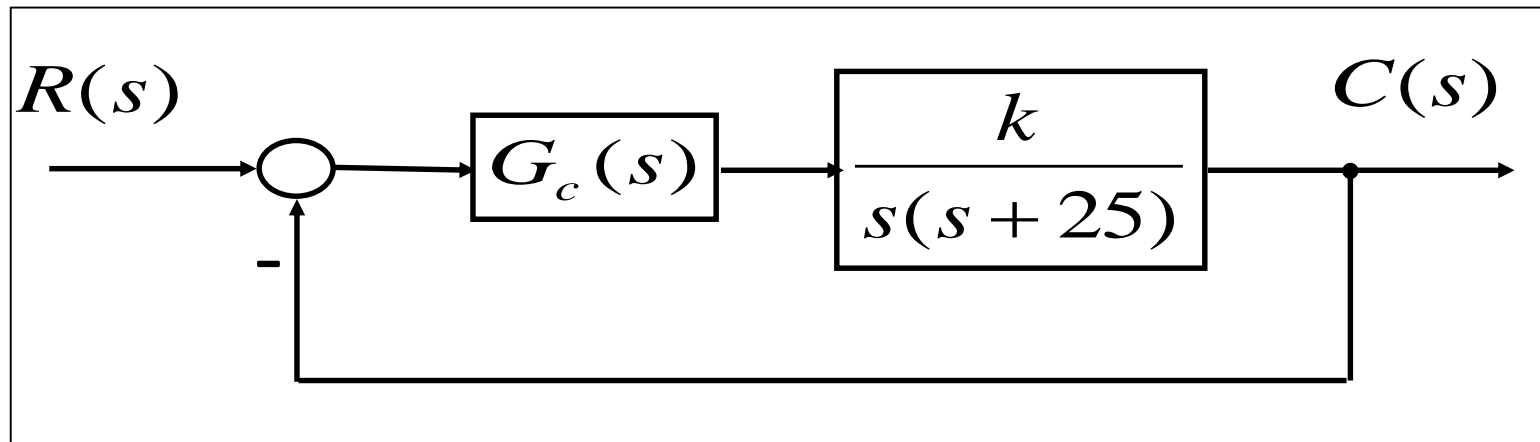
$$\phi_m = \sin^{-1} \frac{a-1}{a+1}$$

Phase -lead compensation Steps:

1. Evaluate the uncompensated system phase margin when error constants are satisfied.
2. Allowing for a small amount of safety , determine the necessary additional phase lead, ϕ_m .
3. Evaluate a form Eq. $\phi_m = \sin^{-1} \frac{a-1}{a+1}$
4. Evaluate $-10 \log a$ and determine the frequency ω_m .
5. Evaluate T from Eq. $\omega_m = \frac{1}{T \sqrt{a}}$
6. Check the result, and repeat the steps if necessary.

Example

Design a lead controller such that the phase margin be 45° and the ramp error constant be 100.



Example

Step 1: Consider $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$ with $a > 1$ as a phase-lead controller.

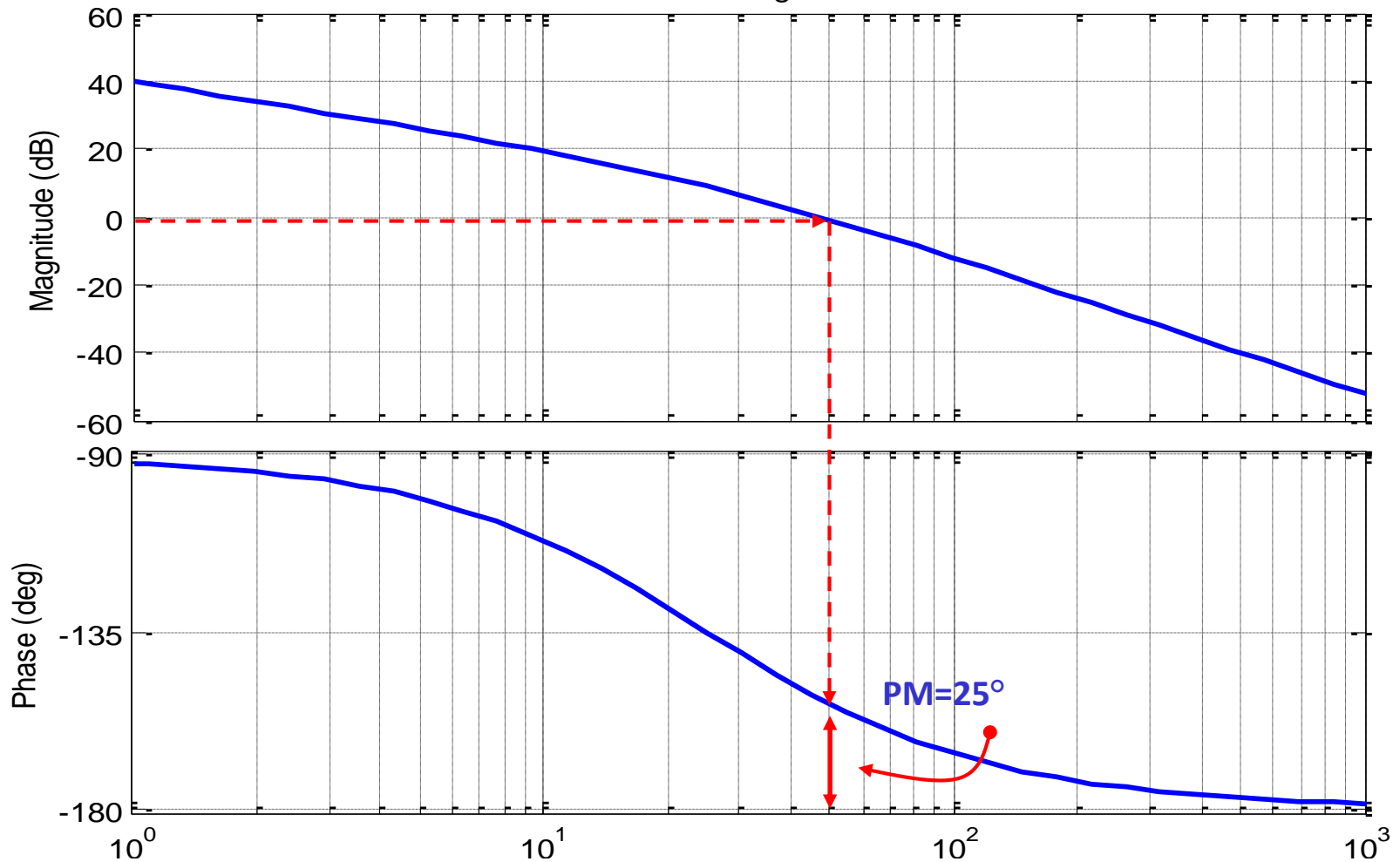
Step 2: find k .

$$k_v = \lim_{s \rightarrow 0} sG_c(s) \frac{k}{s(s + 25)} = 100 \quad k = 2500$$

$$G(s) = \frac{2500}{s(s + 25)} = \frac{100}{s(s/25 + 1)}$$

Step 3: Sketch the Bode plot for $G(s)$.

Bode Diagram

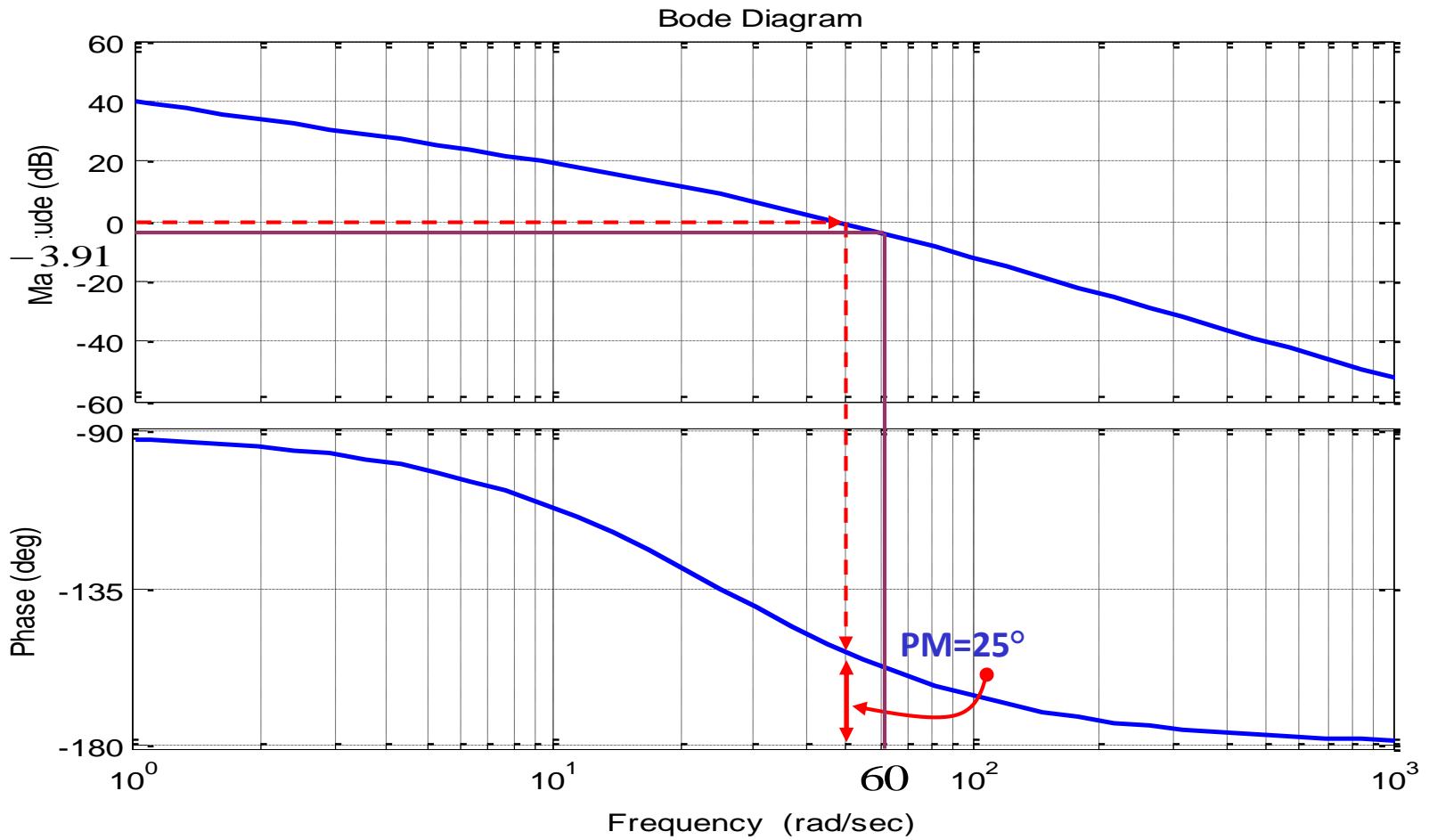


Step 4: Find the system PM and if it is not sufficient choose the required phase by: $\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$

$$\varphi_m = 45^\circ - 25^\circ + 5^\circ = 25^\circ \quad \sin 25 = \frac{a-1}{a+1} \quad a = 2.46$$

Step 5: Put the center of the controller in the new gain

crossover frequency: $20 \log |G(j\omega)|_{\omega=\omega_c^{new}} = -10 \log(a) = -3.91$



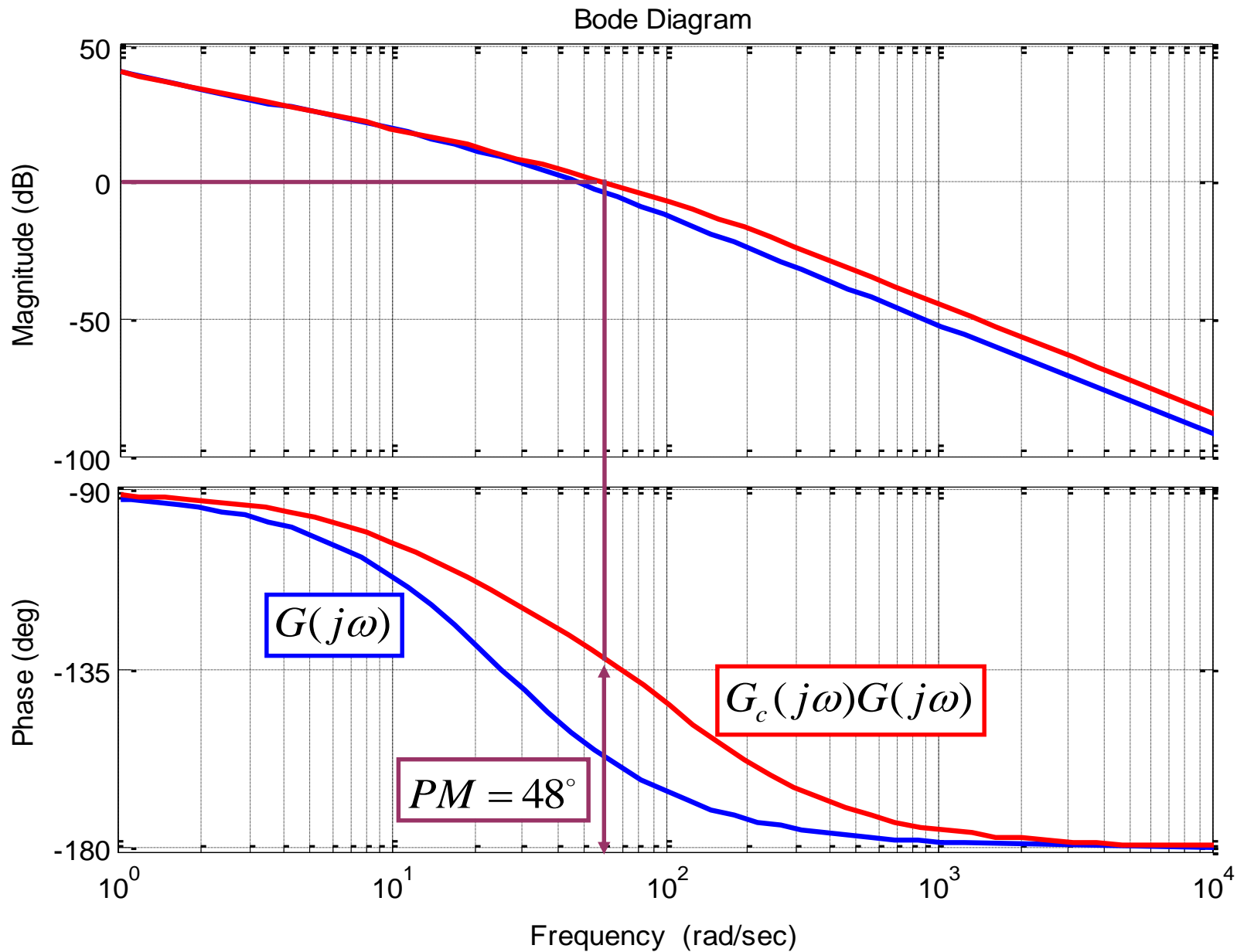
$$\omega_c^{new} = 60$$

$$\omega_c^{new} = \frac{1}{\tau \sqrt{a}}$$

$$\tau = 0.0106$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.0261s + 1}{0.0106s + 1}$$

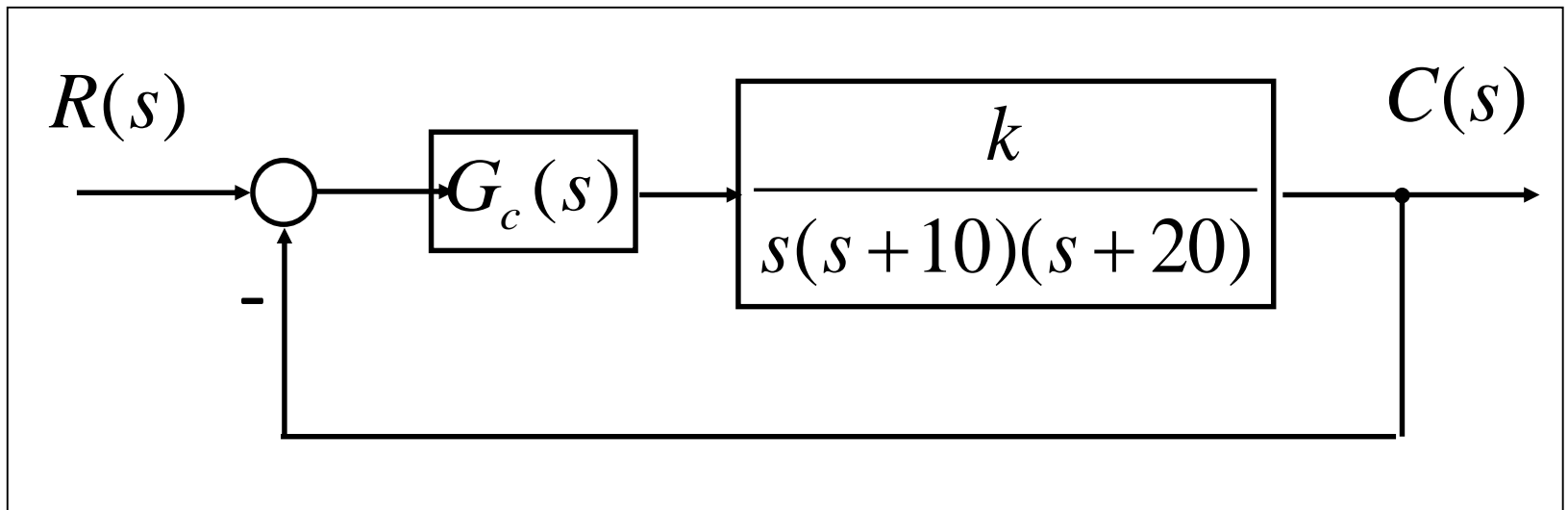
Step 6: Check the controller.



phase-lag compensation

Example 2

Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 1: Consider controller $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$ with $a < 1$ as a phase-lag controller.

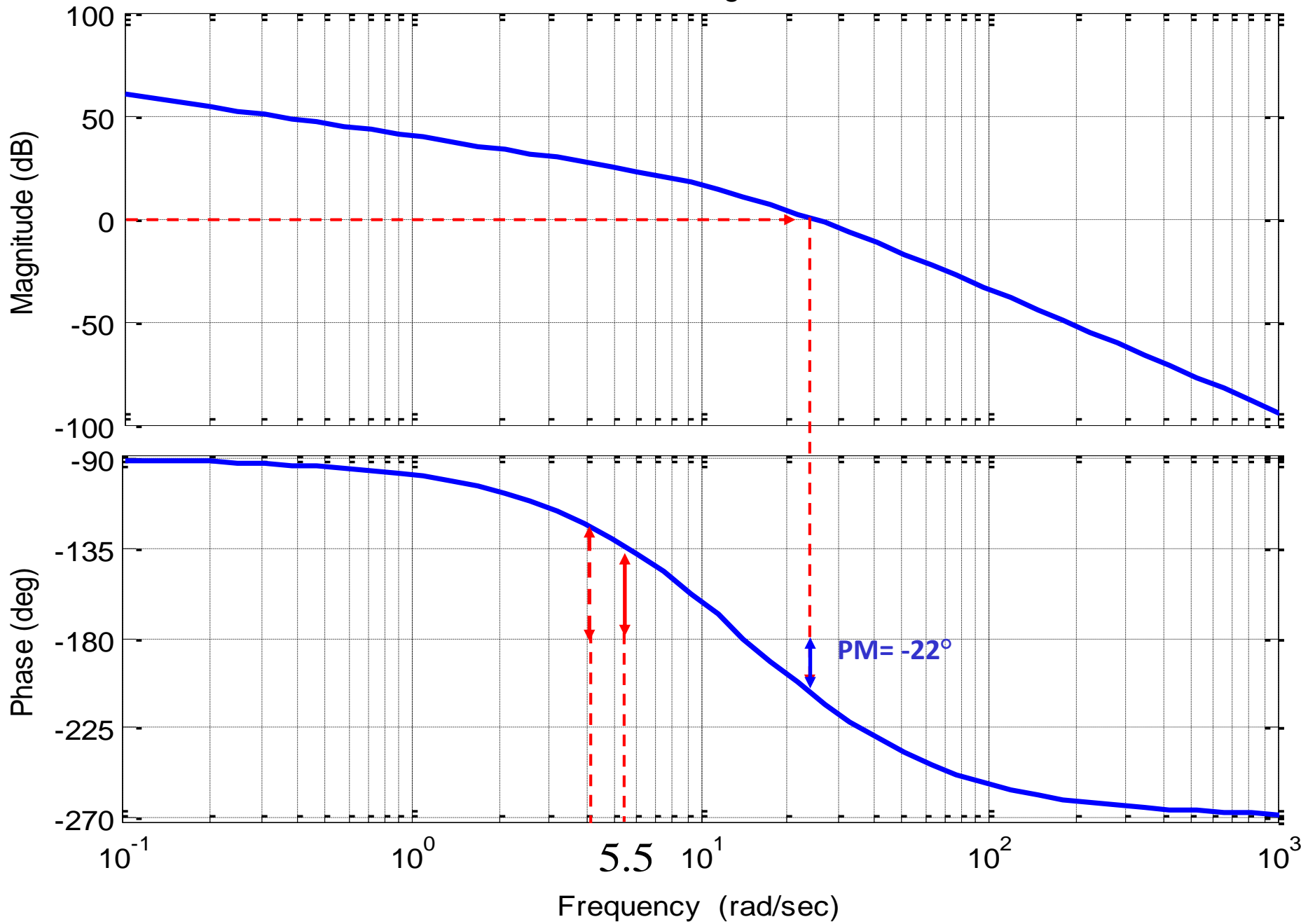
Step 2: Find k

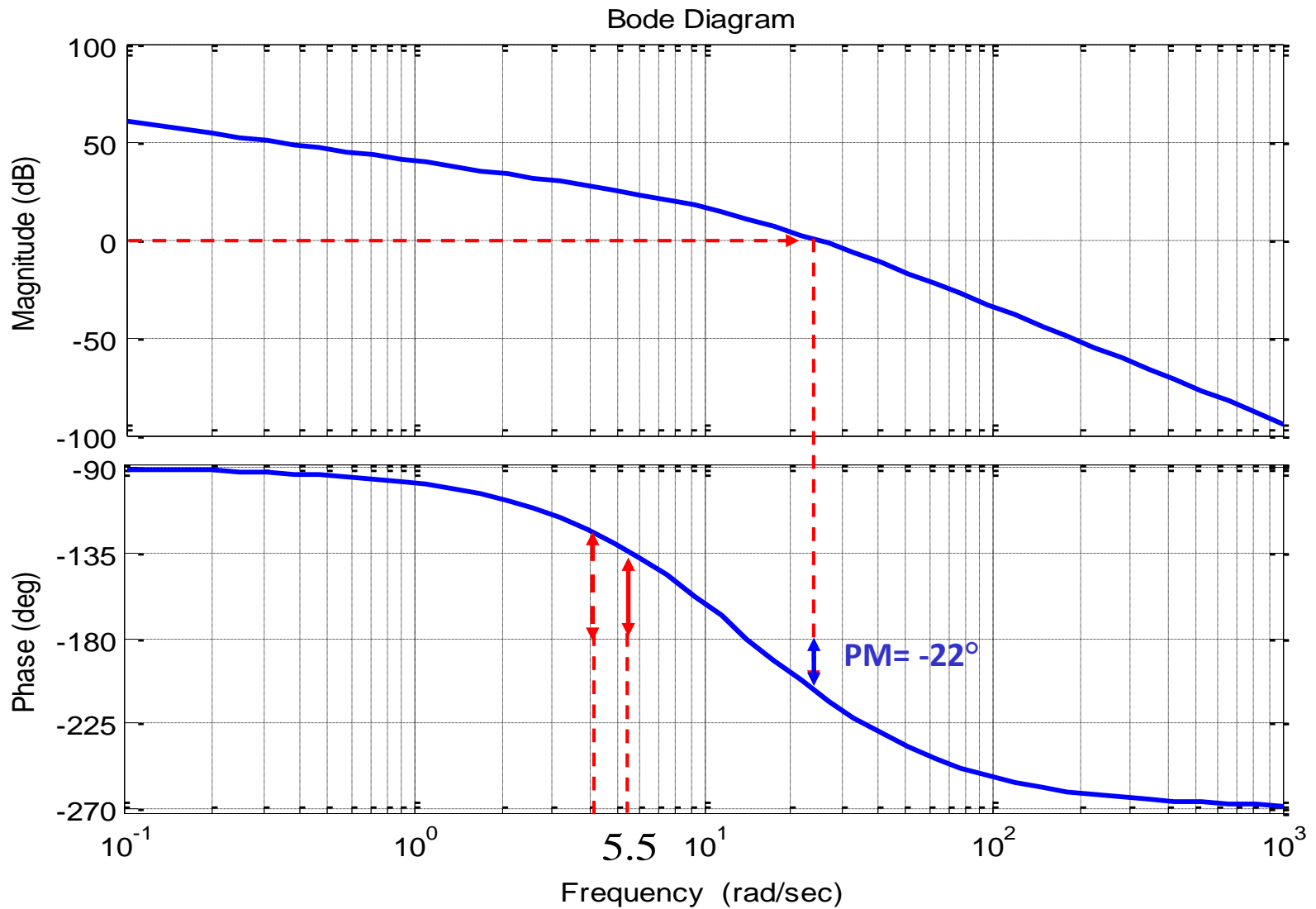
$$k_v = \lim_{s \rightarrow 0} sG_c(s) \frac{k}{s(s+10)(s+20)} = 100 \quad k = 20000$$

Step 3: Sketch the Bode plot for G(s).

$$G(s) = \frac{20000}{s(s+10)(s+20)} = \frac{100}{s(s/10+1)(s/20+1)}$$

Bode Diagram





Step 4: Find the system PM and if it is not sufficient choose the new gain crossover

frequency ω_c^{new} such that the PM is ok (reduce it a little).

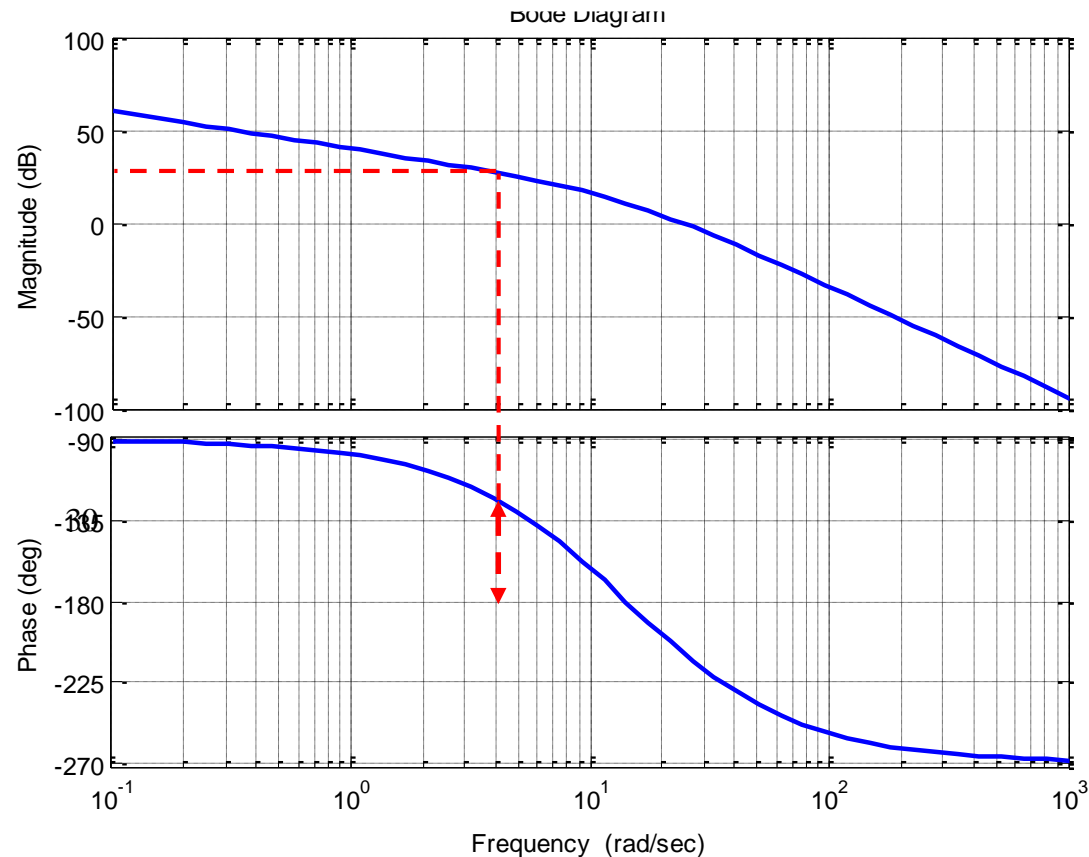
$$\cancel{\omega_c^{new} = 5.5} \quad \omega_c^{new} = 4$$

Step 5: Find the gain of the system at the new gain crossover frequency and let:

$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} + 20 \log(a) = 0$$

$$30 + 20 \log(a) = 0$$

$$a = 10^{-\frac{30}{20}} = 0.0316$$



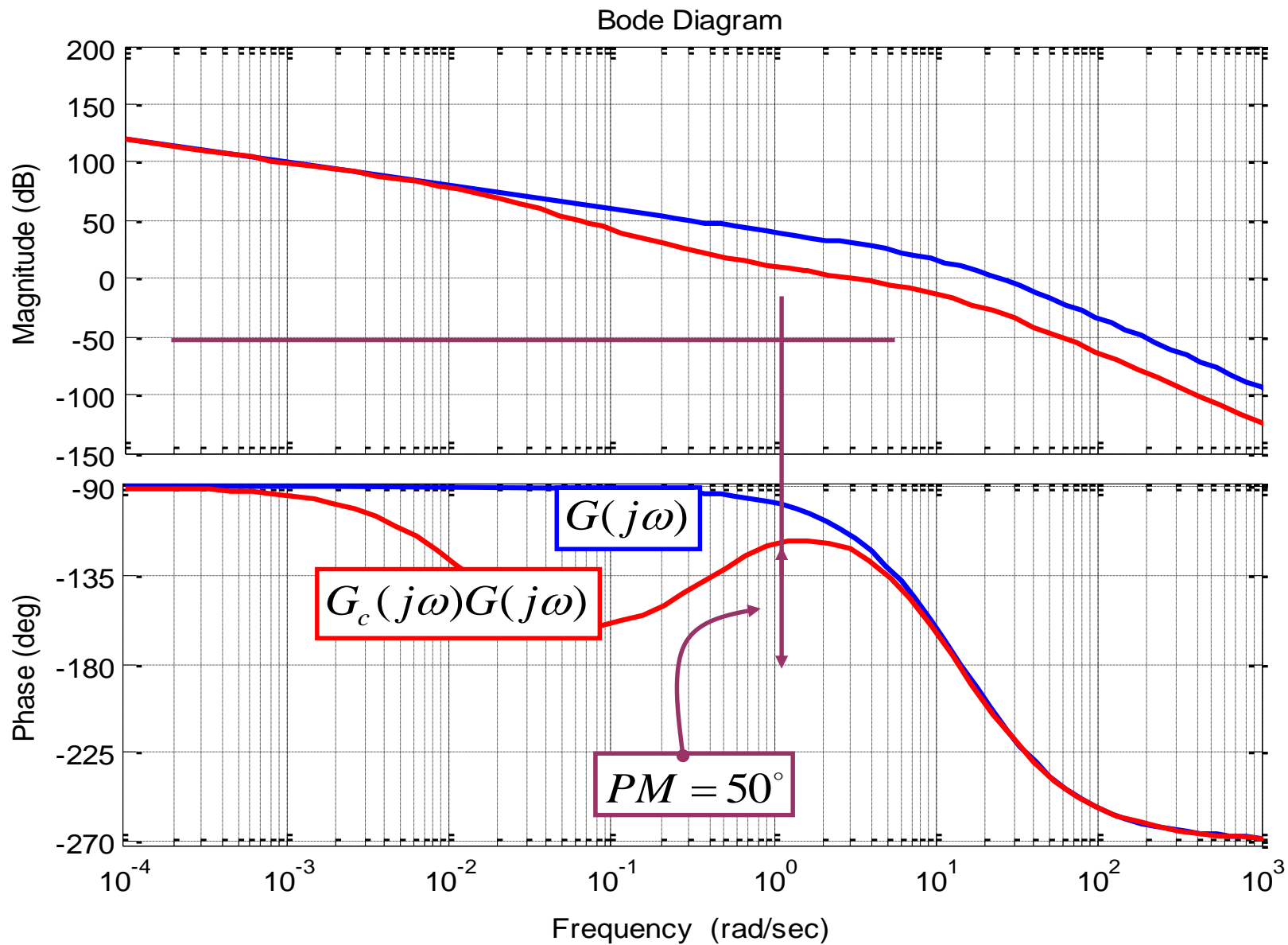
Step 6: Put the right corner of the controller sufficiently far from ω_c^{new}

$$\frac{1}{a\tau} = \frac{\omega_c^{new}}{10} = 0.4$$

$$\tau = 79$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{2.5s + 1}{79s + 1}$$

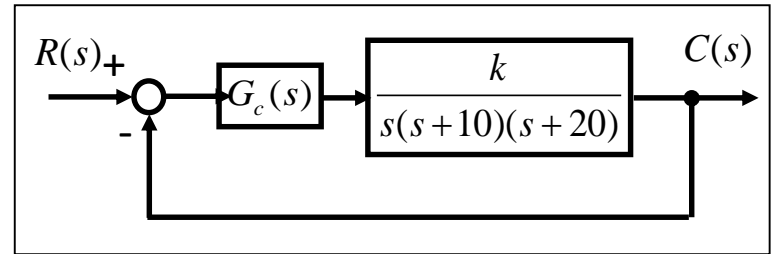
Step 7: Check the designed controller.



End of Lec

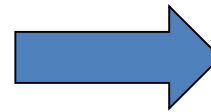
Example 6:

Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a > 1$ as a phase-lead controller.

Note: If the plant has another gain k , let



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} \quad a > 1$$

Step 2: Try to fix k according to performance request, otherwise let $k=1$

$$k_v = \lim_{s \rightarrow 0} s G_c(s) \frac{k}{s(s+10)(s+20)} = 100$$

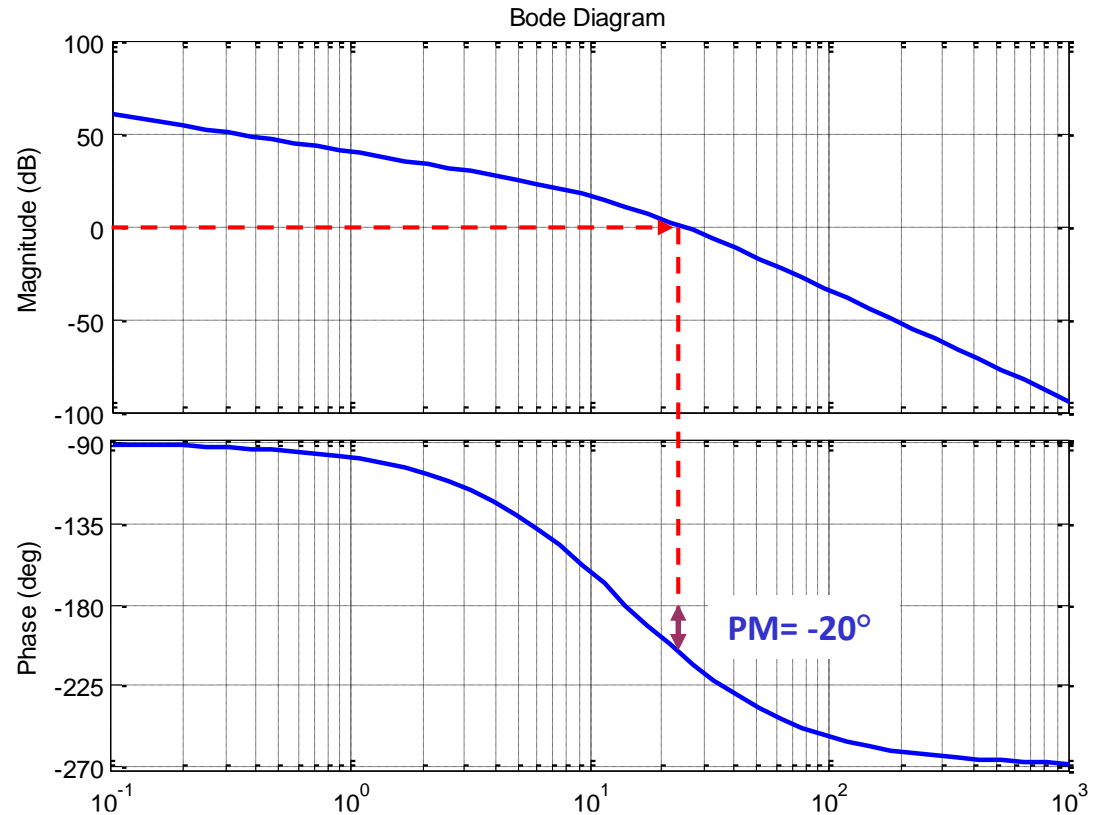


$$k = 20000$$

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s) = \frac{20000}{s(s+10)(s+20)}$$

$$= \frac{100}{s(s/10+1)(s/20+1)}$$



Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (-20) + 5^\circ = 70^\circ \quad \longrightarrow \quad \sin 70 = \frac{a-1}{a+1} \quad \longrightarrow \quad a = 32$$

Step 5: Put the center of controller in the new gain crossover frequency:

$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} + 10 \log(a) = 0$$

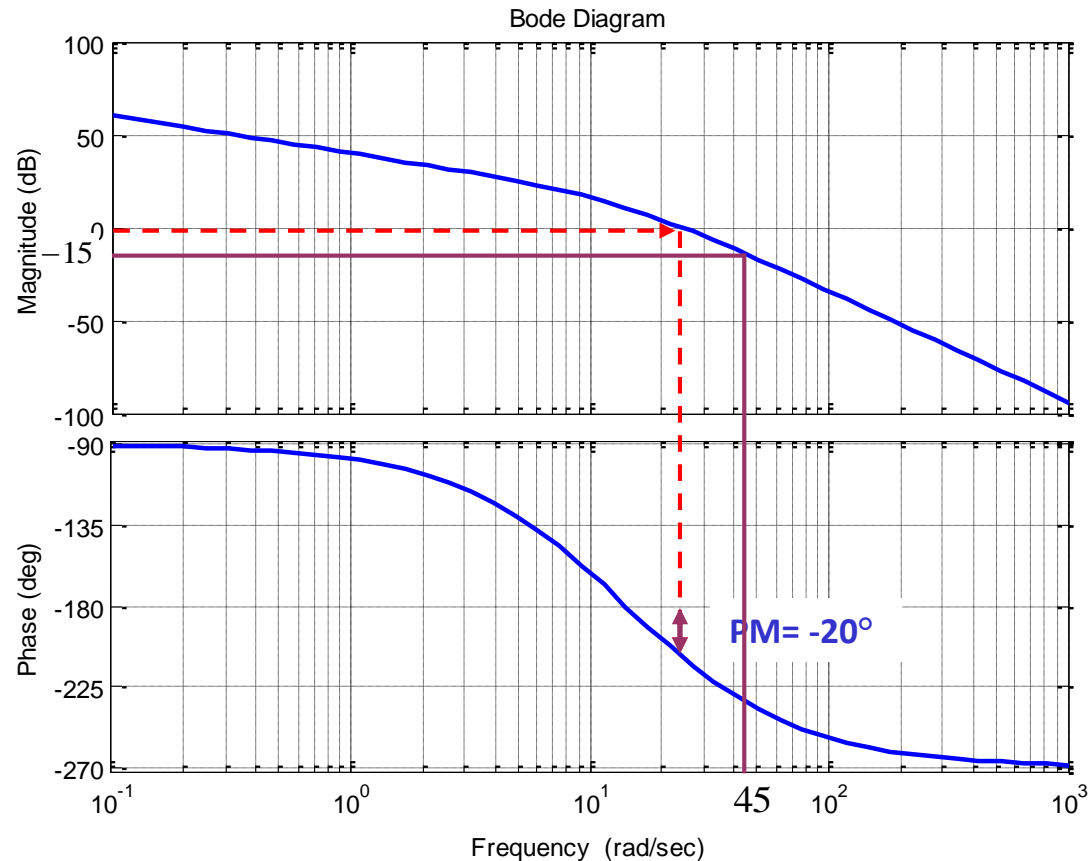
$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} = -10 \log(a) = -15$$

$$\omega_c^{new} = 45$$

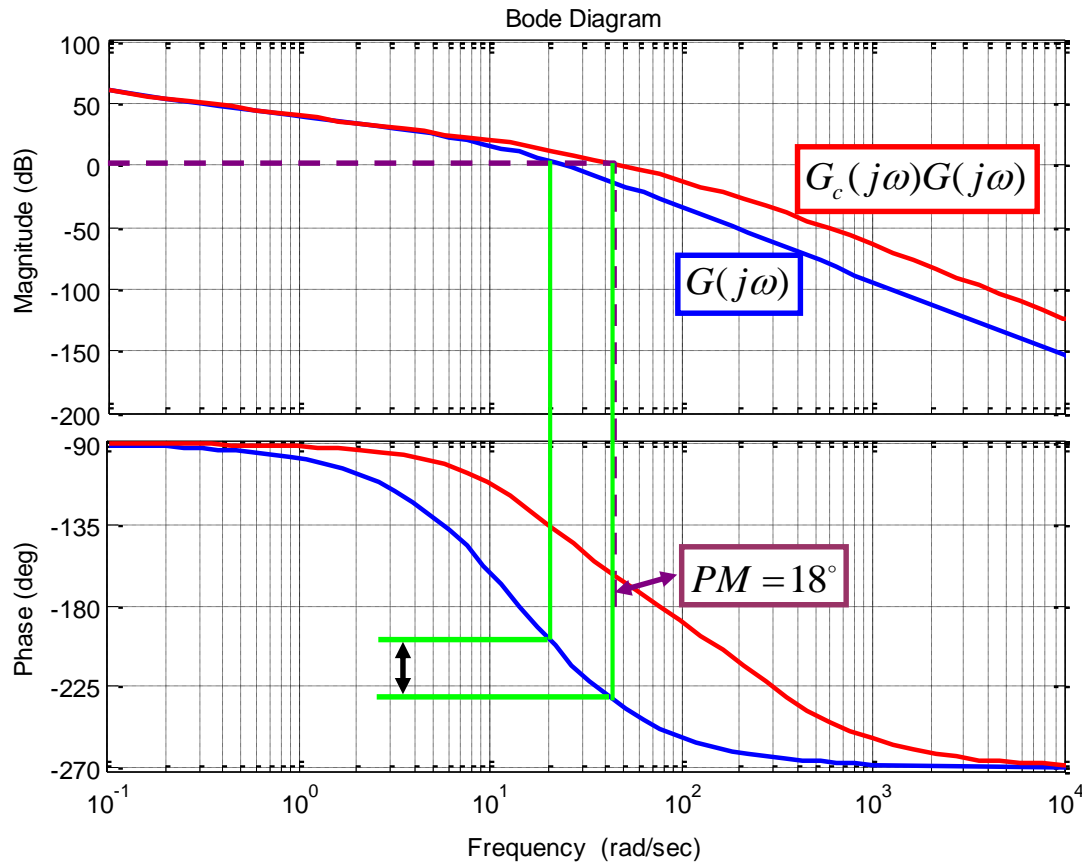
$$\omega_c^{new} = \frac{1}{\tau \sqrt{a}}$$

$$\tau = 0.0039$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.1248s + 1}{0.0039s + 1}$$



Step 6: Check the controller.

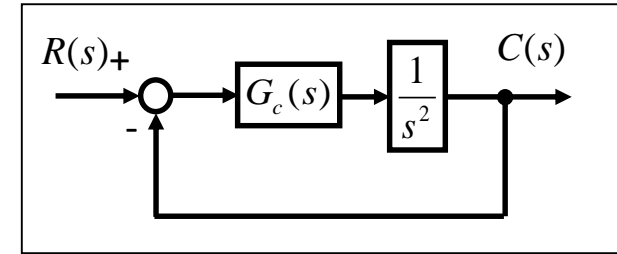


Note: Design is not possible!

Why?

Example 7:

Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a > 1$ as a phase-lag controller.

Note: If the plant has another gain k , let



$$G_c(s) = k \frac{a\tau s + 1}{\tau s + 1} \quad a > 1$$

Step 2: Try to fix k according to performance request, otherwise let $k=1$

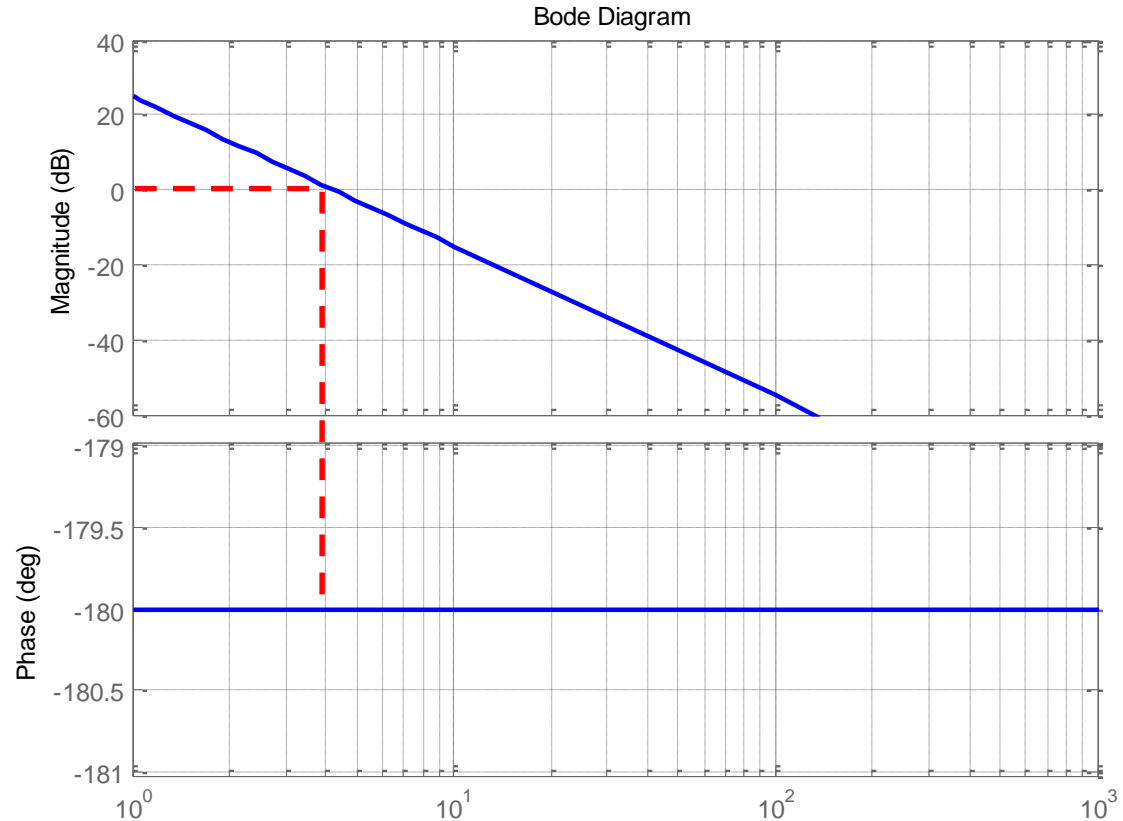
$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=5} = -3 \quad \text{Why?}$$



$$k = 17.7$$

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s) = \frac{17.7}{s^2}$$



Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ \quad \longrightarrow \quad \sin 45^\circ = \frac{a-1}{a+1} \quad \longrightarrow \quad a = 5.8$$

Step 5: Put the center of controller in the new gain crossover frequency:

$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} + 10 \log(a) = 0$$

$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} = -10 \log(a) = -7.63$$

$$\omega_c^{new} = 6.7$$



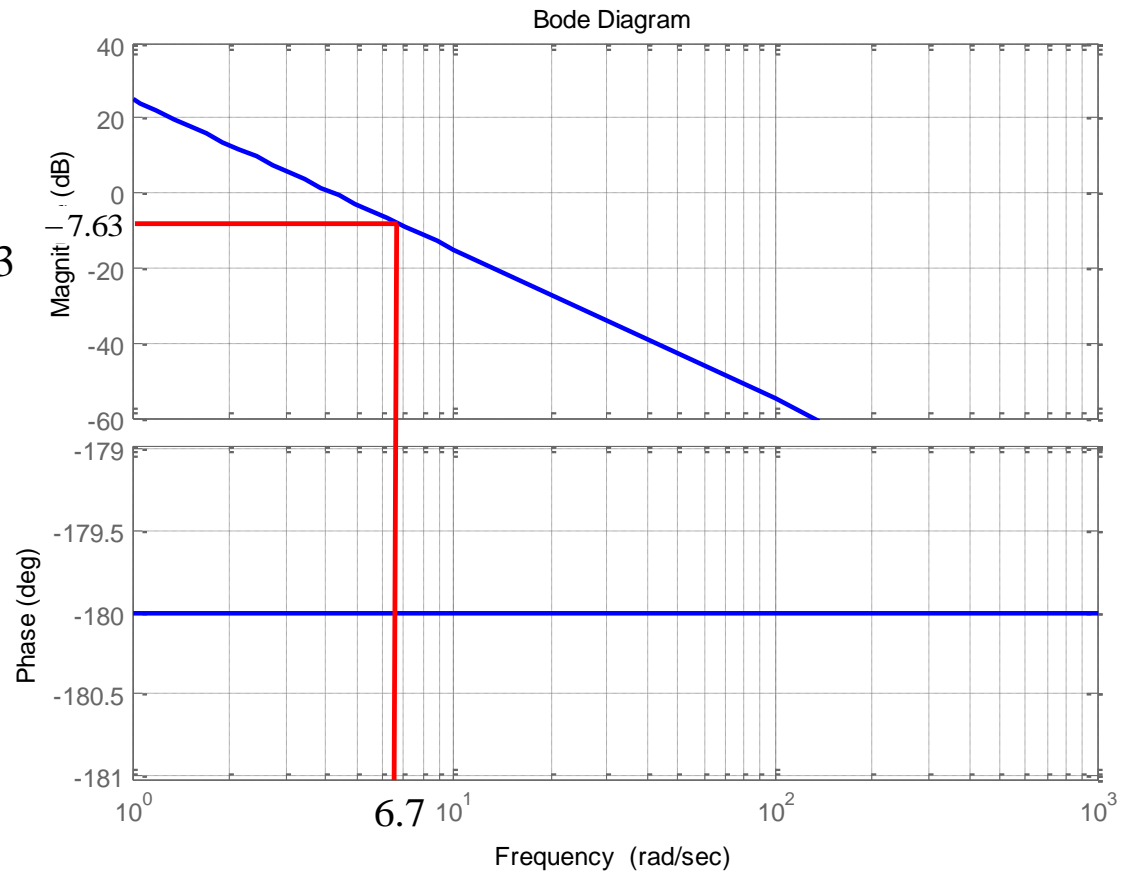
$$\omega_c^{new} = \frac{1}{\tau \sqrt{a}}$$



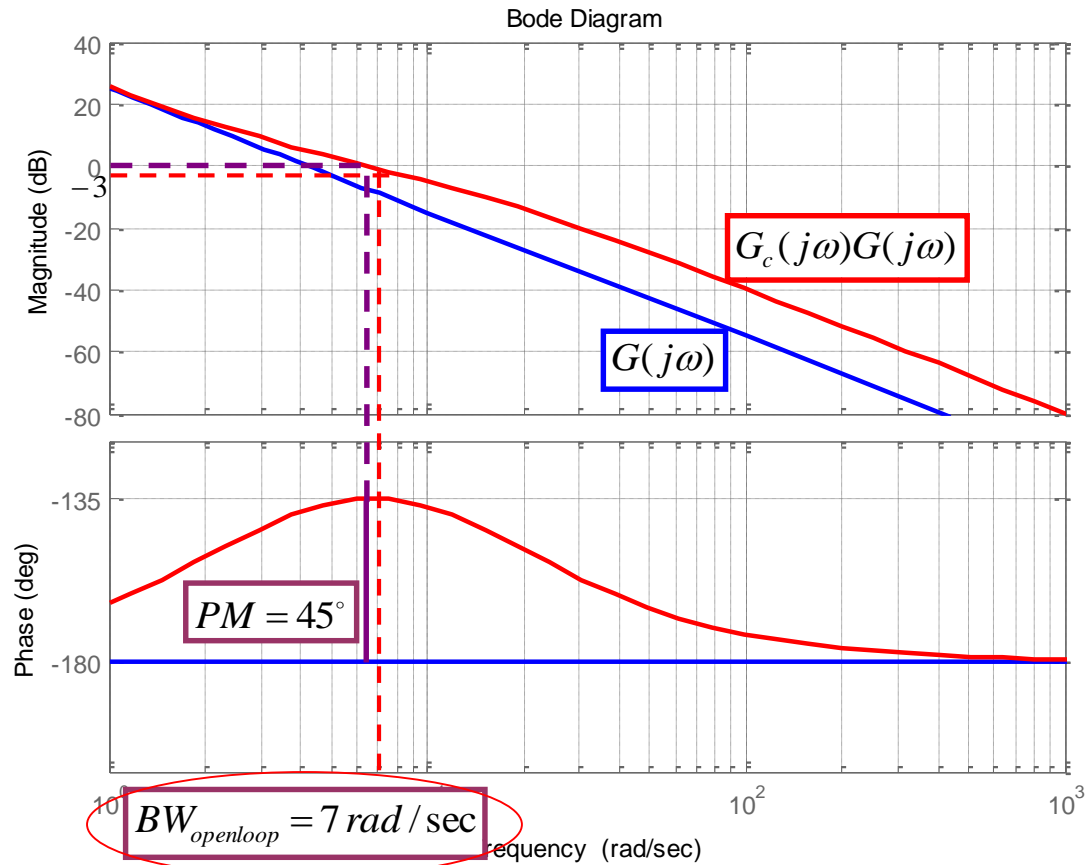
$$\tau = 0.062$$



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.3596s + 1}{0.062s + 1}$$



Step 6: Check the controller.

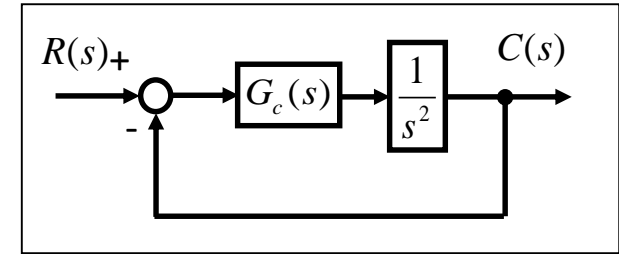


Controller is not ok

Try again

Example 7(Continue):

Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 2: Try to fix k according to performance request, otherwise let k=1

Open loop bandwidth is near to gain crossover frequency so:

$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=5} = -3$$



$$k = 17.7$$

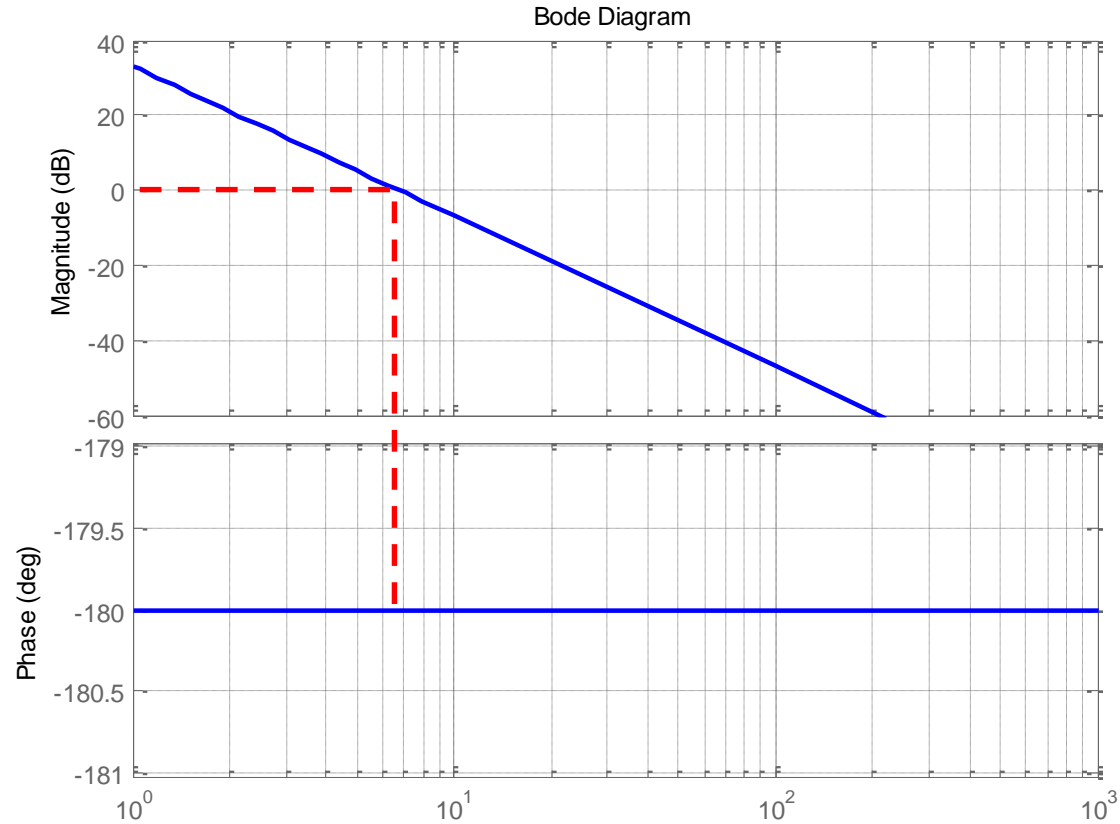
$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=7} = -3$$



$$k = 35.7$$

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s) = \frac{35.7}{s^2}$$



Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ \quad \longrightarrow \quad \sin 45^\circ = \frac{a-1}{a+1} \quad \longrightarrow \quad a = 5.8$$

Step 5: Put the center of controller in the new gain crossover frequency:

$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} + 10 \log(a) = 0$$

$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} = -10 \log(a) = -7.63$$

$$\omega_c^{new} = 9$$



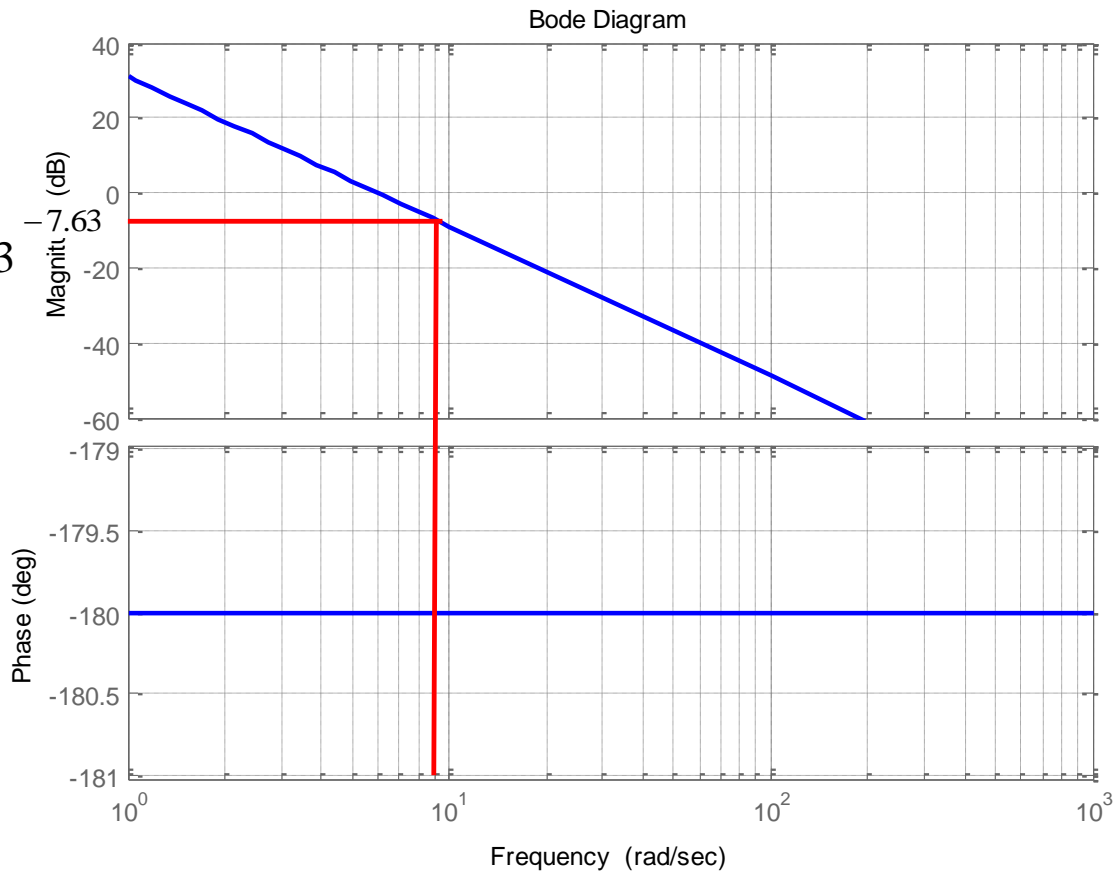
$$\omega_c^{new} = \frac{1}{\tau \sqrt{a}}$$



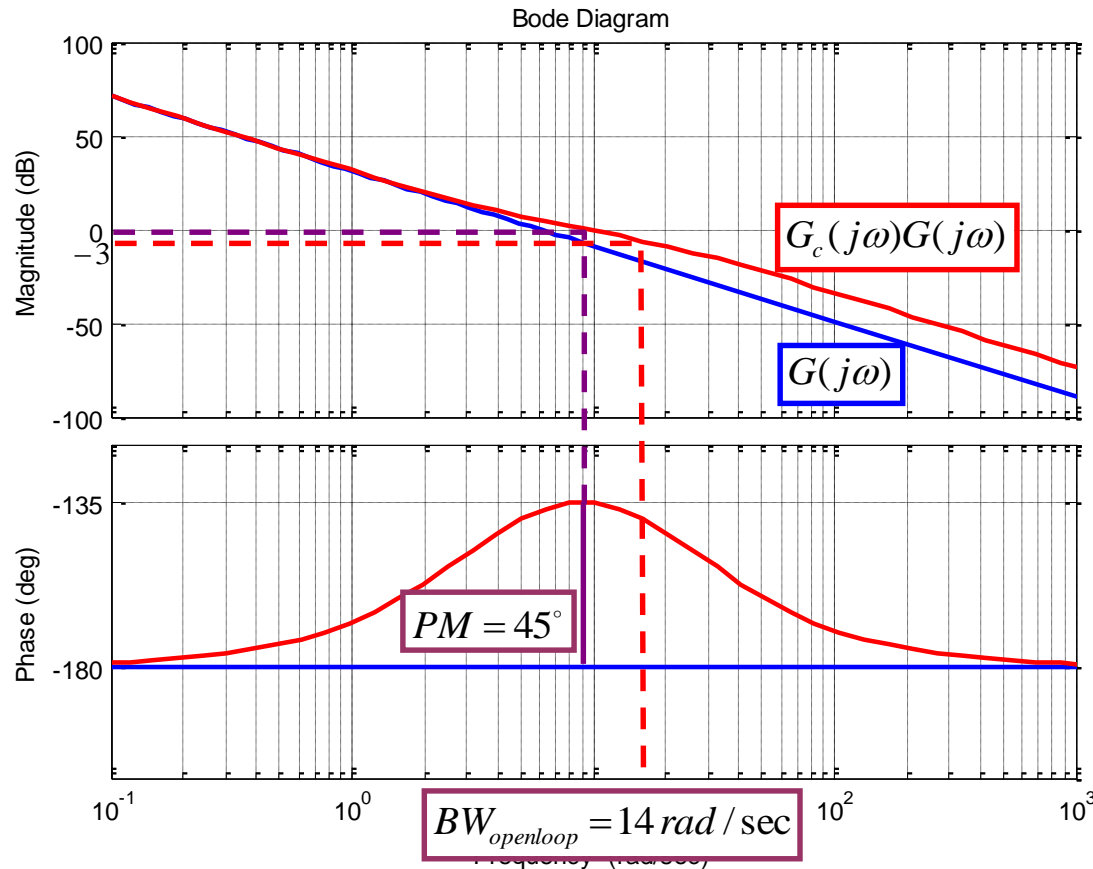
$$\tau = 0.0461$$



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.2674s + 1}{0.0461s + 1}$$



Step 6: Check the controller.

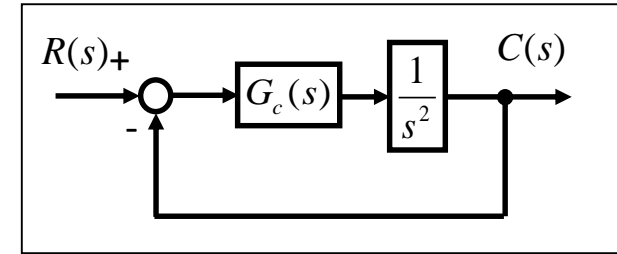


Controller is not ok

Try again

Example 7(Continue):

Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 2: Try to fix k according to performance request, otherwise let k=1

Open loop bandwidth is near to gain crossover frequency so:

$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=7} = -3$$



$$k = 35.7$$

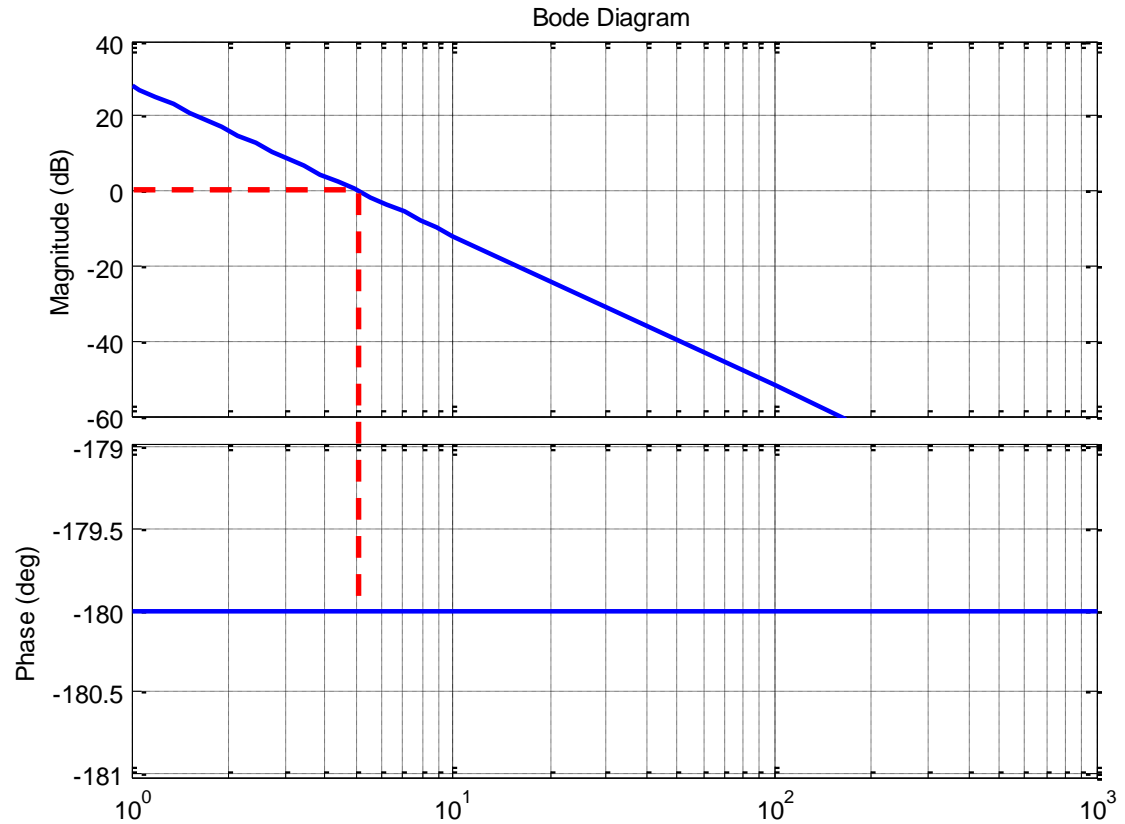
$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=6} = -3$$



$$k = 25.5$$

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s) = \frac{25.5}{s^2}$$



Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ \quad \longrightarrow \quad \sin 45^\circ = \frac{a-1}{a+1} \quad \longrightarrow \quad a = 5.8$$

Step 5: Put the center of controller in the new gain crossover frequency:

$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} + 10 \log(a) = 0$$

$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} = -10 \log(a) = -7.63$$

$$\omega_c^{new} = 8$$



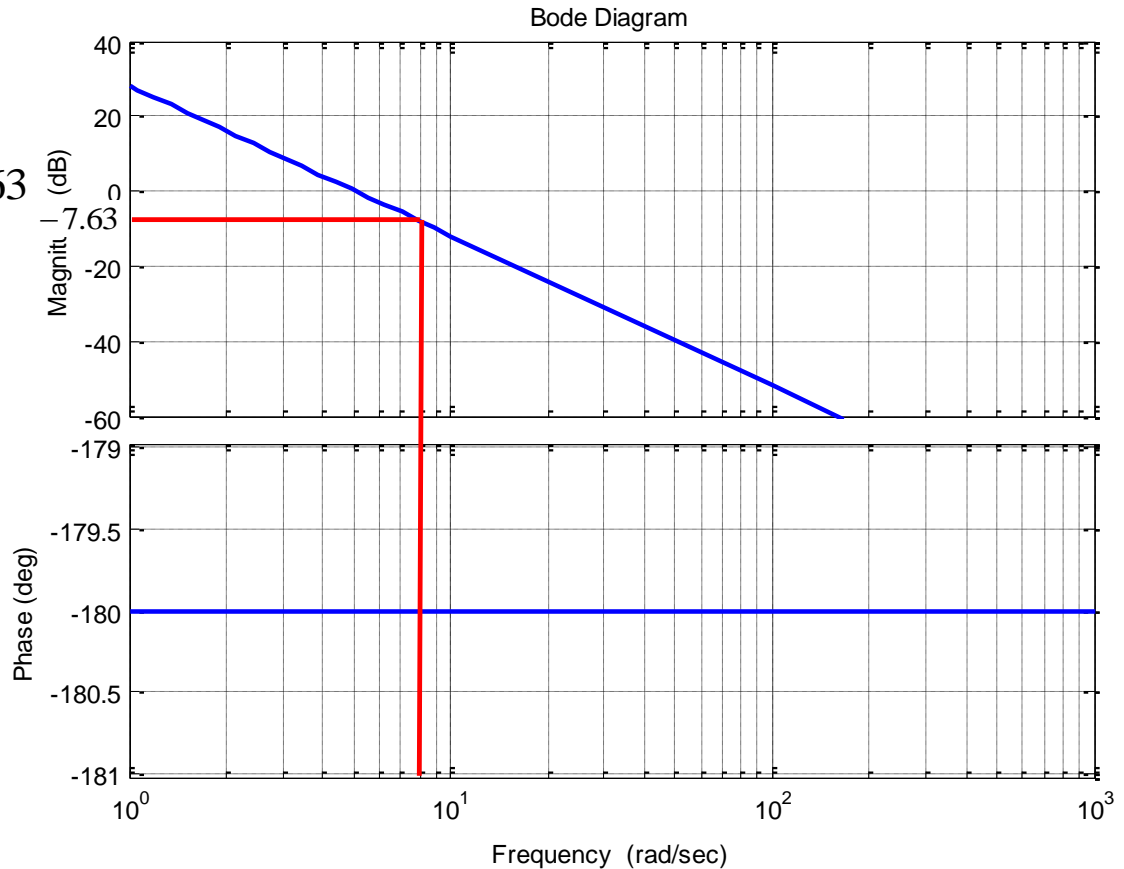
$$\omega_c^{new} = \frac{1}{\tau \sqrt{a}}$$



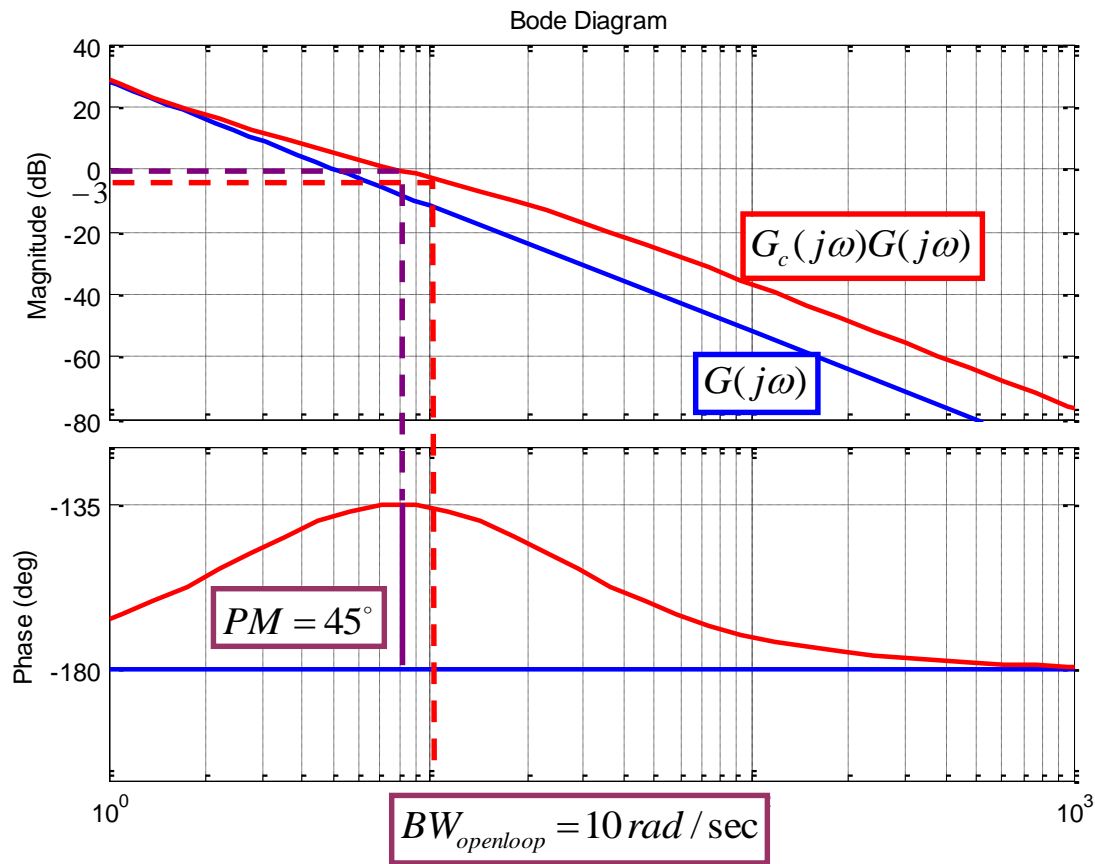
$$\tau = 0.0519$$



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.301s + 1}{0.0519s + 1}$$



Step 6: Check the controller.



Controller is ok

