Computer Control of Dynamic Systems

Lecture-5

STATE ESTIMATION

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STATE ESTIMATION

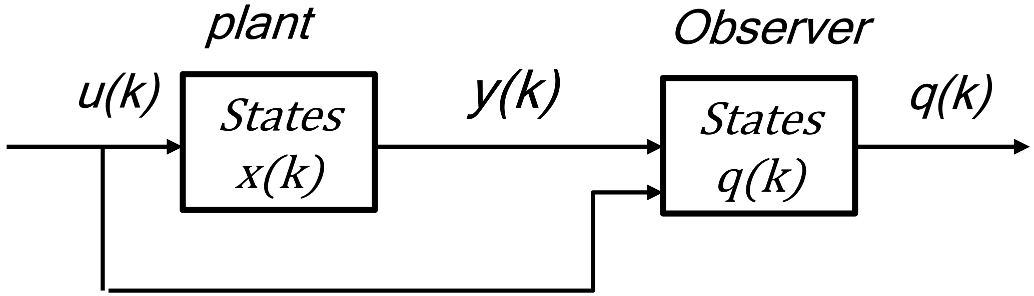
- In the previous Sections a design technique was developed which requires that all the plant states be measured.
- In general, the measurement of all the plant states is impractical, if not impossible, in all but the simplest of systems.
- In this section we develop a technique for estimating the states of a plant from the information that is available concerning the plant.
- The system that estimates the states of another system is generally called an ***observer***, or a ***state estimator***.

STATE ESTIMATION

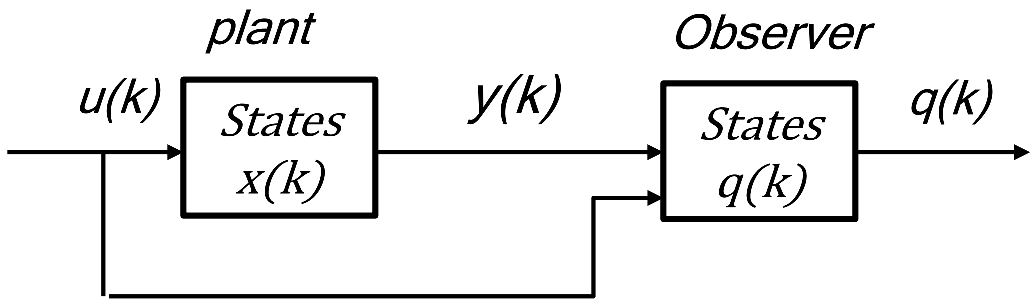
- The plant is described by:

$$x\,(k+1) = A\,x\,(k) + B\,u(k)$$
$$y(k) = C\,x\,(k)$$

- *Y(k):* the plant signals that will be measured.
- A, B, C, *Y(k)* and *U(k)* are known.
- The problem of observer design can be depicted as shown in Figure.



- where the observer is a set of difference equations to be solved by a digital computer.



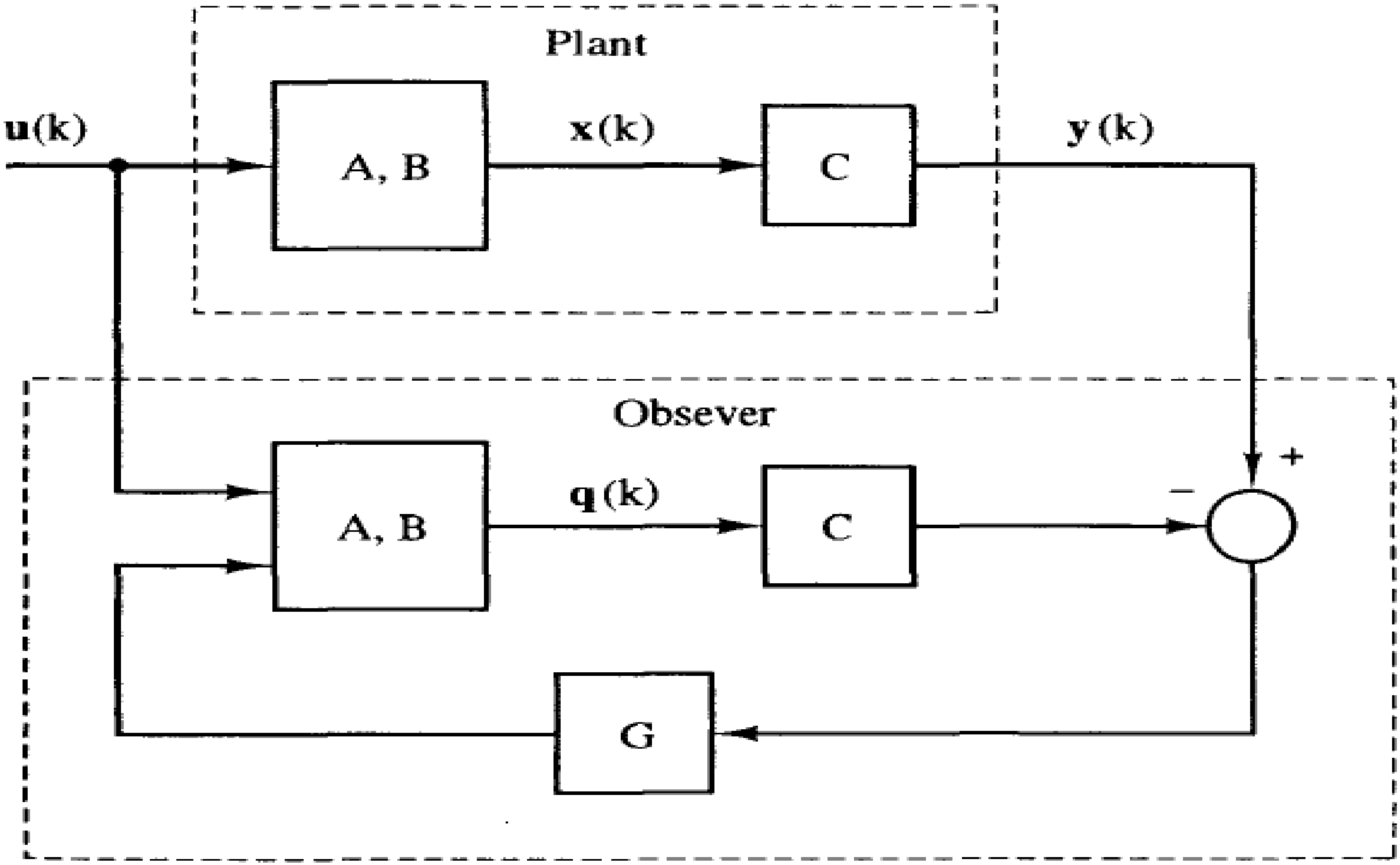
- *x(k):* The states of the system to be observed.

- *q(k):* the states of the observer.

- We desire that *q(k)* be approximately equal to *x(k).*

- Since the observer will be implemented on a computer, the signals *q(k)* are then available for feedback calculations.

Observer Model



Observer Model

- Equations describing observers can be developed in several different ways.

- We will choose a ***transfer-function approach***.

- The observer design criterion to be used is that the

  TF from $u(k)$ to $q_i(k)$ = ***TF*** from $u(k)$ to $x_i(k)$

  for $i = 1,2, \ldots ,n.$

Original system state equations

- Original system

$$x\,(k+1) = A\,x\,(k) + B\,u(k)$$

- Z transform

$$Z\,X\,(z) = A\;X(z) + B\,U(z)$$

$$X\,(z) = (ZI - A)^{-1}\,B\,U(z) \quad .....(1)$$

Observer state equations

- Since the observer has two inputs, $y(k)$ and $u(k)$, we can write the observer state equations as

$$q(k+1) = F\,q(k) + G\,y(k) + H\,u(k)$$

- Z transform

$$Z\,Q(z) = F\,Q(z) + GY(z) + H\,U(z)$$

$$Q(z) = (ZI - F)^{-1}\,[GY(z) + H\,U(z)]$$

$$Q(z) = (ZI - F)^{-1} [G\,Y(z) + H\,U(z)]$$

- *From original Sys.*

$$Y(z) = C\,X(z)$$

$$X(z) = (ZI - A)^{-1} B\,U(z) \ldots\ldots (1)$$

- *so*

$$Q(z) = (ZI - F)^{-1} [G\,C\,X(z) + H\,U(z)]$$

$$Q(z) = (ZI - F)^{-1} [G\,C\,(ZI - A)^{-1} B + H]\,U(z) \ldots. (2)$$

- *Equate (1) and (2) {we need $X(z) = Q(z)$}*

- *Equate (1) and (2) {we need X (z)= Q (z)}*

$$(ZI - A)^{-1} B = (ZI - F)^{-1} [GC (ZI - A)^{-1} B + H]$$

$$= (ZI - F)^{-1} GC (ZI - A)^{-1} B + (ZI - F)^{-1} H$$

$$[I - (ZI - F)^{-1} GC] (ZI - A)^{-1} B = (ZI - F)^{-1} H$$

$$(ZI - F)^{-1} [ZI - (F+GC)] (ZI - A)^{-1} B = (ZI - F)^{-1} H$$

$$(ZI - A)^{-1} B = [ZI - (F+GC)]^{-1} H$$

$$(ZI - A)^{-1} B = [ZI - (F+GC)]^{-1} H$$

- *For the observer states q(k) to be equal to the system states x(k), the following conditions must be satisfied*

➢ $H = B$

➢ $A = F + GC$

$$F = A - GC$$

- *State dynamic equation of observer*

$$q(k+1) = F\, q(k) + G\, y(k) + H\, u(k)$$

$$q(k+1) = (A-G\,C)\, q(k) + G\, y(k) + B\, u(k)$$

Prediction observer

- *System matrix of observer* $= A_{Fo} = A - G\,C$
- *The characteristic equation* $|z\,I - A + G\,C| = 0$

Errors in Estimation

- *Define the error vector e(k) as:*

$$e(k) = x(k) - q\,(k)$$

$$e(k+1) = x(k+1) - q\,(k+1)$$

$$= A\,x(k) + Bu(k) - [\,(A - G\,C)\,q(k) + G\,C\,x(k) + B\,u(k)]$$

$$= (A - G\,C)\,[x(k) - q(k)]$$

$$e(k+1) = (A - GC)\,e(k)$$

- *Hence the error dynamics have a characteristic equation given by* $|z\,I - (A - GC)| = 0$ *which is the same as that of the observer.*

Example

$$x\,(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} x(k)$$

$$y\,(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \; x(K)$$

- *designed an observer for the system with the gain matrix*

$$K = \begin{bmatrix} 4.52 & 1.12 \end{bmatrix}$$

- *choose T = 0.1 sec.*
- *choose* τ *of observer poles as 0.5 sec. so real poles* $\rightarrow$ $S_{1,2}$ *= -2 , -2*

$$Z = e^{+TS}\ ,\ \ T = 0.1\ sec.$$

$$Z_{1,2} = e^{0.1(-2)} = e^{-0.2} = 0.819$$

- *Desired c/c equation in Z.*

$$(Z - 0.819)^2 = Z^2 - 1.638\,Z + 0.671 = 0 \ldots\ldots(1)$$

- *observer c/cs equation* $|ZI - A + GC| = 0$

$$G\,C = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} G_1 & 0 \\ G_2 & 0 \end{bmatrix}$$

$$| ZI - (A + GC) | =$$

$$\left| \begin{bmatrix} Z - 1 + G_1 & -0.0952 \\ +G_2 & Z - 0.905 \end{bmatrix} \right|$$

$$(Z - 1 + G_1)(Z - 0.905) + 0.0952\, G_2 = 0 \ldots\ldots.(2)$$

- *Equate the coefficients of (1) & (2) we get* G_1 *,* G_2

Using Ackermann's Formula

$$G = \propto_e (A)\, Q_0^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- *Closed loop c/c equation (from last example) .*

$$\propto (Z) = Z^2 - 1.776\, Z + 0.819 = 0$$

$$\propto_e (A) = \propto_e (Z)|_{Z \rightarrow A} = A^2 - 1.638A + 0.671\, I$$

$$= \begin{bmatrix} 0.033 & 0.0254 \\ 0 & 0.00763 \end{bmatrix}$$

$$Q_0^{-1} = \begin{bmatrix} C \\ CA \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 0.0952 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -10.51 & 10.51 \end{bmatrix}$$

$$G = \propto_e (A)\ Q_0^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.033 & 0.0254 \\ 0 & 0.00763 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -10.51 & 10.51 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.033 & 0.0254 \\ 0 & 0.00763 \end{bmatrix} \begin{bmatrix} 0 \\ 10.51 \end{bmatrix}$$

$$= \begin{bmatrix} 0.267 \\ 0.0802 \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

- *The system matrix of the estimator is given by*

$$F = A - GC = \begin{bmatrix} 0.733 & 0.0952 \\ -0.0802 & 0.905 \end{bmatrix}$$

- *The estimator's state equation*

$$q(k+1) = (A\text{-}G\,C)\,q(k) + G\,y(k) + B\,u(k)$$

$$= \begin{bmatrix} 0.733 & 0.0952 \\ -0.0802 & 0.905 \end{bmatrix} q(k) + \begin{bmatrix} 0.267 \\ 0.0802 \end{bmatrix} y(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

- *But*

$$u(k) = -K\,q(k) = -\begin{bmatrix} 4.52 & 1.12 \end{bmatrix} q(k)$$

$$q(k+1) = \begin{bmatrix} 0.711 & 0.0898 \\ -0.510 & 0.798 \end{bmatrix} q(k) + \begin{bmatrix} 0.267 \\ 0.0802 \end{bmatrix} y(k)$$

End Of Lecture