



AAST-Cairo
EC210
Solid State Electronics
Spring 2014

Electron Equivalent Wavelength and
Introduction to Schrodinger Wavefunction



Kasap:

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Young's Experiment

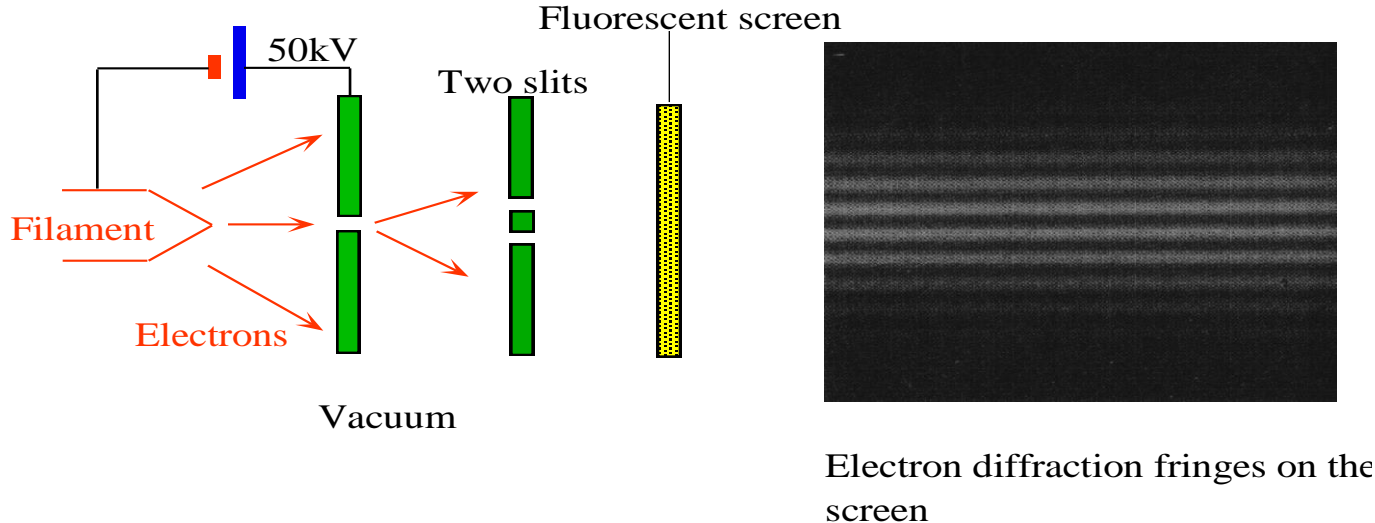
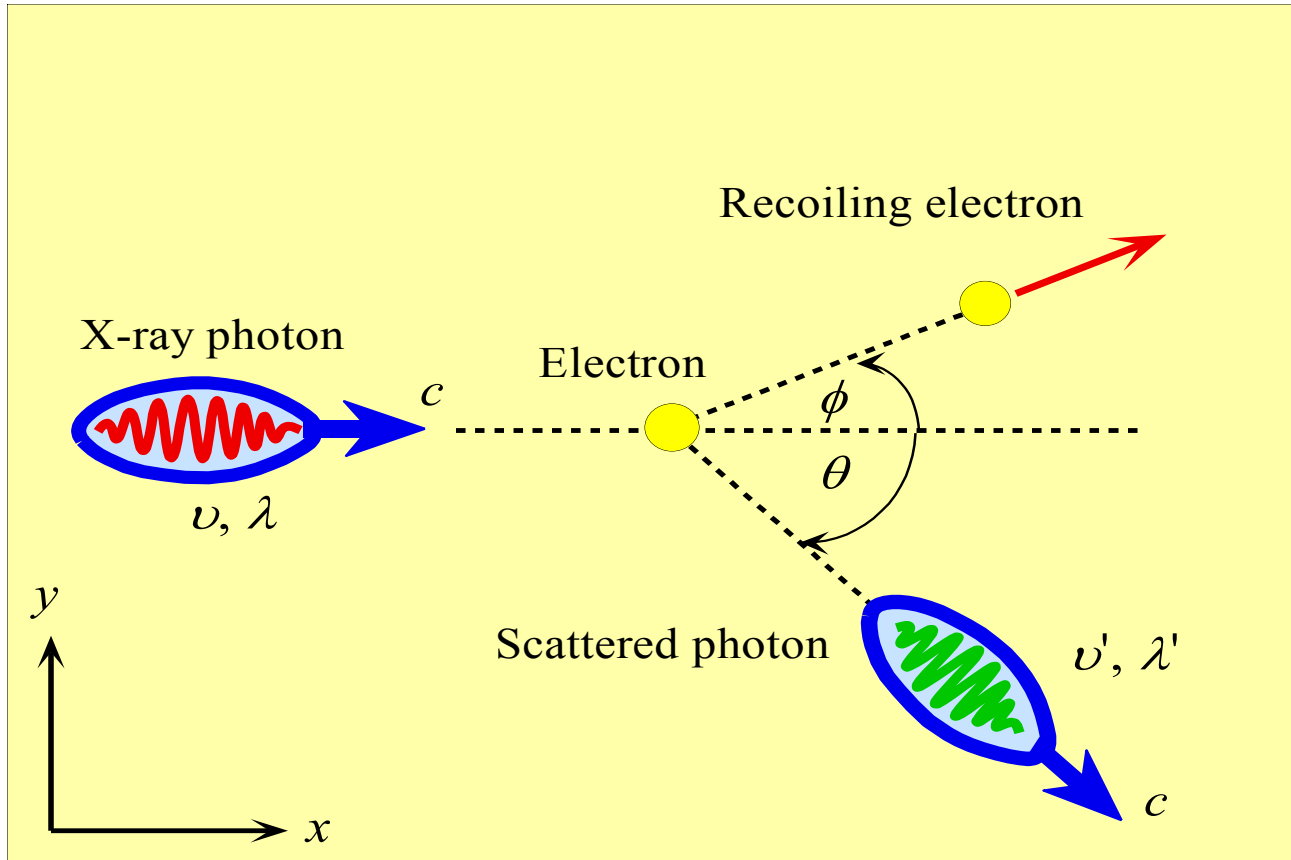


Fig 3.12: Young's double slit experiment with electrons involves an electron gun and two slits in a cathode ray tube (CRT) (hence in vacuum). Electrons from the filament are accelerated by a 50 kV anode voltage to produce a beam which is made to pass through the slits. The electrons then produce a visible pattern when they strike a fluorescent screen (e.g. a TV screen) and the resulting visual pattern is photographed (pattern from C. Jönsson, D. Brandt, S. Hirschi, *Am. J. Physics*, **42**, Fig. 8, p. 9, 1974).

From *Principles of Electronic Materials and Devices, Second Edition*, S.O. Kasap (© McGraw-Hill, 2002)
<http://Materials.Usask.ca>

Compton's Scattering



Scattering of an x-ray photon by a "free" electron in a conductor.

Fig 3.9



Compton Scattering

- Kinetic Energy of Scattered *electron*
 - $KE = \frac{1}{2} m v_e^2 = h\nu - h\nu'$
 - ν is the frequency of light f , v_e is the electron velocity
- Momentum of *photon*
 - $p = h / \lambda$
 - $\lambda = 2\pi / k$
- Energy of photon
 - $E = h\nu = (h/2\pi) \omega$



Schrödinger equation: Wavefunction

- The Schrödinger equation plays the role of Newton's laws in classical mechanics
- Explains the wave-like nature of electrons using photon-like concepts

Assuming:

$$\psi(x, t) = \psi(x) \phi(t) = \psi(x) e^{-i\omega t} = \psi(x) e^{-i\frac{E}{\hbar}t}$$

$$e^{-i\theta} = \cos \theta + i \sin \theta$$



1-D, time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

- **$V(x)$: Potential Energy**
- **E : Total energy**
- **Equivalent of energy conservation equation in classical mechanics.**
- **Predicts the shape of the wave function.**
- **System is defined by potential energy, boundary conditions**



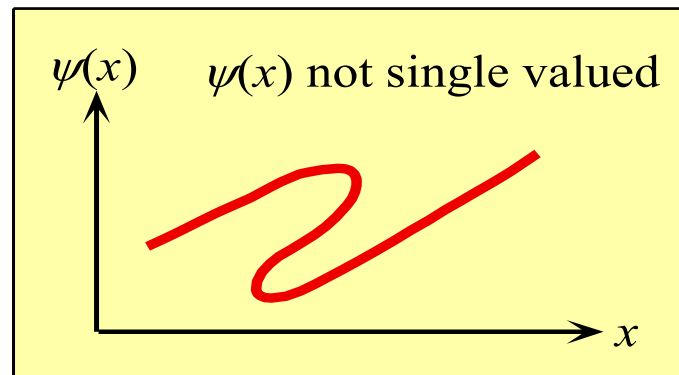
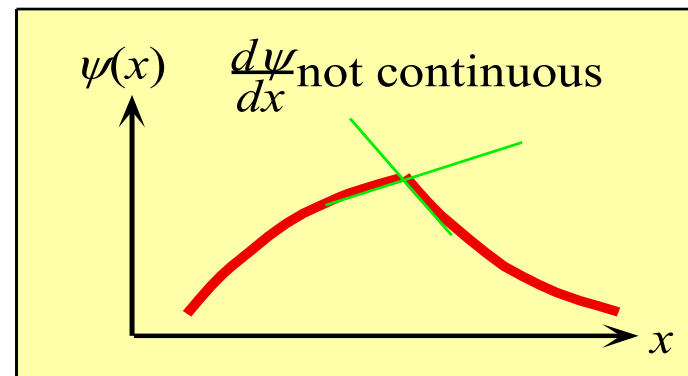
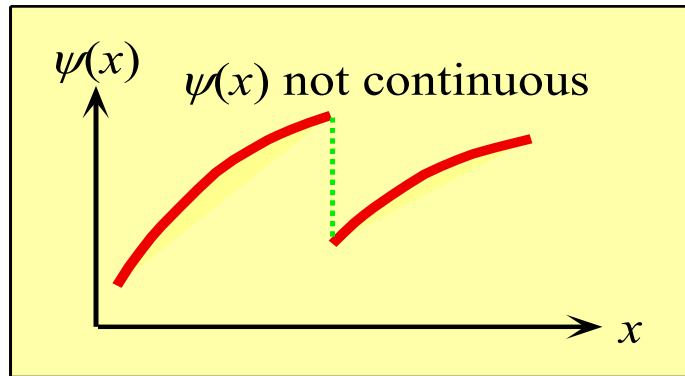
Stationary Wavefunction

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

Normalization Condition



Unacceptable Forms of Wavefunctions



Unacceptable forms of $\psi(x)$

Fig 3.14



Applications

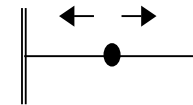
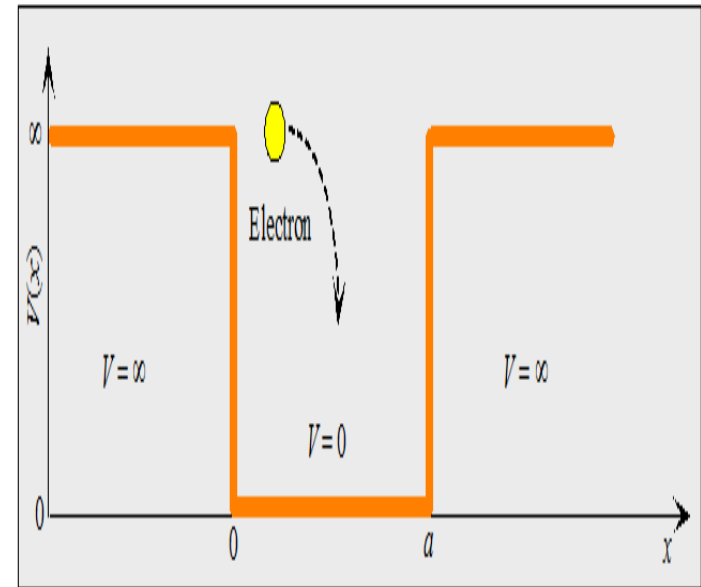
(1) Particle in an Infinite Potential Well: Particle in a Box (1-D):

A particle of mass m is completely trapped in a region and is moving freely between $x=0$ and $x=L$

- $V(x)=0$, if $0 \leq x \leq a$
- $V(x)=\infty$, if $x < 0$ or $x > a$
- **Boundary conditions on ψ :**

Ψ must be continuous across any boundary

$$\Psi(0)=0=\Psi(L)$$





To find the value of A:

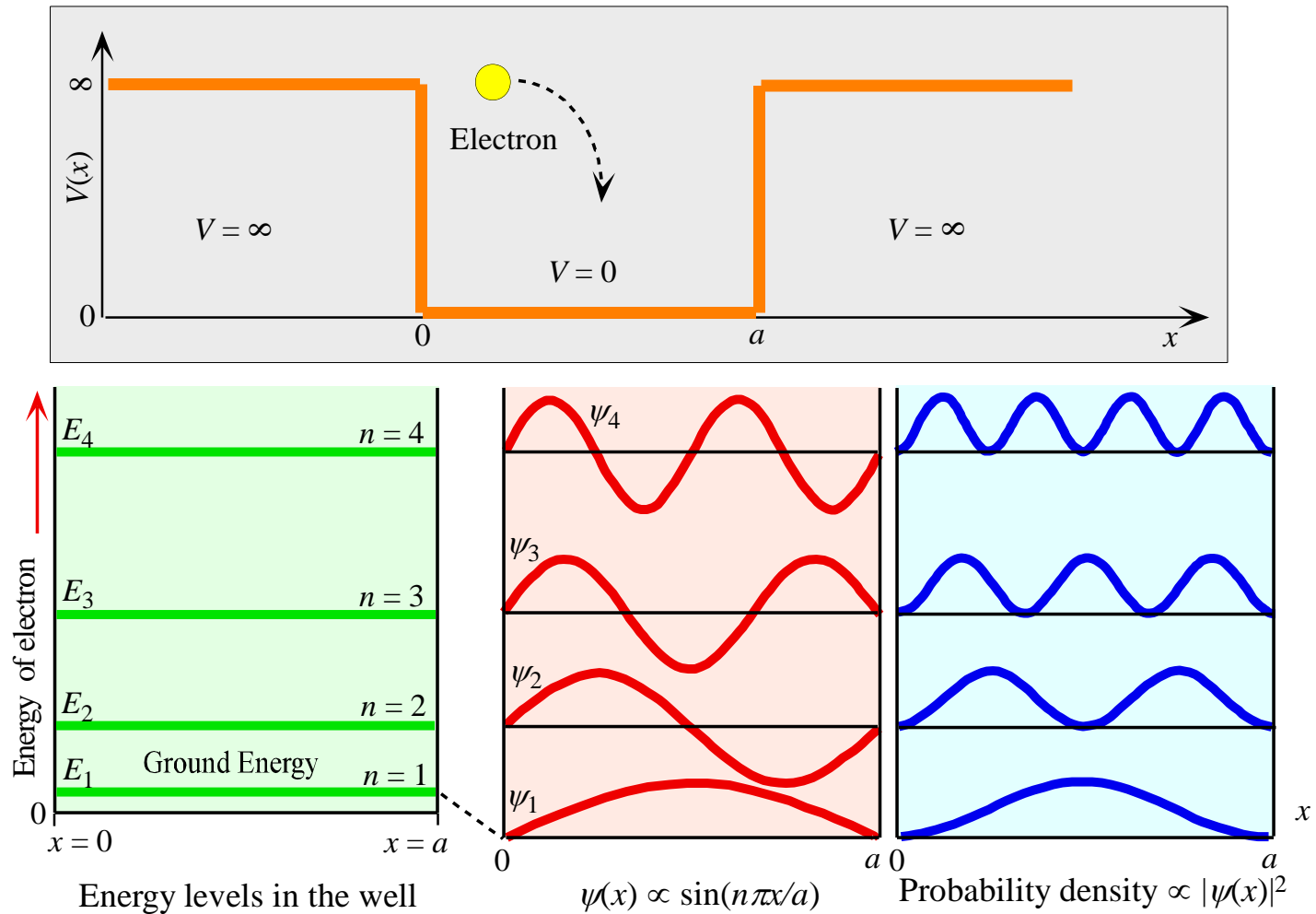
Probability to find the particle in $0 \leq x \leq a$ is 1:

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = \int_0^a |\psi(x)|^2 dx = 1$$

$$\psi_n(x) = j \sqrt{\frac{2}{a}} \sin(k_n x)$$

wave function

$$k_n = \frac{n\pi}{a}$$



Electron in a one-dimensional infinite PE well. The energy of the electron is quantized. Possible wavefunctions and the probability distributions for the electron are shown.

Fig 3.15