

**Table (3): The Fourier Transform Pairs & Properties**

**Table (3-a): Summary of Properties of the Fourier Transform**

Property	Mathematical Description
1. Linearity	$a g_1(t) + b g_2(t) \iff a G_1(f) + b G_2(f)$ where $a$ and $b$ are constants
2. Time Scaling	$g(at) \iff \frac{1}{ a } G\left(\frac{f}{a}\right)$ where $a$ is a constant
3. Duality	If $g(t) \iff G(f)$ , then $G(t) \iff g(-f)$
4. Time shifting	$g(t - t_0) \iff G(f) e^{-j2\pi f t_0}$
5. Frequency shifting	$g(t) e^{j2\pi f_c t} \iff G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t) dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f) df$
8. Differentiation in time domain	$\frac{d}{dt} g(t) = j2\pi f G(f)$
9. Integration in time domain	$\int_{-\infty}^t g(\tau) d\tau = \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$
10. Conjugate functions	If $g(t) \iff G(f)$ , then $g^*(t) \iff G^*(-f)$
11. Multiplication in the time domain	$g_1(t) g_2(t) \iff \int_{-\infty}^{\infty} G_1(\lambda) G_2(f - \lambda) d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau) d\tau \iff G_1(f) G_2(f)$

Notes :  $u(t)$  = Unit Step function  
 $\delta(t)$  = Dirac Delta function  
 $\text{rect}(t)$  = Rectangular function  
 $\text{Tri}(t)$  = Triangular function  
 $\text{sgn}(t)$  = Signum function  
 $\text{sinc}(t)$  = sinc function

**Table (3-b): Fourier Transform Pairs**

Time Function	Fourier Transform
$\text{rect}\left(\frac{t}{T}\right)$	$T \text{sinc}(f T)$
$\text{sinc}(2Wt)$	$\frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right)$
$e^{-at} u(t), a > 0$	$\frac{1}{a + j2\pi f}$
$e^{-a t }, a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$e^{-\pi t^2}$	$e^{-\pi f^2}$
$\text{tri}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T}, &  t  < T \\ 0, &  t  \geq T \end{cases}$	$T \text{sinc}^2(f T)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$e^{-j2\pi f t_0}$
$e^{j2\pi f_c t}$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2} [\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j} [\delta(f - f_c) - \delta(f + f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j \text{sgn}(f)$
$u(t)$	$\frac{1}{2} \delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$