Electrical Machines II

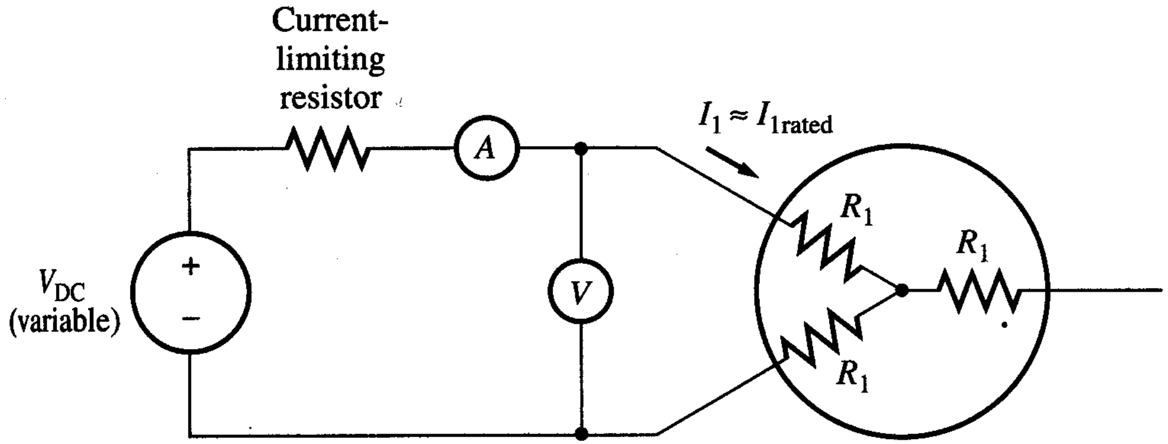
Week 8: Induction motor tests, Maximum power, maximum torque and maximum efficiency criterion

Determination of motor parameters

- Due to the similarity between the induction motor equivalent circuit and the transformer equivalent circuit, same tests are used to determine the values of the motor parameters.
  - DC test: determine the stator resistance R_s
  - No-load test: determine the rotational losses and magnetization current (similar to no-load test in Transformers).
  - Locked-rotor test: determine the rotor and stator impedances (similar to short-circuit test in Transformers).

DC test

- The purpose of the DC test is to determine R_s. A variable DC voltage source is connected between two stator terminals.
- The DC source is adjusted to provide approximately rated stator current, and the resistance between the two stator leads is determined from the voltmeter and ammeter readings.



$$R_{DC} = \frac{V_{DC}}{I_{DC}}$$

- If the stator is Y-connected, the per phase stator resistance is

$$R_1 = \frac{R_{DC}}{2}$$

- If the stator is delta-connected, the per phase stator resistance is

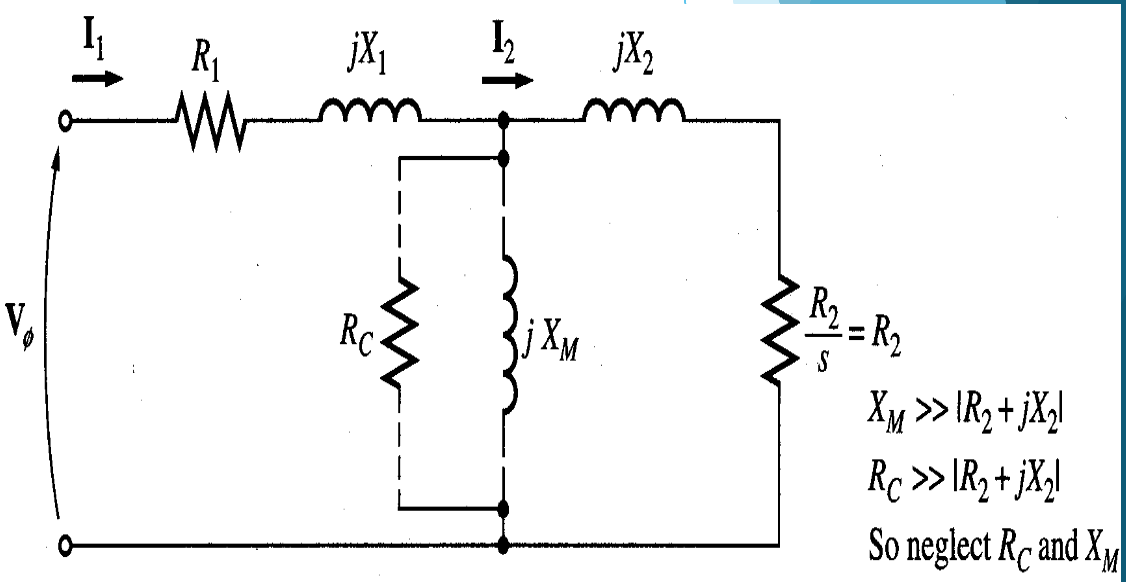
$$R_1 = \frac{3}{2} R_{DC}$$

The measured value of the resistance may be multiplied by a factor ranging from 1.05 to 1.25 in order to convert it from its dc value to its ac value. This is done to account for the skin effect.

Blocked-rotor test

- In this test, the rotor is locked or blocked so that it cannot move.
- The stator field winding is connected to a variable three-phase supply. The voltage is carefully increased from zero to a level at which the motor draws the rated current. At this time, the readings of the line current, the applied line voltage, and the power input are taken by using the two-wattmeter method,

Since the rotor-circuit impedance is relatively small under blocked-rotor condition (s = 1), the applied voltage is considerably lower than the rated voltage of the motor. Thus, the excitation current is quite small and can be neglected. Under this assumption, the approximate equivalent circuit of the motor is given:



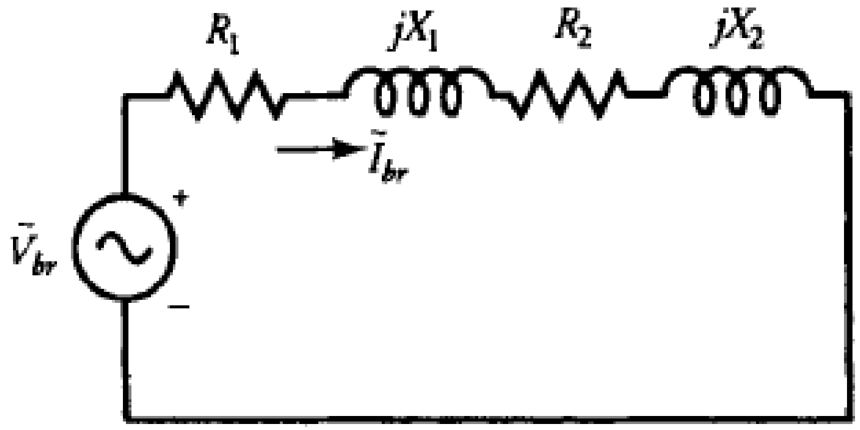
$$Z_e = R_s + R_r + j(X_s + X_r) = R_e + jX_e$$

$$Z_e = R_1 + R_2 + j(X_1 + X_2) = R_e + jX_e$$

Blocked-rotor test

Since R_s is already known from the stator-resistance test, the equivalent rotor resistance is

$$R_r = R_2 = R_e - R_s$$



However,

$$Z_{br} = \frac{V_{br}}{I_{br}}$$

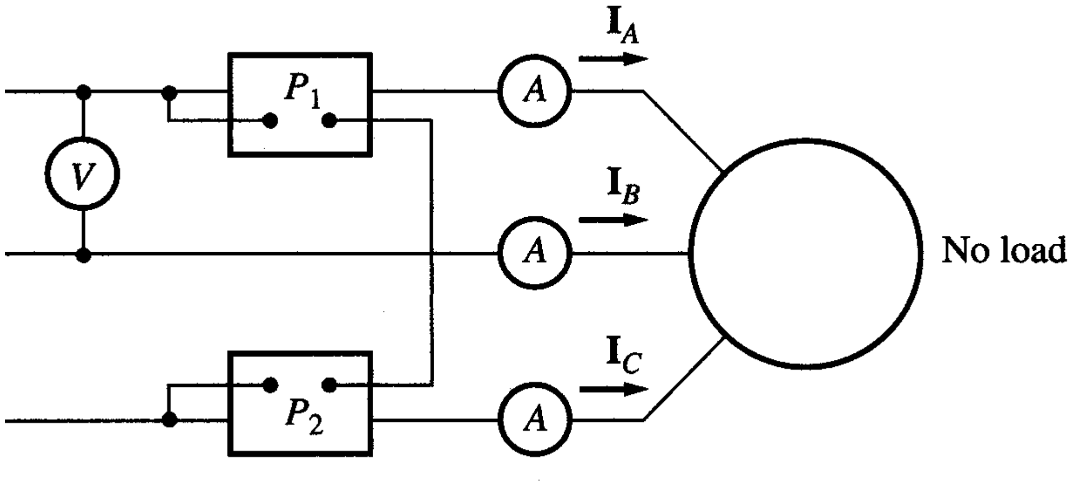
Therefore,

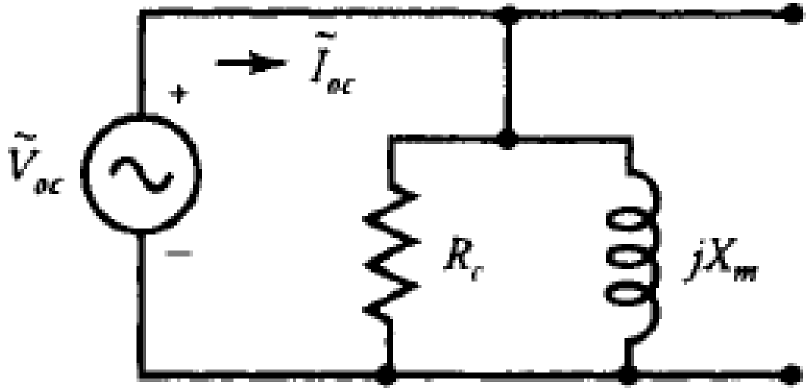
$$X_e = \sqrt{{Z_{br}}^2 - {R_e}^2}$$

It is rather difficult to isolate the leakage reactances X_1 and X_2. For all practical purposes, these reactances are usually assumed to be equal. That is

$$X_1 = X_2 = 0.5X_e$$

No-load test





- In this case the rated voltage is impressed upon the stator windings and the motor operates freely without any load. This test, therefore, is similar to the open-circuit test on the transformer except that friction and windage loss is associated with an induction motor.
- Since the slip is nearly zero, the impedance of the rotor circuit is almost infinite. The per-phase approximate equivalent circuit of the motor with the rotor circuit open is shown.

No-load test

- In order to represent the core loss by an equivalent resistance R_c **we must subtract the friction and windage loss from the power input**. The friction and windage loss can be measured by coupling the motor under test to another motor with a calibrated output and running it at the no-load speed of the induction motor. Let $P_{fw\varphi}$ be the friction and windage loss on a per-phase basis. Then, the power loss in R_c, is:

$$P_{oc} = W_{oc} - P_{fw\varphi}$$

Type equation here.

Hence, the core-loss resistance is

$$R_c = \frac{V_{oc}^2}{P_{oc}}$$

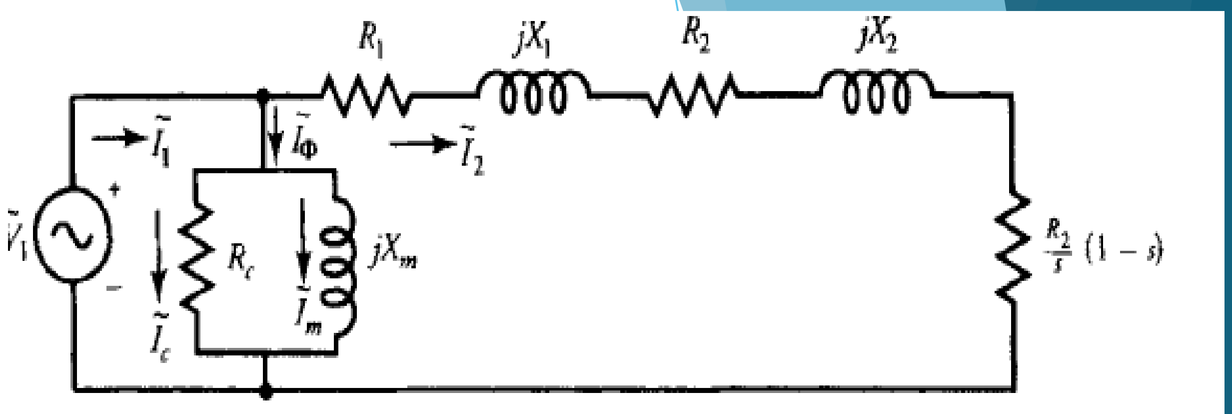
The power factor under no load is

$$\cos \varphi_{oc} = \frac{W_{oc}}{V_{oc} I_{oc}}$$

The magnetization reactance is

$$X_m = \frac{V_{oc}}{I_{oc} \sin \theta_{oc}}$$

Maximum Power Criterion



From the equivalent circuit, the rotor current is

$$I_2 = I_r' = \frac{V_1}{R_e + jX_e + R_2\frac{(1-s)}{s}}$$

The power developed by the three-phase induction motor:

$$P_{dev} = \frac{3V_1^2 R_2 \frac{(1-s)}{s}}{R_e^2 + X_e^2 + \left[R_2\frac{(1-s)}{s}\right]^2 + 2R_eR_2\frac{(1-s)}{s}}$$

From the above equation it is evident that the power developed by a three phase induction motor is a function of slip.

Therefore, we can determine the slip s_p at which the power developed by the motor is maximum by differentiating the above equation and setting the derivative equal to zero. After differentiating and canceling most of the terms, we obtain

$$R_e^2 + jX_e^2 = \left[\frac{R_2}{s_p}\left(1 - s_p\right)\right]^2$$

Maximum Power Criterion

$$Z_e = \frac{R_2}{s_p}(1 - s_p)$$

- where Z_e, is the magnitude of the equivalent impedance of the stator and the rotor windings at rest. That is:

$$Z_e = |R_e + jX_e|$$

- the power developed by a three-phase induction motor is maximum **when the equivalent load (dynamic) resistance is equal to the magnitude of the standstill impedance of the motor**. This, of course, is the well known result we obtained from the maximum power transfer theorem during the study of electrical circuit theory.

- The slip at which the induction motor develops maximum power as: $$s_p = \frac{R_2}{R_2 + Z_e}$$

- Substituting for the slip ,we obtain an expression for the maximum power developed by a three-phase induction motor as

$$P_{dev} = \frac{3}{2}\left[\frac{V_1^2}{R_e + jX_e}\right]$$

The net power output, however, is less than the power developed by an amount equal to the rotational loss of the motor.

Maximum Torque Criterion

- The torque developed by a three-phase induction motor

$$T_{dev} = \frac{3V_1^2 R_2 \dfrac{(1-s)}{s}}{\omega_s \left[R_e^2 + jX_e^2 + \left[R_2 \dfrac{(1-s)}{s}\right]^2 + 2R_e R_2 \dfrac{(1-s)}{s}\right]}$$

Differentiating the above equation with respect to s and setting it equal to zero we obtain an expression for the breakdown slip s_b at which the motor develops the maximum (breakdown) torque as :

$$s_b = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}$$

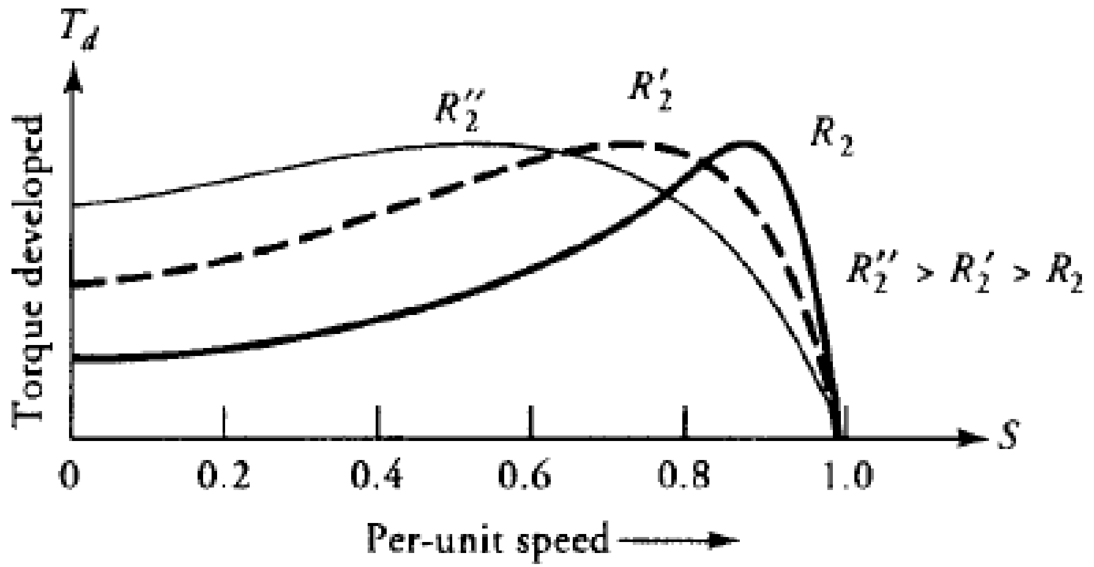
- ❑ Note that the **breakdown slip** is directly proportional to the **rotor** resistance.
- ❑ Since the rotor resistance can be easily adjusted in a wound-rotor induction motor by means of an external resistor, we can obtain the maximum torque at any desired speed, including the zero speed (starting). Substituting the above expression for the breakdown slip we obtain an expression for the maximum torque by the motor as

$$T_{devmax} = \frac{3V_1^2}{2\omega_s}\left[\frac{1}{R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2}}\right]$$

Does not contain rotor resistance term

Maximum Torque Criterion

Note that the maximum torque developed by the motor is independent of the rotor resistance. In other words, the motor develops the same maximum torque regardless of its rotor resistance. The rotor resistance affects only the breakdown slip (or breakdown speed) at which the torque is maximum



Maximum Torque Criterion: Further Approximation

When the **stator impedance is so small** that it can be neglected in comparison with the rotor impedance at standstill, we obtain a very useful expression for the breakdown slip as:

$$s_b = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \xrightarrow{\text{stator impedance neglected}} s_b = \frac{R_2}{X_2}$$

This equation states that the breakdown slip is simply a ratio of **rotor resistance to rotor reactance**. When the rotor resistance is made equal to the rotor reactance, the breakdown slip is unity. In this case, the motor develops the maximum torque at starting. The approximate expression for the breakdown torque becomes:

$$T_{devmax} = \frac{3V_1^2}{2\omega_s}\left[\frac{1}{X_2}\right] = \frac{3V_1^2}{2\omega_s}\left[\frac{s_b}{R_2}\right]$$

The rotor current at any speed when the stator impedance is neglected is:

$$I_2 = \frac{V_1}{\frac{R_2}{s} + jX_2}$$

Maximum Torque Criterion: Further Approximation

The torque developed by the motor at any slip s is

$$T_{dev} = \frac{3I_1^2 R_2}{s\omega_s} = \left[\frac{3V_1^2}{\left({R_2}/{s}\right)^2 + X_2^2}\right]\frac{R_2}{s\omega_s}$$

The ratio of the torque developed at any slip s to the breakdown torque is

$$\frac{T_{dev}}{T_{devmax}} = \frac{2ss_b}{s^2 + s_b^2}$$

Maximum Efficiency Criterion

When the core loss is considered a part of the rotational loss the power input to the motor using the approximate equivalent circuit, is

$$P_{in} = 3V_1 I_2 \cos\theta$$

where θ is the power-factor angle between the applied voltage $V_1$and the rotor current I_2The power output is:

$$P_{op} = 3V_1 I_2 \cos\theta - 3I_2^2\,(R_1 + R_2) - P_r$$

The motor efficiency is

$$\eta = \frac{3V_1 I_2 \cos\theta - 3I_2^2\,(R_1 + R_2) - P_r}{3V_1 I_2 \cos\theta}$$

Differentiating η with respect to I_2, and setting the derivative equal to zero, we obtain

$$3I_2^2(R_1 + R_2) = P_r$$

as the criterion for the maximum efficiency of an induction motor. It simply states that the efficiency of an induction motor is maximum when the sum of the stator and the **rotor copper losses is equal to the rotational loss.**

Torque – Speed Characteristics

$$T_L = \frac{P_{out}}{\omega_m} \qquad T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{P_{gap}}{\omega_s} = \frac{3I_r'^2 R_r'}{\omega_s s}$$

$R_r \Uparrow, I_r \Downarrow$ *at the same rotor emf*, $T_{dev} \Downarrow$

Substituting for I_r' from the equivalent circuit:

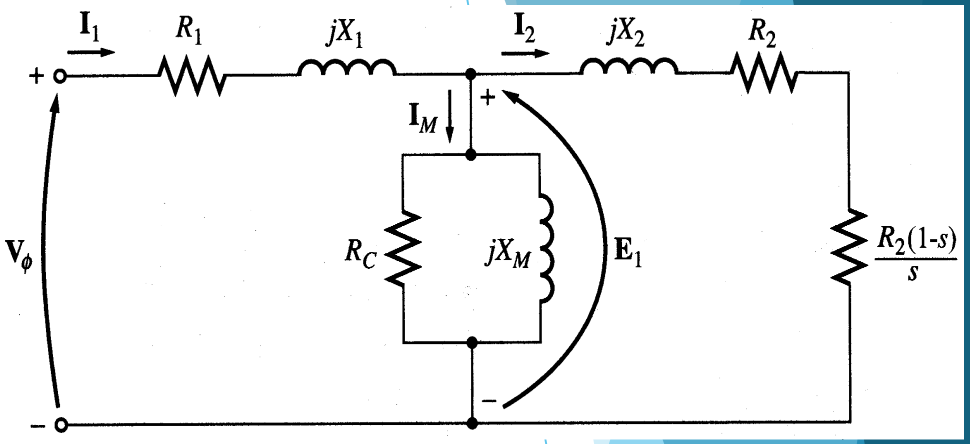
$$T_e = \frac{3R_r'}{s\omega_s} \frac{V_s^{\,2}}{\left[\left(R_s + \frac{R_r'}{s}\right)^2 + \left(X_{ls} + X_{lr}\right)^2\right]}$$

- The equation reveals that the torque developed by an induction motor is directly proportional to the **square of the current** in the rotor circuit and **the equivalent hypothetical resistance** of the rotor.
- However, the two quantities, the rotor current and the hypothetical rotor resistance, **are inversely related** to each other.
- For instance, if the rotor resistance is increased, we expect the torque developed by the motor to increase linearly. But any increase in the rotor resistance is accompanied by a decrease in the rotor current for the same induced emf in the rotor.
- A decrease in the rotor current causes a reduction in the torque developed. Whether the overall torque developed increases or decreases depends upon which parameter plays a dominant role.

Torque – Speed Characteristics

At standstill, the rotor slip is unity (s=1) and the effective rotor resistance is R'_r,. The magnitude of the rotor current is:

$$\left|I_r^{'}\right| = \frac{E_1}{\sqrt{R_r^{'2} + X_r^{'2}}}$$



per-phase exact equivalent circuit of a balanced three-phase induction motor

Note that the rotor winding resistance R'_r, **is usually very small** compared with its **leakage reactance X'_r**,. That is, R'_r, <<X'_r

The starting torque developed by the motor is:

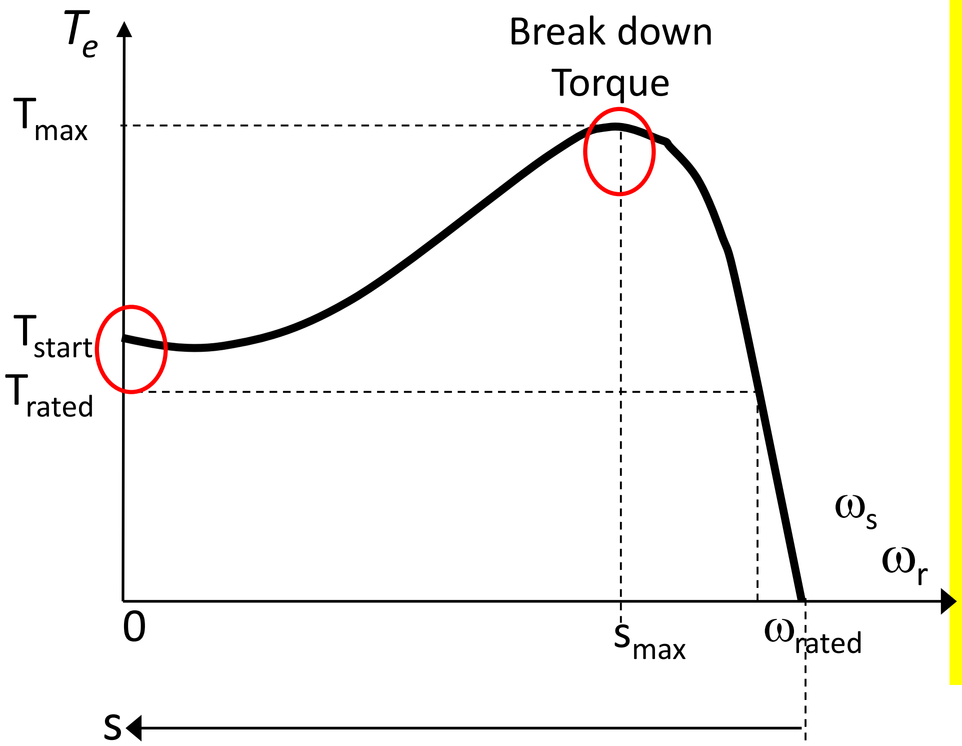
At start, rotor speed is 0, slip is 1

$$T_{start} = \frac{3I_r^{'2} R_r^{'}}{\omega_s}$$

As the rotor starts rotating, an increase in its speed is accompanied by a decrease in its slip. As *s* decreases, speed increases. **As long as R'_r/s is smaller than X'_r, the reduction in the rotor current is minimal**. Thus, in this speed range, the rotor current may be approximated as

$$I_r^{'} \approx \frac{E_1}{X_r^{'}}$$

Torque – Speed Characteristics



- Since the rotor current is almost constant, the torque developed by the motor increases with the increase in the effective resistance R'_r/s. **Thus, the torque developed by the motor keeps increasing with the decrease in the slip as long as the rotor resistance has little influence on the rotor current.**
- When the slip falls below a certain value called the breakdown slip s_b, the hypothetical resistance becomes the dominating factor. In this range, $R'_r/s >> X'_r$, and the rotor current can be approximated as

$$I'_r \approx \frac{sE_1}{R'_r}$$

- The **torque developed by the motor is now proportional to the slip s**. As the slip decreases, so does the torque developed.
- At no load, the slip is almost zero, the hypothetical rotor resistance is nearly infinite, **the rotor current** is approximately **zero**, and the **torque developed is virtually zero**. With this understanding, we are able to sketch the speed-torque curve of an induction motor. Such a curve is depicted in Figure