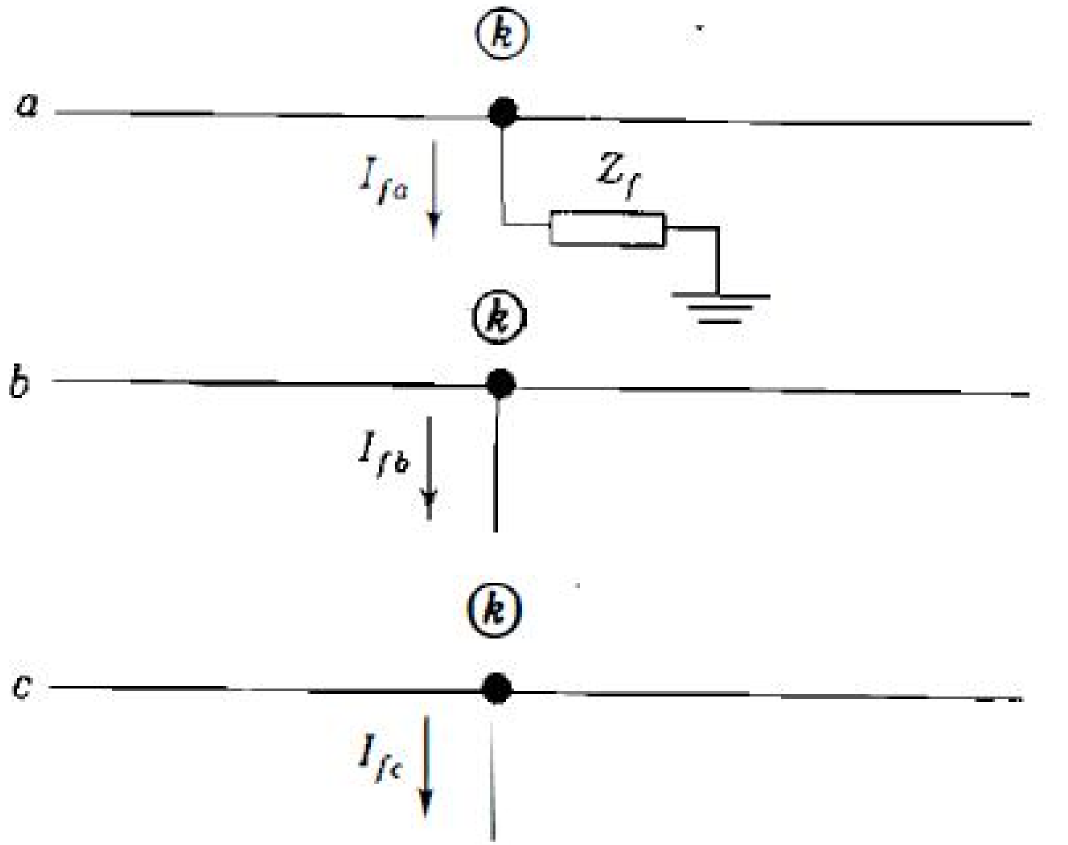
Single Line to Ground Fault



$$I_{fb} = 0 \qquad I_{fc} = 0 \qquad V_{ka} = Z_f I_{fa}$$

$$\begin{bmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix}$$

$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{I_{fa}}{3}$$

$$V_{ka}^{(0)} = \quad - Z_{kk}^{(0)} I_{fa}^{(0)}$$

$$V_{ka}^{(1)} = V_f - Z_{kk}^{(1)} I_{fa}^{(0)}$$

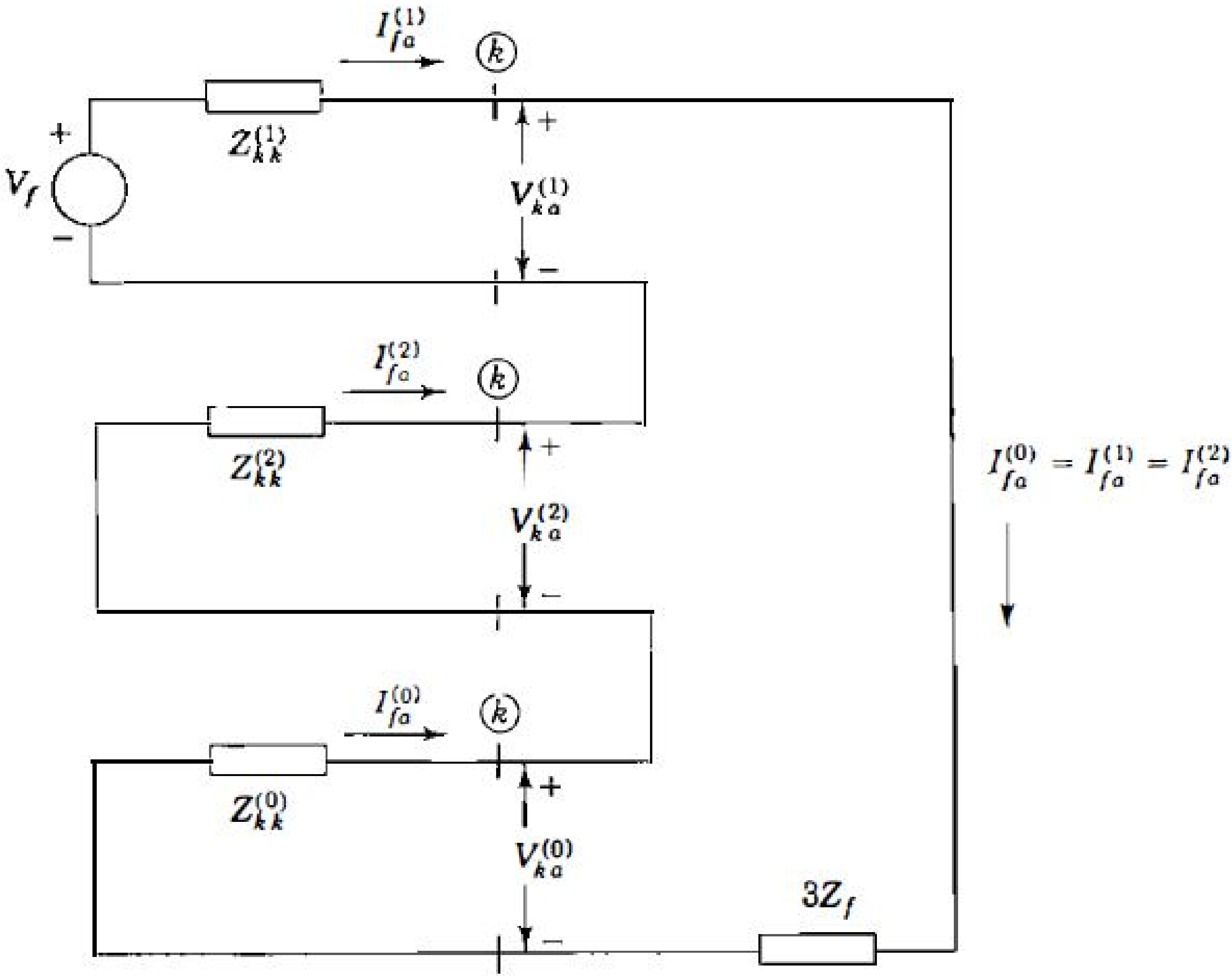
$$V_{ka}^{(2)} = \quad - Z_{kk}^{(2)} I_{fa}^{(0)}$$

Summing these equations and noting that $V_{ka} = 3Z_f I_{fa}^{(0)}$ give

$$V_{ka} = V_{ka}^{(0)} + V_{ka}^{(1)} + V_{ka}^{(2)} = V_f - \left(Z_{kk}^{(0)} + Z_{kk}^{(1)} + Z_{kk}^{(2)}\right) I_{fa}^{(0)} = 3Z_f I_{fa}^{(0)}$$

Solving for $I_{fa}^{(0)}$

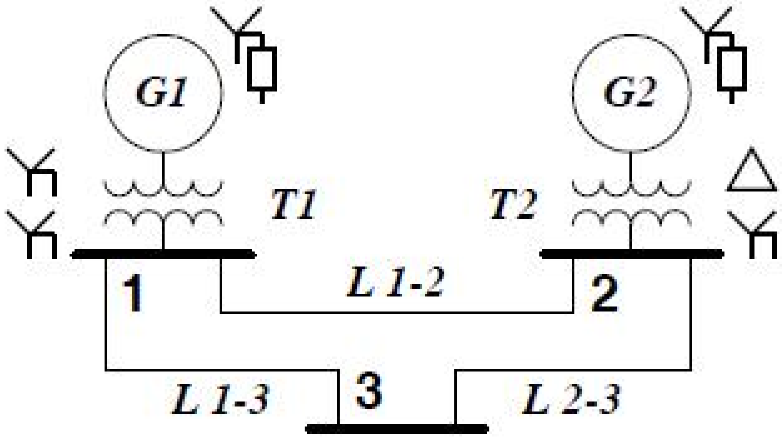
$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}$$



$I_{fa}^{(1)}$
k
$Z_{kk}^{(1)}$
+
V_f
−
+
$V_{ka}^{(1)}$
−
$I_{fa}^{(2)}$
k
$Z_{kk}^{(2)}$
+
$V_{ka}^{(2)}$
−
$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)}$
$I_{fa}^{(0)}$
k
$Z_{kk}^{(0)}$
+
$V_{ka}^{(0)}$
−
$3Z_f$

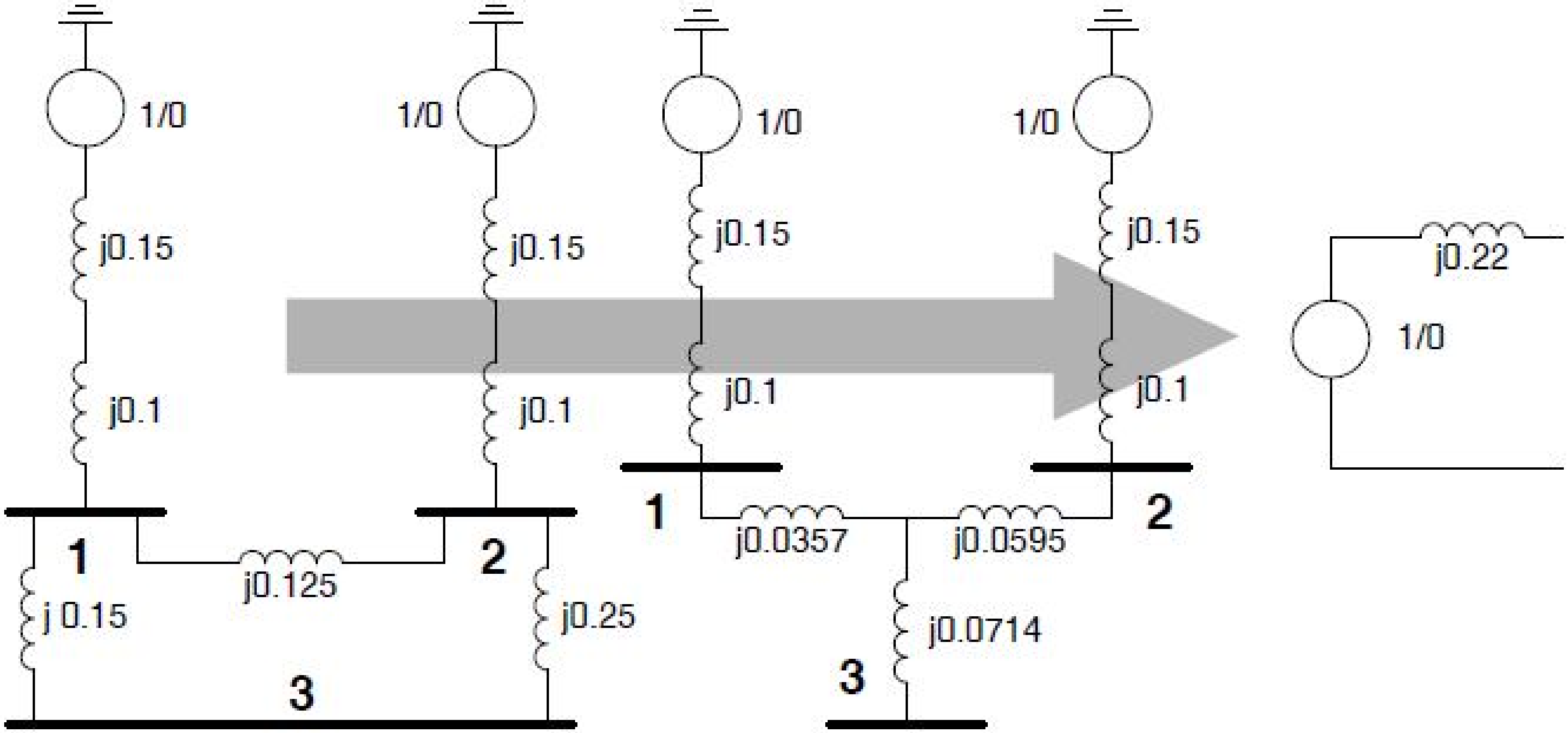
Example

- The neutral of each generator is grounded through a current limiting resistor of 8.333 % on a 100 MVA base
- generators are running at no-load at rated voltage and in phase
- all network data is expressed on a 100 MVA base
- Find the fault current for 3-phs, 1-phs, At Bus Bar 3

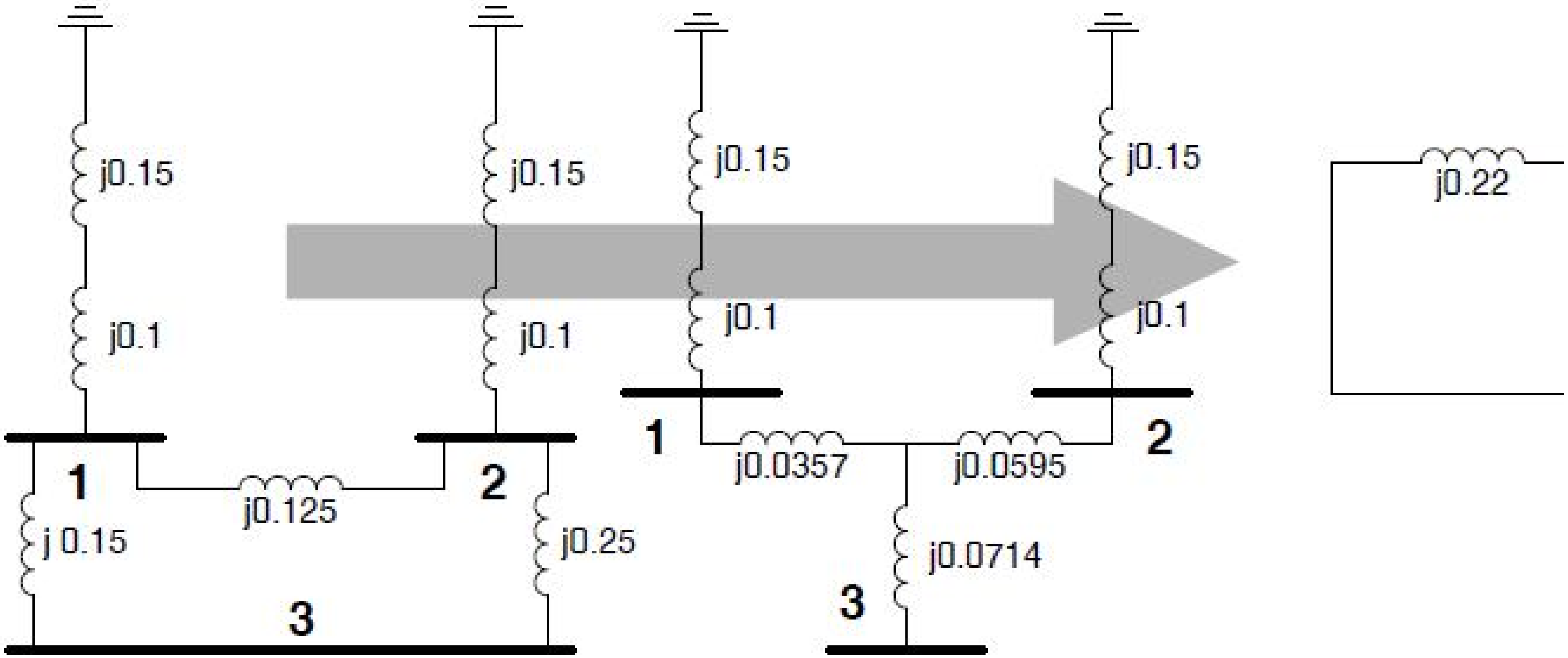


Item	V Rating	X1	X2	X0
G1	20 kV	15%	15%	5%
G2	20 kV	15%	15%	5%
T1	20/200 kV	10%	10%	10%
T2	20/200 kV	10%	10%	10%
L 1-2	200 kV	12.5%	12.5%	30%
L 1-3	200 kV	15%	15%	35%
L 2-3	200 kV	25%	25%	71.25%

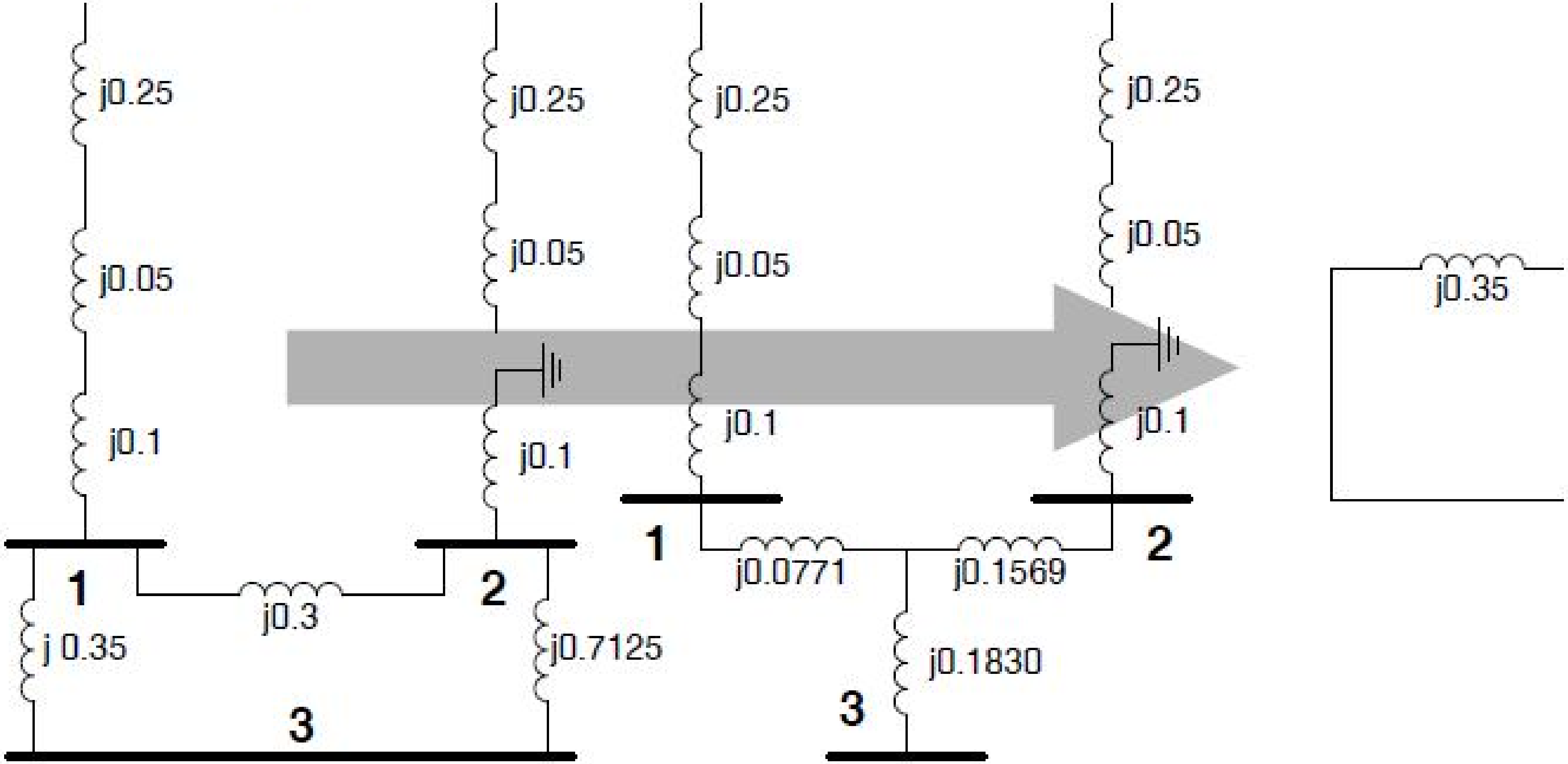
- **Positive Sequence Network**



- **Negative Sequence Network**



- **Zero Sequence Network**



- **3-phase fault**

$$I_3^{a1}(F) = \frac{V_3^{a1}}{Z_{33}^1} = \frac{1.0\angle 0^\circ}{j0.22} = -j4.54 \text{ pu}$$

$$I_{af} = I_3^{a1}(F) = -j4.54 \text{ pu}$$

- **SLG fault**

$$I_3^{a0} = I_3^{a1} = I_3^{a2} = \frac{V_3^{a1}}{Z_{33}^0 + Z_{33}^1 + Z_{33}^2}$$

$$= \frac{1.0\angle 0^\circ}{j0.35 + j0.22 + j0.22} = -j1.266 \text{ pu}$$

$$I_{af} = 3I_3^{a1} = -j3.80 \text{ pu}$$

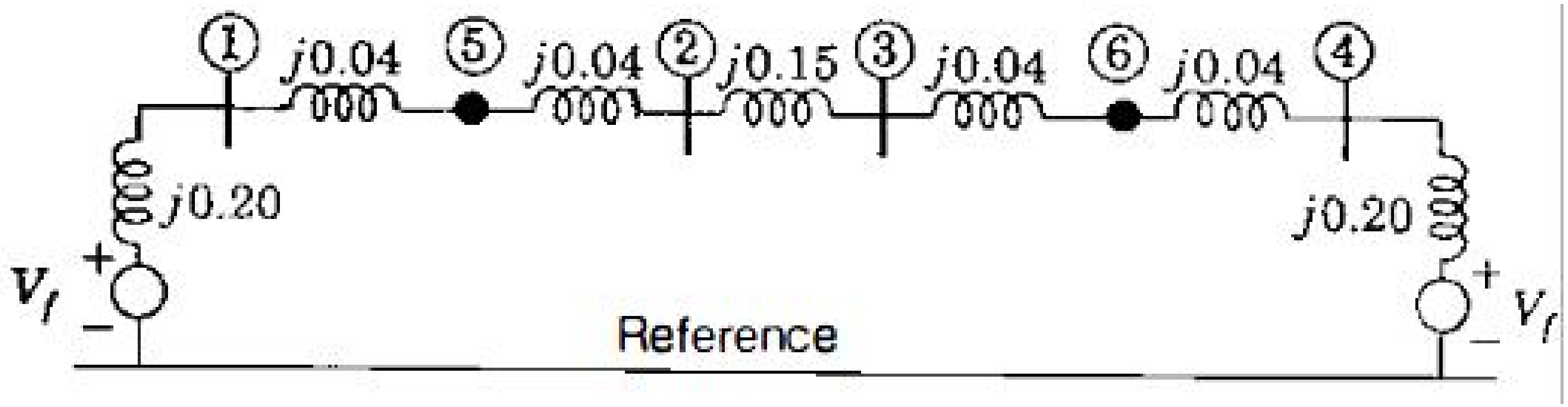
Example 12.1. Two synchronous machines are connected through three-phase transformers to the transmission line shown in Fig. 12.5. The ratings and reactances of the machines and transformers are

Machines 1 and 2: 100 MVA, 20 kV; $X_d'' = X_1 = X_2 = 20\%$,

$X_0 = 4\%$, $X_n = 5\%$

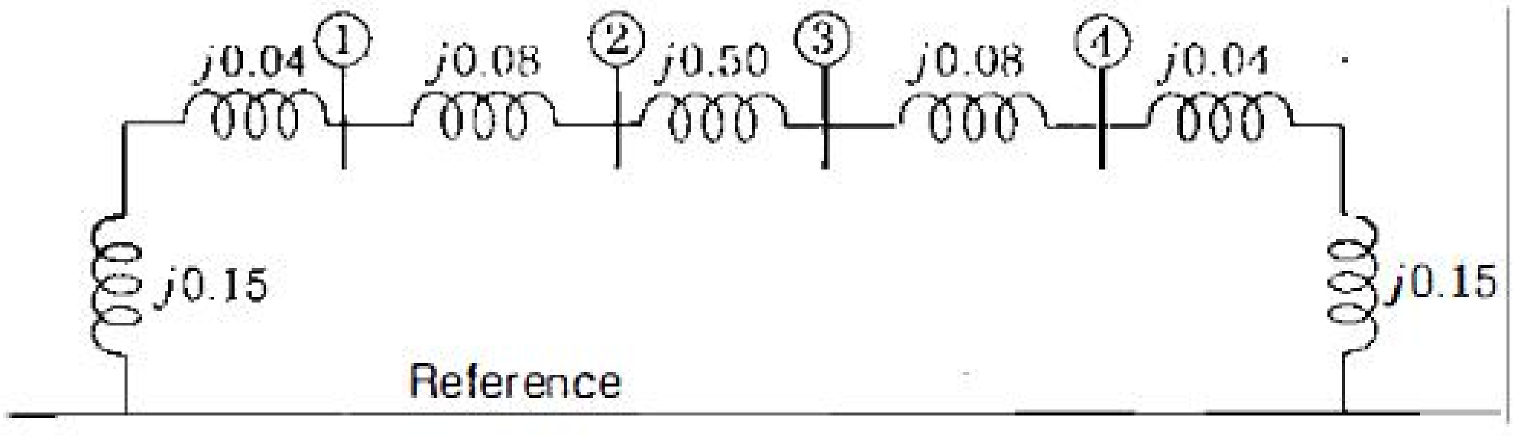
Transformers T_1 and T_2: 100 MVA, 20/345 kV ; $X = 8\%$

Both transformers are solidly grounded on two sides. On a chosen base of 100 MVA, 345 kV in the transmission-line circuit the line reactances are $X_1 = X_2 =$ 15% and $X_0 = 50\%$. The system is operating at nominal voltage without prefault currents when a bolted ($Z_f = 0$) single line-to-ground fault occurs on phase A at bus ③. Using the bus impedance matrix for each of the three sequence networks, determine the subtransient current to ground at the fault, the line-to-ground voltages at the terminals of machine 2,



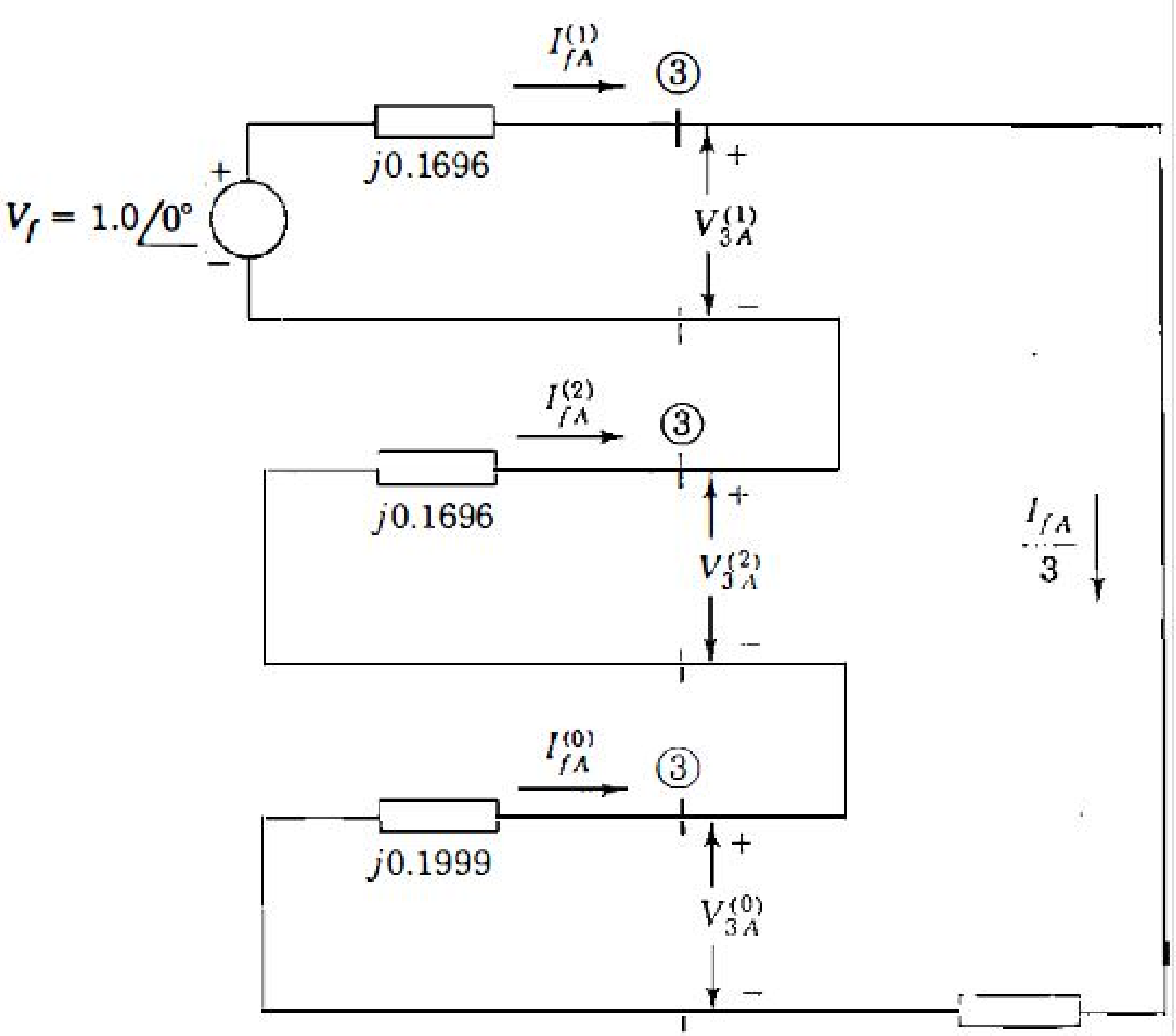
Positive Sequence Network

Z(positive sequence)=Z(negative sequence)=j0.1696 p.u.



Zero Sequence Network

Z(zero sequence)=j0.1999 p.u.



$I_{fA}^{(1)}$
③
$j0.1696$
$V_f = 1.0\angle 0°$
+
−
+
$V_{3A}^{(1)}$
−
$I_{fA}^{(2)}$
③
$j0.1696$
+
$V_{3A}^{(2)}$
−
$\frac{I_{fA}}{3}$
$I_{fA}^{(0)}$
③
$j0.1999$
+
$V_{3A}^{(0)}$
−

$$I_{fA}^{(0)} = I_{fA}^{(1)} = I_{fA}^{(2)} = \frac{V_f}{Z_{33}^{(1)} + Z_{33}^{(2)} + Z_{33}^{(0)}}$$

$$= \frac{1.0\angle 90^\circ}{j(0.1696 + 0.1696 + 0.1999)} = -j1.8549 \text{ per unit}$$

The total current in the fault is

$$I_{fA} = 3I_{fA}^{(0)} = -j5.5648 \text{ per unit}$$

and since the base current in the high-voltage transmission line is $100{,}000/\sqrt{3} \times 345 = 167.35$ A, we have

$$I_{fA} = -j5.5648 \times 167.35 = 931\angle 270^\circ \text{ A}$$

Find the line to ground voltages at the terminals of machine 2

The terminals of machine 2 is Bus number 4

So we have to use the following equations

$$V_{4a}^{(0)} = \quad - Z_{43}^{(0)} I_{fA}^{(0)}$$

$$V_{4a}^{(1)} = V_f - Z_{43}^{(1)} I_{fA}^{(1)}$$

$$V_{4a}^{(2)} = \quad - Z_{43}^{(2)} I_{fA}^{(2)}$$

$$
\mathbf{Z}_{\text{bus}}^{(1)} = \mathbf{Z}_{\text{bus}}^{(2)} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \begin{bmatrix} j0.1437 & j0.1211 & j0.0789 & j0.0563 \\ j0.1211 & j0.1696 & j0.1104 & j0.0789 \\ j0.0789 & j0.1104 & j0.1696 & j0.1211 \\ j0.0563 & j0.0789 & j0.1211 & j0.1437 \end{bmatrix} \end{array}
$$

$$
\mathbf{Z}_{\text{bus}}^{(1)} = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{cccc} \quad ① \quad & \quad ② \quad & \quad ③ \quad & \quad ④ \quad \end{array} \\ \begin{bmatrix} j0.1553 & j0.1407 & j0.0493 & j0.0347 \\ j0.1407 & j0.1999 & j0.0701 & j0.0493 \\ j0.0493 & j0.0701 & j0.1999 & j0.1407 \\ j0.0347 & j0.0493 & j0.1407 & j0.1553 \end{bmatrix} \end{array}
$$

$$V_{4a}^{(0)} = -Z_{43}^{(0)} I_{fA}^{(0)} = -(j0.1407)(-j1.8549) = -0.2610 \text{ per unit}$$

$$V_{4a}^{(1)} = V_f - Z_{43}^{(1)} I_{fA}^{(1)} = 1 - (j0.1211)(-j1.8549) = 0.7754 \text{ per unit}$$

$$V_{4a}^{(2)} = -Z_{43}^{(2)} I_{fA}^{(2)} = -(j0.1211)(-j1.8549) = -0.2246 \text{ per unit}$$

$$\begin{bmatrix} V_{4a} \\ V_{4b} \\ V_{4c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.2610 \\ 0.7754 \\ -0.2246 \end{bmatrix} = \begin{bmatrix} 0.2898 + j0.0 \\ -0.5364 - j0.8660 \\ -0.5364 + j0.8660 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2898\angle 0^\circ \\ 1.0187\angle -121.8^\circ \\ 1.0187\angle 121.8^\circ \end{bmatrix}$$

To express the line-to-ground voltages of machine 2 in kilovolts, we multiply by $20/\sqrt{3}$, which gives

$$V_{4a} = 3.346\angle 0^\circ \text{ kV} \qquad V_{4b} = 11.763\angle -121.8^\circ \text{ kV} \qquad V_{4c} = 11.763\angle 121.8^\circ \text{ kV}$$