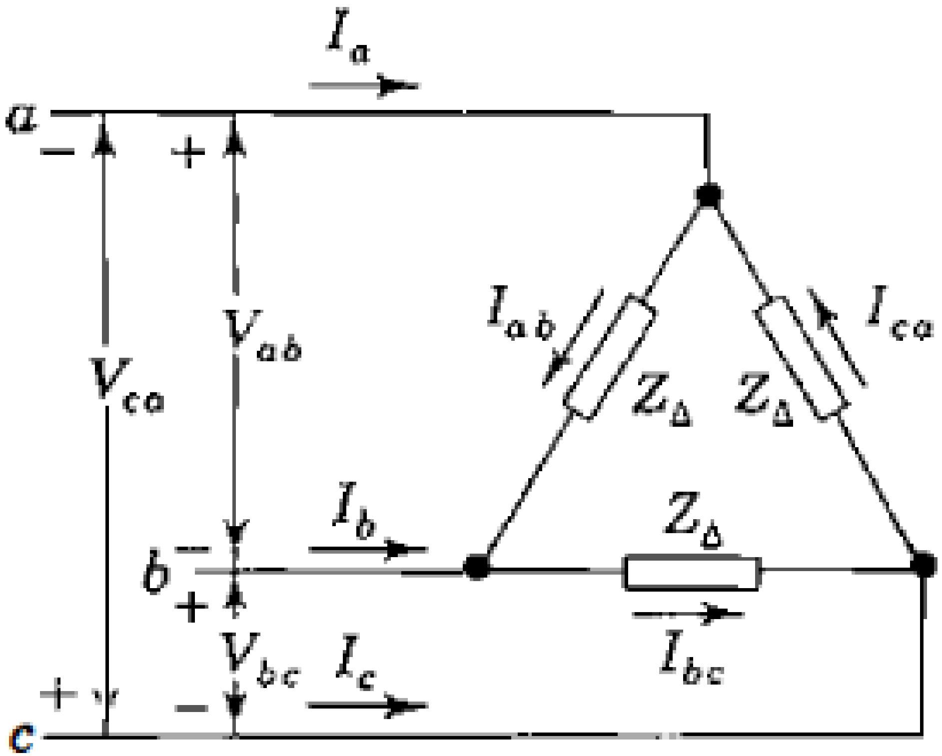
Symmetrical Delta Circuits



$$I_a = I_{ab} - I_{ca}$$

$$I_b = I_{bc} - I_{ab}$$

$$I_c = I_{ca} - I_{bc}$$

Adding all three equations together and invoking the definition of **zero-sequence current**, we obtain $I_a^{(0)} = (I_a + I_b + I_c)/3 = 0$, which means that *line currents into a Δ-connected circuit have no zero-sequence currents*. Substituting components of current in the equation for I_a yields

$$I_a^{(1)} + I_a^{(2)} = \left(I_{ab}^{(0)} + I_{ab}^{(1)} + I_{ab}^{(2)}\right) - \left(I_{ca}^{(0)} + I_{ca}^{(1)} + I_{ca}^{(2)}\right)$$

$$= \underbrace{\left(I_{ab}^{(0)} - I_{ca}^{(0)}\right)}_{0} + \left(I_{ab}^{(1)} - I_{ca}^{(1)}\right) + \left(I_{ab}^{(2)} - I_{ca}^{(2)}\right) \qquad (11.18)$$

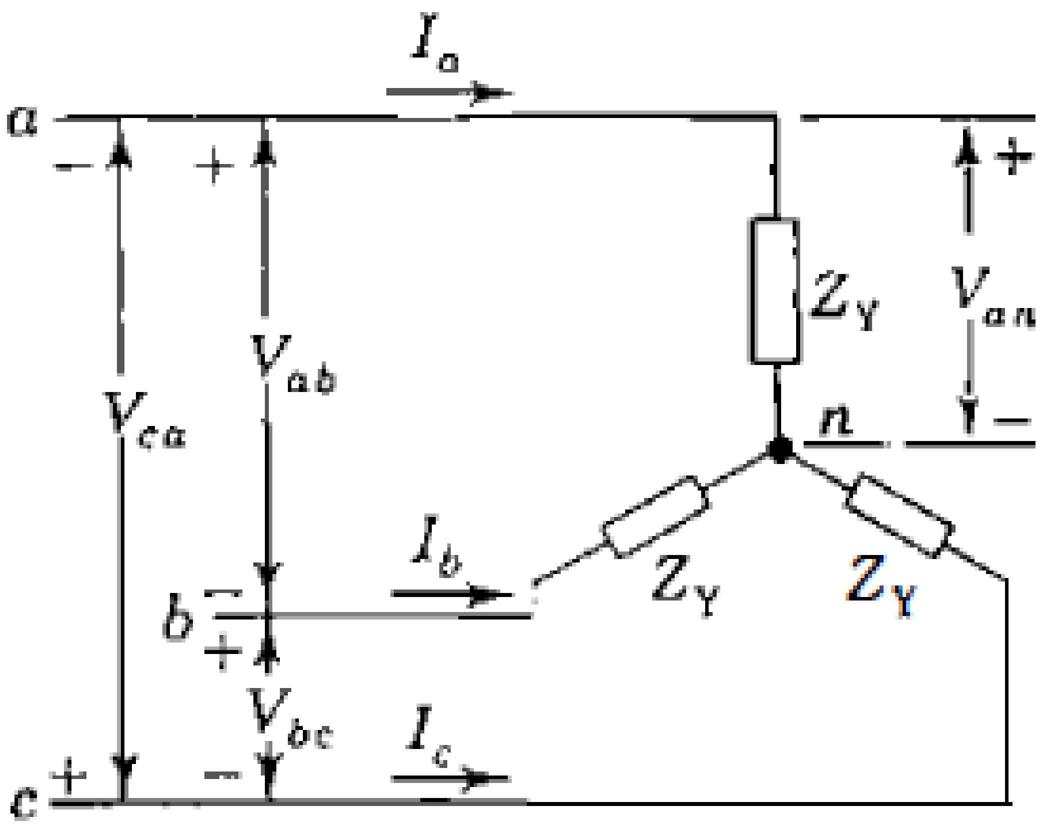
Evidently, if a nonzero value of circulating current $I_{ab}^{(0)}$ exists in the Δ circuit, it cannot be determined from the line currents alone. Noting that $I_{ca}^{(1)} = aI_{ab}^{(1)}$ and $I_{ca}^{(2)} = a^2 I_{ab}^{(2)}$, we now write Eq. (11.18) as follows:

$$I_a^{(1)} + I_a^{(2)} = (1 - a) I_{ab}^{(1)} + (1 - a^2) I_{ab}^{(2)} \tag{11.19}$$

A similar equation for phase b is $I_b^{(1)} + I_b^{(2)} = (1 - a)I_{bc}^{(1)} + (1 - a^2)I_{bc}^{(2)}$, and expressing $I_b^{(1)}$, $I_b^{(2)}$, $I_{bc}^{(1)}$, and $I_{bc}^{(2)}$ in terms of $I_a^{(1)}$, $I_a^{(2)}$, $I_{ab}^{(1)}$, and $I_{ab}^{(2)}$, we obtain a resultant equation which can be solved along with Eq. (11.19) to yield the important results

$$I_a^{(1)} = \sqrt{3}\,\underline{/-30^\circ} \times I_{ab}^{(1)} \qquad I_a^{(2)} = \sqrt{3}\,\underline{/30^\circ} \times I_{ab}^{(2)} \tag{11.20}$$

Symmetrical Star Circuits



$$V_{ab} = V_{an} - V_{bn}$$

$$V_{bc} = V_{bn} - V_{cn}$$

$$V_{ca} = V_{cn} - V_{an}$$

Adding together all three equations shows that $V_{ab}^{(0)} = (V_{ab} + V_{bc} + V_{ca})/3 = 0$. In words, *line-to-line voltages have no zero-sequence components.* Substituting components of the voltages in the equation for V_{ab} yields

$$V_{ab}^{(1)} + V_{ab}^{(2)} = \left(V_{an}^{(0)} + V_{an}^{(1)} + V_{an}^{(2)}\right) - \left(V_{bn}^{(0)} + V_{bn}^{(1)} + V_{bn}^{(2)}\right)$$

$$= \underbrace{\left(V_{an}^{(0)} - V_{bn}^{(0)}\right)}_{0} + \left(V_{an}^{(1)} - V_{bn}^{(1)}\right) + \left(V_{an}^{(2)} - V_{bn}^{(2)}\right) \quad (11.22)$$

Therefore, a nonzero value of the zero-sequence voltage $V_{an}^{(0)}$ cannot be determined from the line-to-line voltages alone. Separating positive- and negative-sequence quantities in the manner explained for Eq. (11.19), we obtain the important voltage relations

$$\begin{aligned} V_{ab}^{(1)} &= (1 - a^2)V_{an}^{(1)} = \sqrt{3}\,\angle 30^\circ \times V_{an}^{(1)} \\ V_{ab}^{(2)} &= (1 - a)V_{an}^{(2)} = \sqrt{3}\,\angle -30^\circ \times V_{an}^{(2)} \end{aligned} \quad (11.23)$$

Example 11.2. Three identical Y-connected resistors form a load bank with a three-phase rating of 2300 V and 500 kVA. If the load bank has applied voltages

$$|V_{ab}| = 1840 \text{ V} \qquad |V_{bc}| = 2760 \text{ V} \qquad |V_{ca}| = 2300 \text{ V}$$

find the line voltages and currents in per unit into the load. Assume that the neutral of the load is not connected to the neutral of the system and select a base of 2300 V, 500 kVA.

Solution. The rating of the load bank coincides with the specified base, and so the resistance values are 1.0 per unit. On the same base the given line voltages in per unit are

$$|V_{ab}| = 0.8 \qquad |V_{bc}| = 1.2 \qquad |V_{ca}| = 1.0$$

Assuming an angle of 180° for V_{ca} and using the law of cosines to find the angles of the other line voltages, we find the per-unit values

$$V_{ab} = 0.8\angle 82.8^\circ \qquad V_{bc} = 1.2\angle -41.4^\circ \qquad V_{ca} = 1.0\angle 180^\circ$$

Law of Cosines

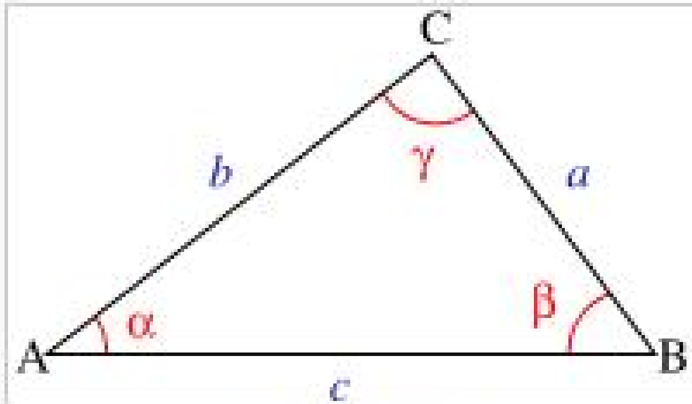


Figure 1 – A triangle. The angles α (or *A*), β (or *B*), and γ (or *C*) are respectively opposite the sides *a*, *b*, and *c*.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

The symmetrical components of the line voltages are

$$V_{ab}^{(1)} = \tfrac{1}{3}\left(0.8\angle 82.8^\circ + 1.2\angle 120^\circ - 41.4^\circ + 1.0\angle 240^\circ + 180^\circ\right)$$

$$= \tfrac{1}{3}(0.1003 + j0.7937 + 0.2372 + j1.1763 + 0.5 + j0.8660)$$

$$= 0.2792 + j0.9453 = 0.9857\angle 73.6^\circ \text{ per unit (line-to-line voltage base)}$$

$$V_{ab}^{(2)} = \tfrac{1}{3}\left(0.8\angle 82.8^\circ + 1.2\angle 240^\circ - 41.4^\circ + 1.0\angle 120^\circ + 180^\circ\right)$$

$$= \tfrac{1}{3}(0.1003 + j0.7937 - 1.1373 - j0.3828 + 0.5 - j0.8660)$$

$$= -0.1790 - j0.1517 = 0.2346\angle 220.3^\circ \text{ per unit (line-to-line voltage base)}$$

The absence of a neutral connection means that zero-sequence currents are not present. Therefore, the phase voltages at the load contain positive- and negative-sequence components only. The phase voltages are found from Eqs. (11.23) with the $\sqrt{3}$ factor omitted since the line voltages are expressed in terms of the base voltage from line to line and the phase voltages are desired in per unit of the base voltage to neutral. Thus,

$$V_{an}^{(1)} = 0.9857\angle\underline{73.6^\circ - 30^\circ}$$

$$= 0.9857\angle\underline{43.6^\circ} \text{ per unit (line-to-neutral voltage base)}$$

$$V_{an}^{(2)} = 0.2346\angle\underline{220.3^\circ + 30^\circ}$$

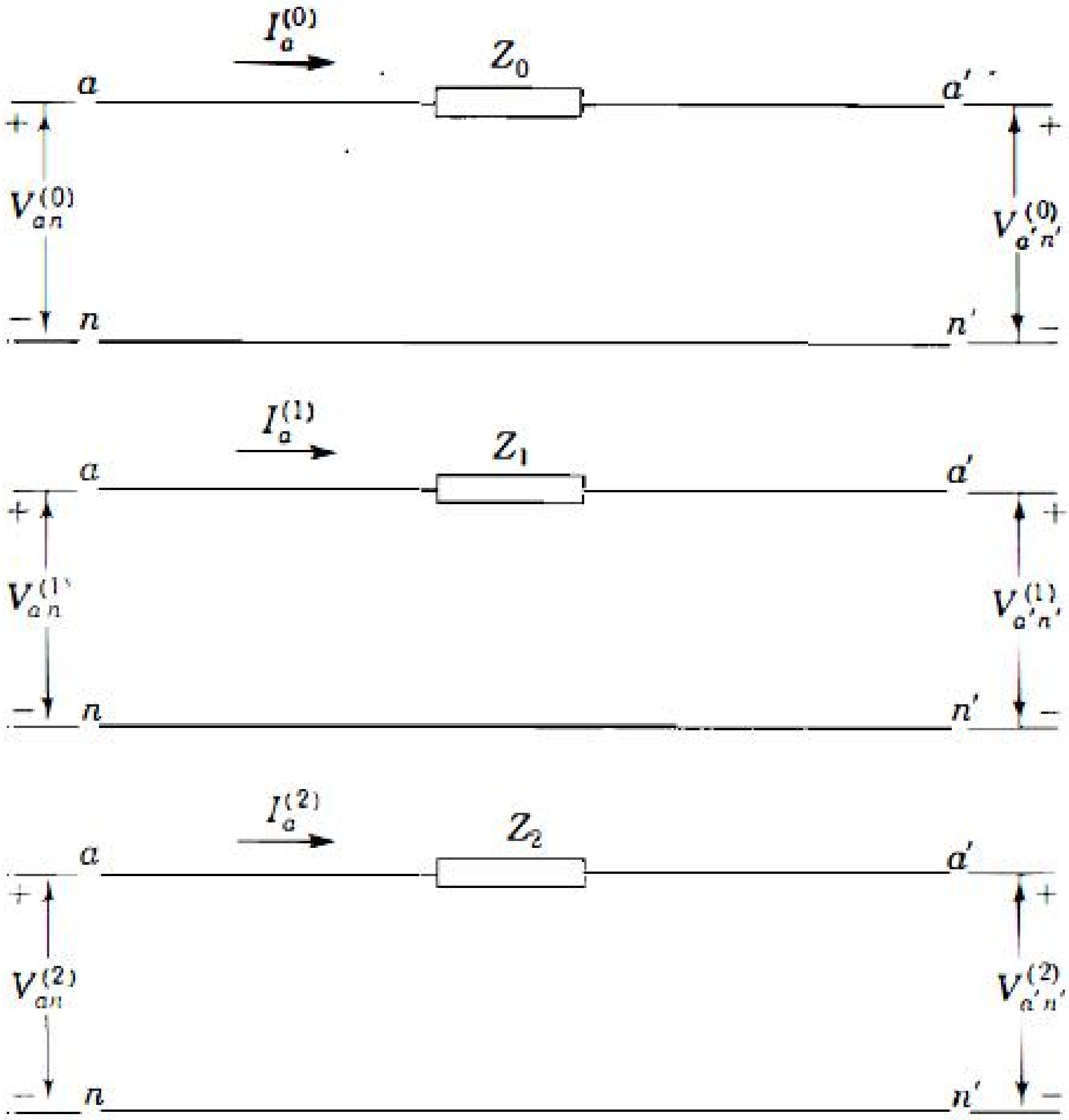
$$= 0.2346\angle\underline{250.3^\circ} \text{ per unit (line-to-neutral voltage base)}$$

Since each resistor has an impedance of $1.0\angle 0^\circ$ per unit,

$$I_a^{(1)} = \frac{V_a^{(1)}}{1.0\angle 0^\circ} = 0.9857\angle 43.6^\circ \text{ per unit}$$

$$I_a^{(2)} = \frac{V_a^{(2)}}{1.0\angle 0^\circ} = 0.2346\angle 250.3^\circ \text{ per unit}$$

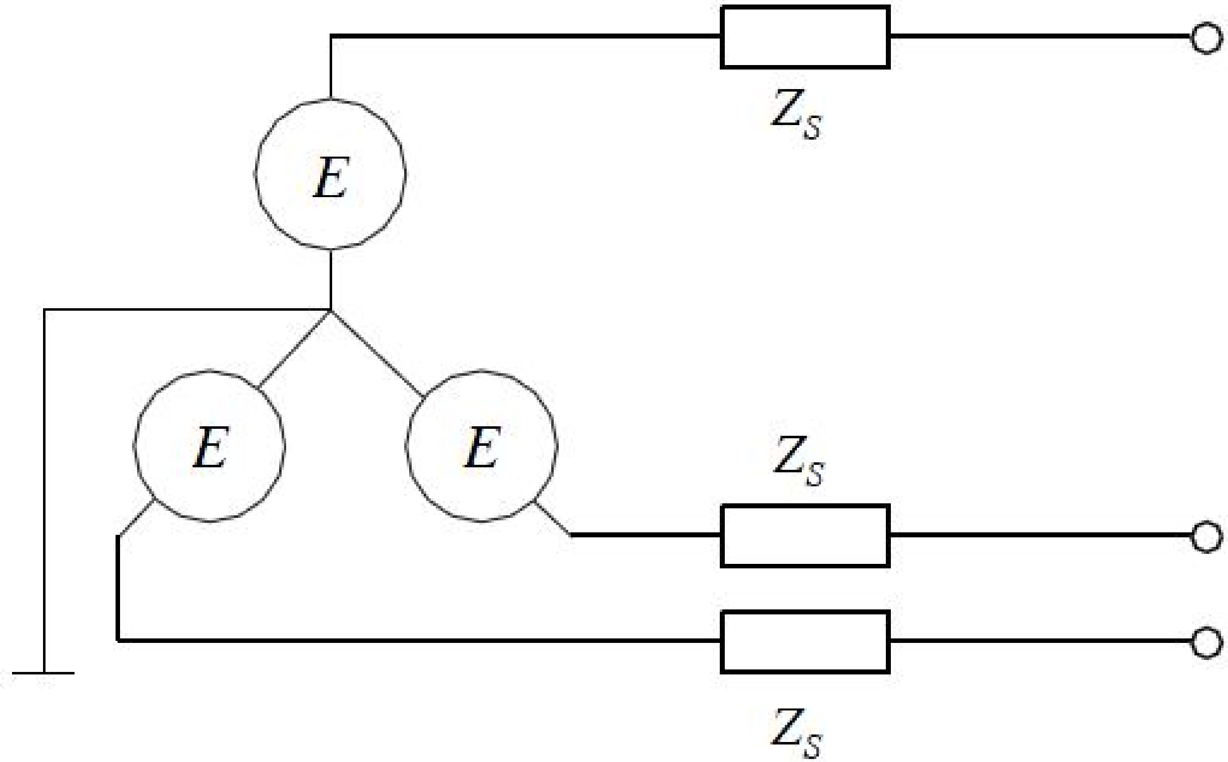
Sequence Circuit of Transmission Line



Generators

- **Synchronous Reactance**
  - Dependent on the time period of the study
    - Steady-state
    - Transient
    - Sub-transient
  - Generator is modeled as a wye-connected machine for the positive- and negative-sequence impedances
  - The zero-sequence is dependent on the winding connections of the machine
    - wye-grounded
    - wye-grounded through an impedance
    - wye
    - delta

Solidly Grounded Generators



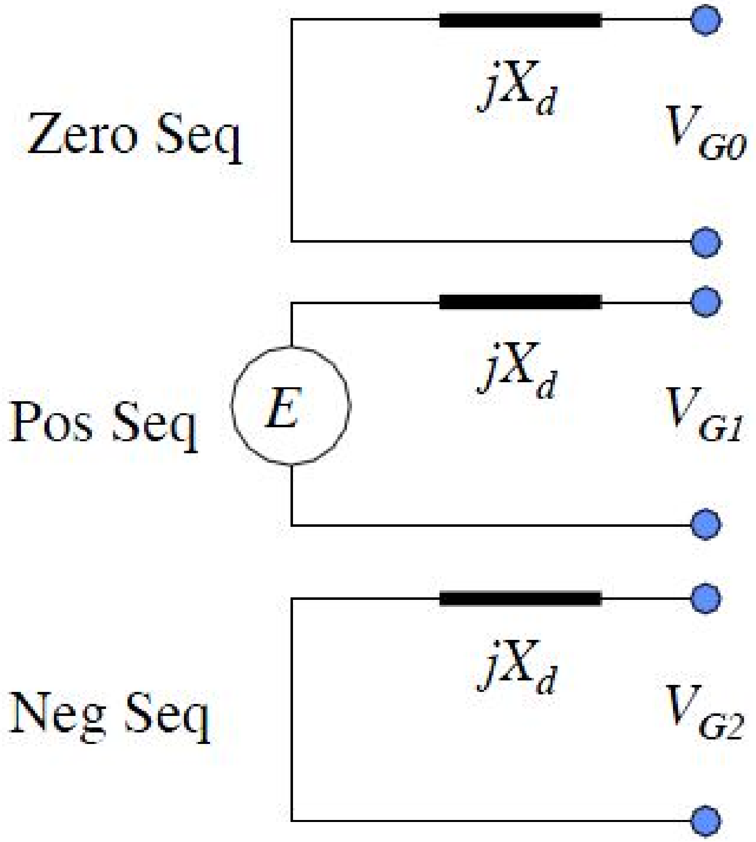
Solidly Grounded Generators

$$\mathbf{A}\,\mathbf{E}_{012} = \mathbf{E}_{abc}$$

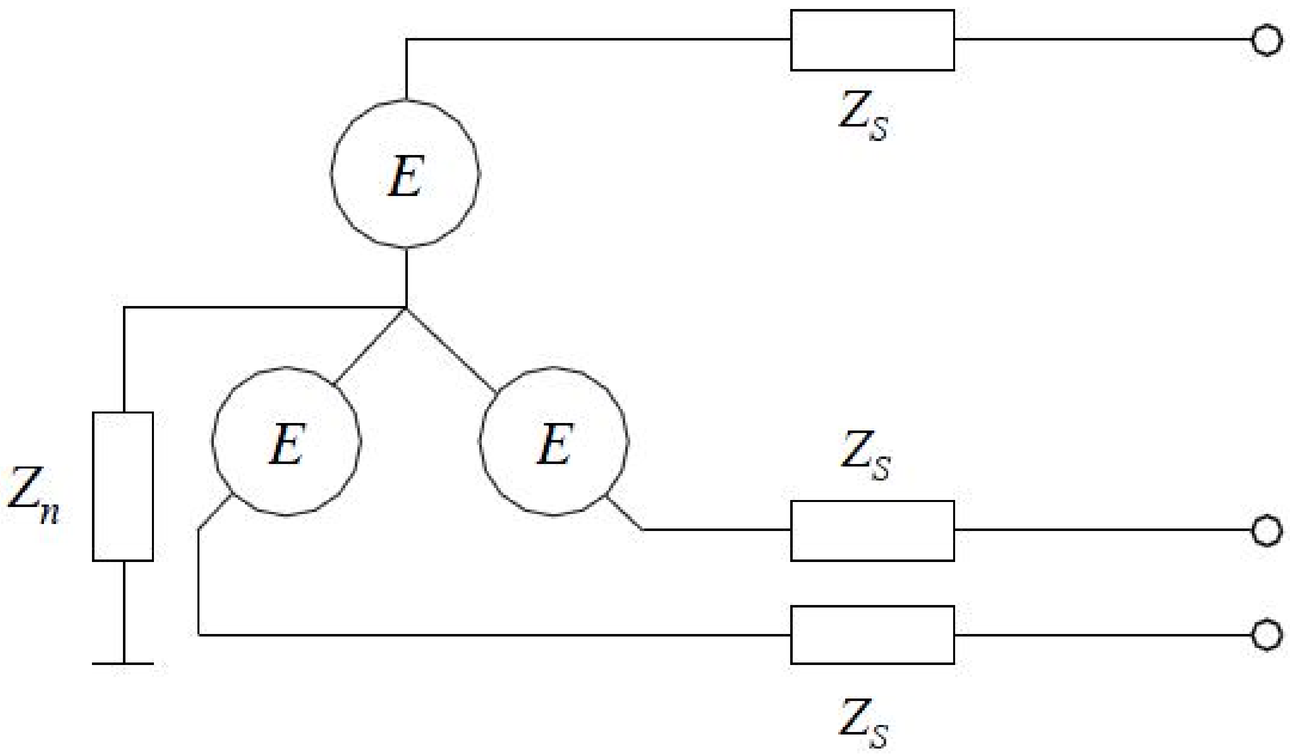
$$\mathbf{E}_{abc} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix}$$

$$\mathbf{E}_{012} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix}$$

$$X_0 = X_1 = X_2 = X_d$$



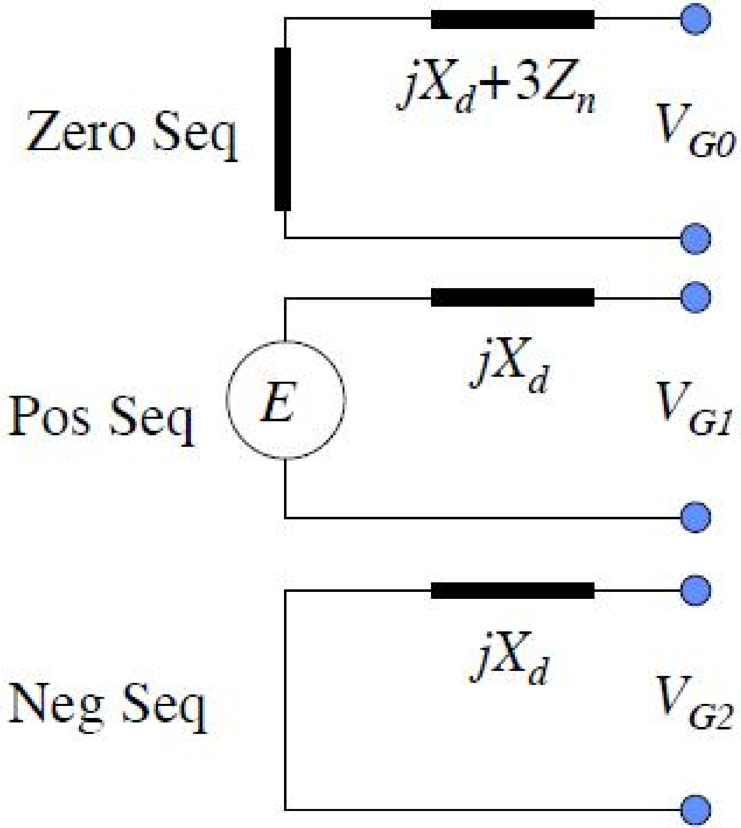
Impedance Grounded Generators



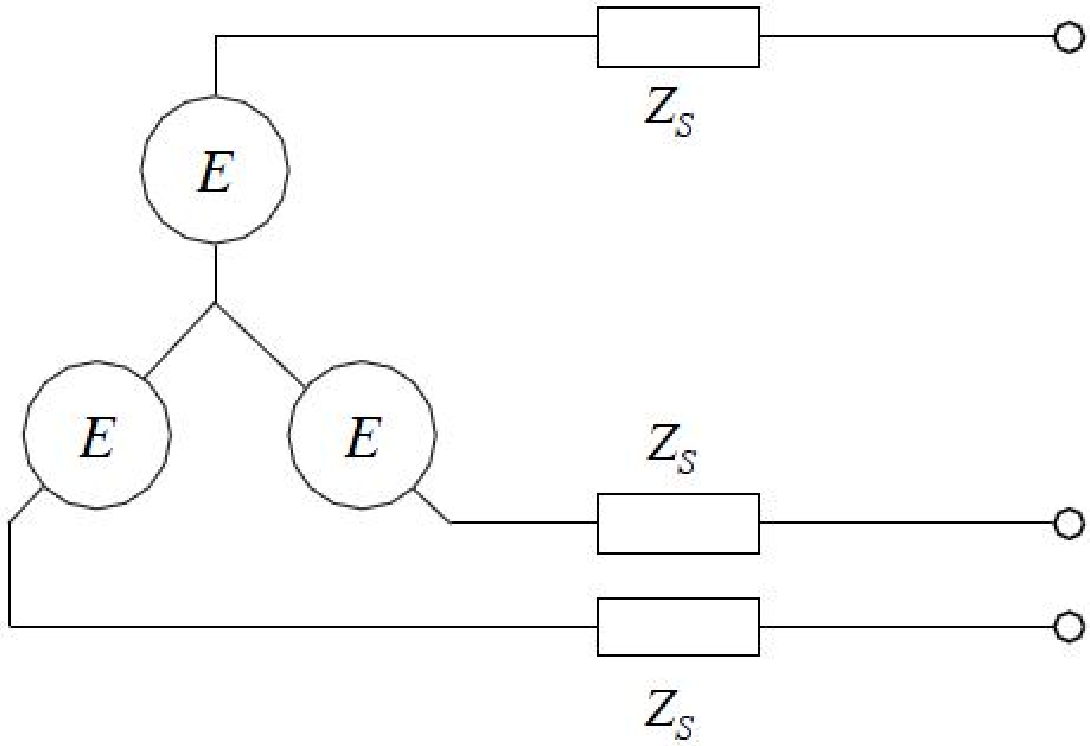
Impedance Grounded Generators

$$\mathbf{E}_{012} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix}$$

$$\mathbf{Z}_{012} = \begin{bmatrix} jX_d + 3Z_n & 0 & 0 \\ 0 & X_d & 0 \\ 0 & 0 & X_d \end{bmatrix}$$



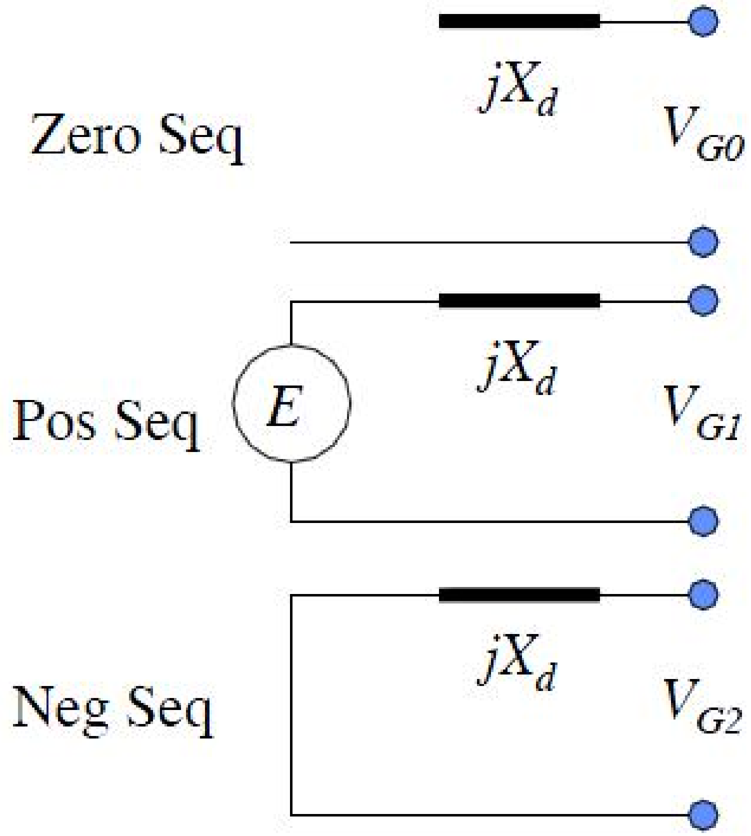
Wye Connected Generators



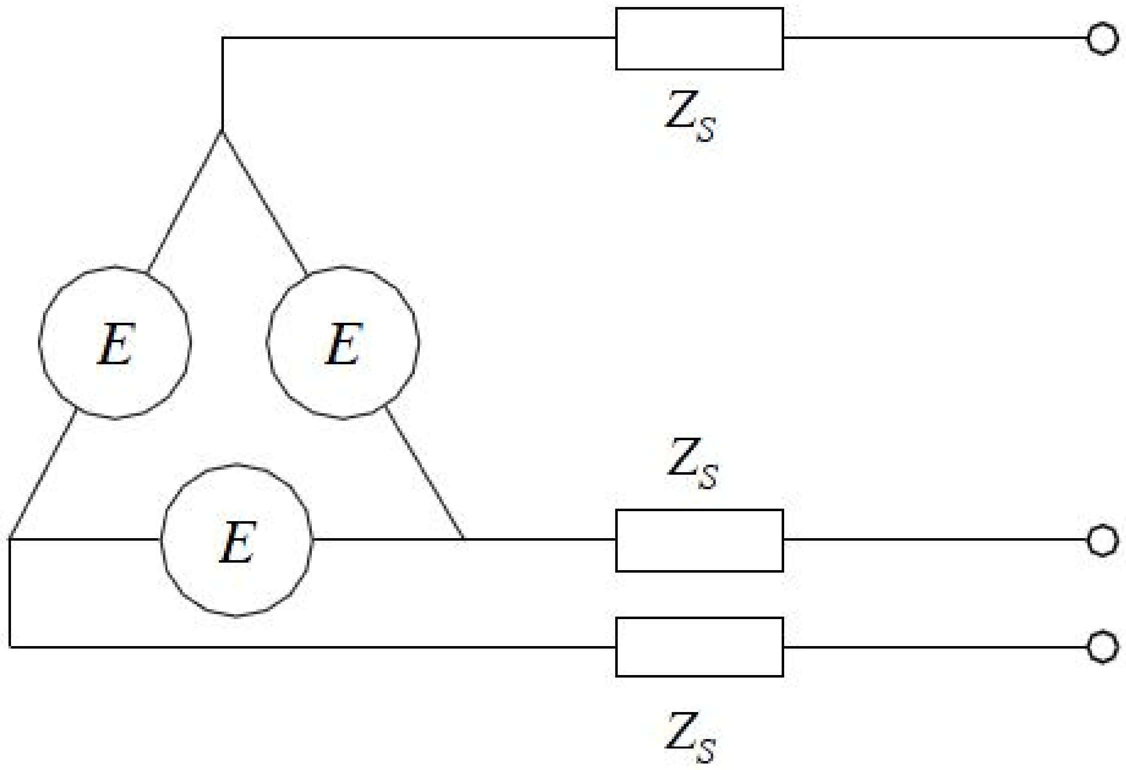
Wye Connected Generators

$$\mathbf{E}_{012} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix}$$

$$\mathbf{Z}_{012} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & X_d & 0 \\ 0 & 0 & X_d \end{bmatrix}$$



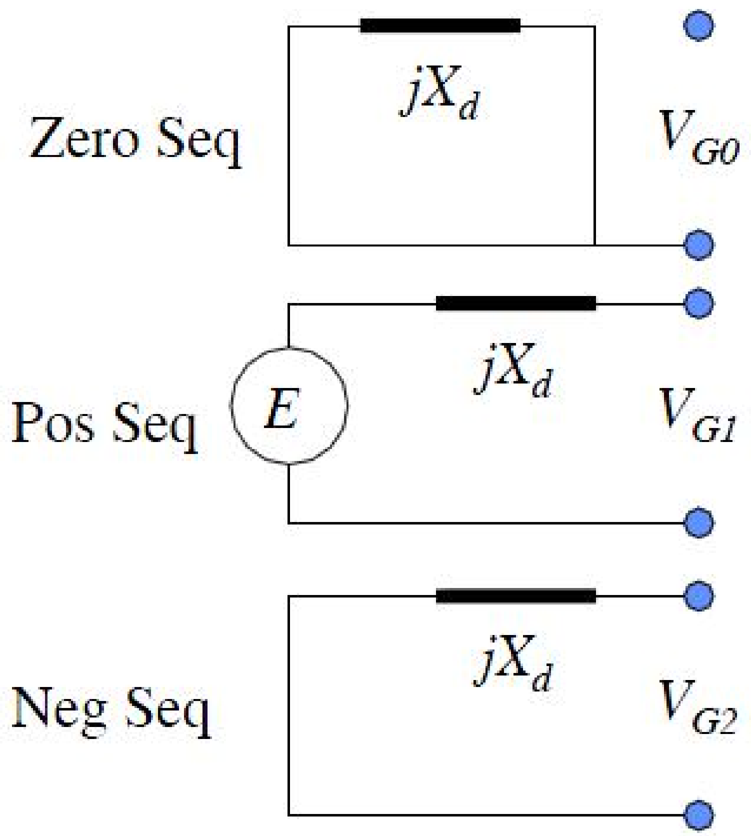
Delta Connected Generators



Delta Connected Generators

$$\mathbf{E}_{012} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix}$$

$$\mathbf{Z}_{012} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & X_d & 0 \\ 0 & 0 & X_d \end{bmatrix}$$



Transformers

- **Equivalent Series Impedance:**
  - Transformer bank of three single-phase transformers

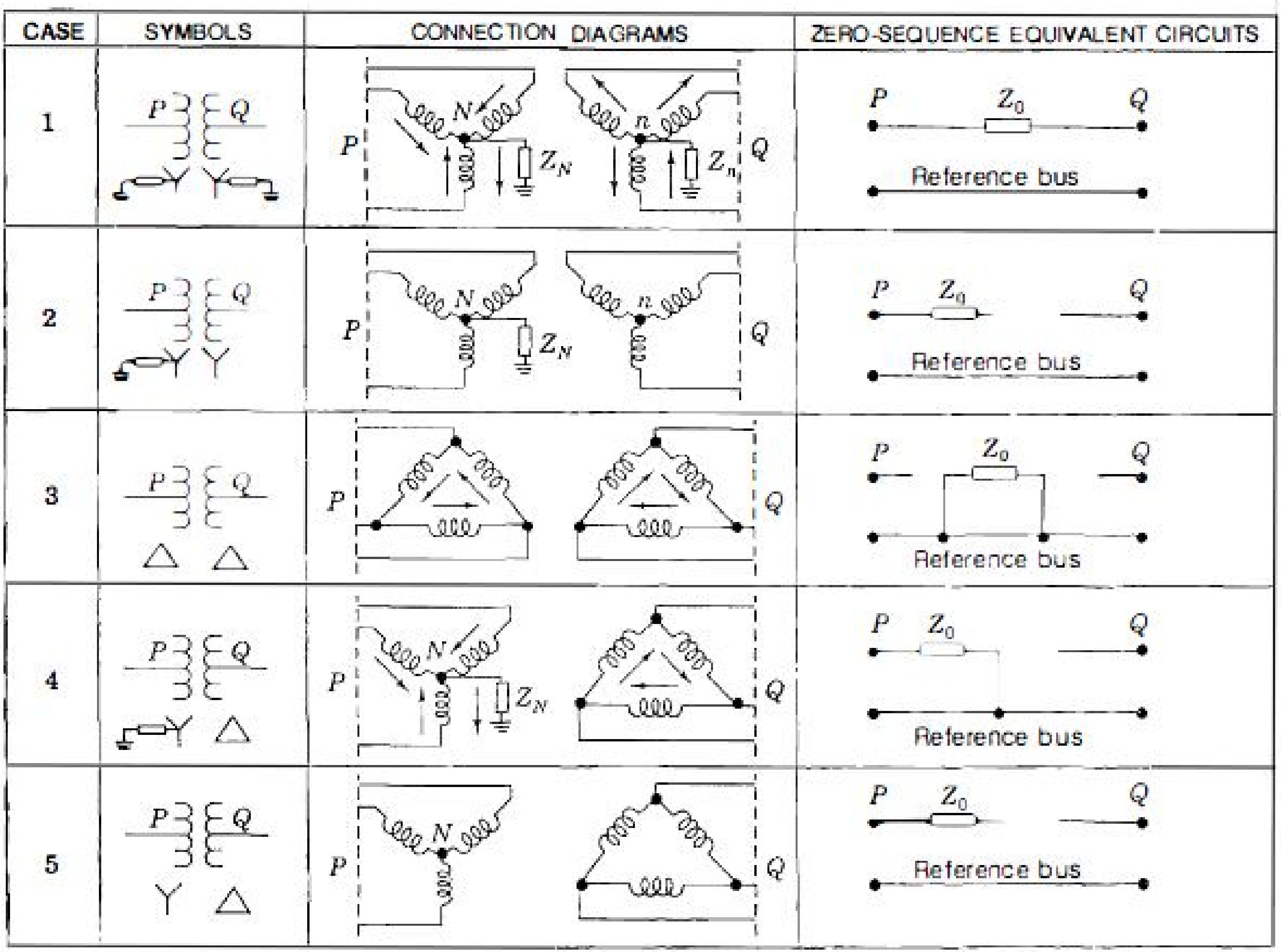
$$Z_0 = Z_1 = Z_2 = Z_\ell$$

  - Three-phase transformer with a three-leg core

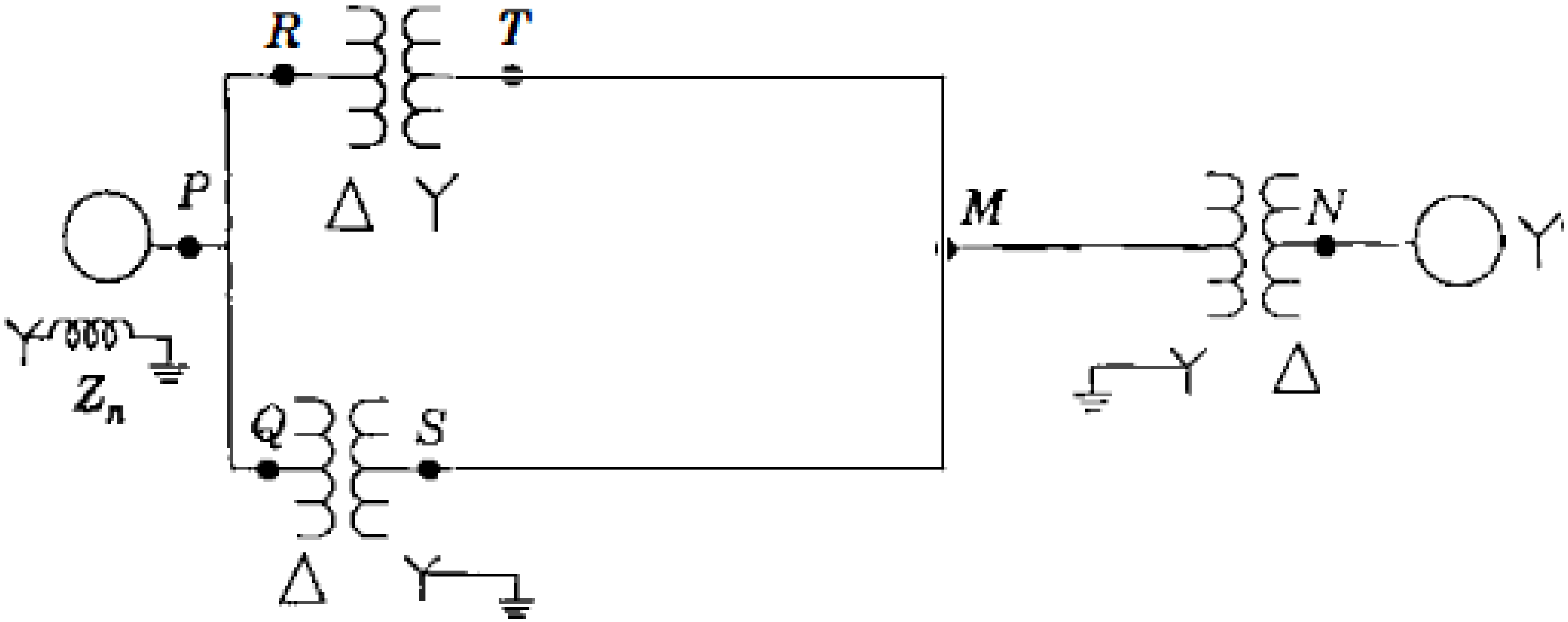
$$Z_1 = Z_2 = Z_\ell \qquad Z_0 > Z_\ell$$

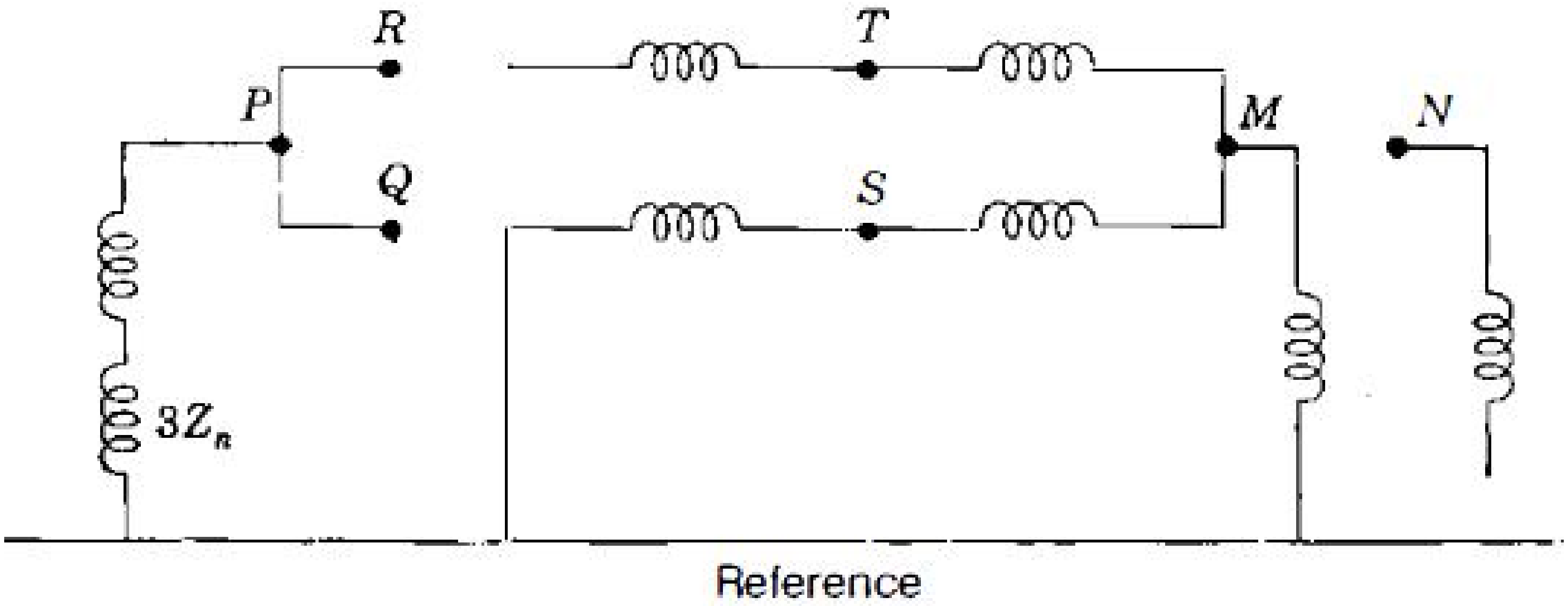
- **Wye-Delta Wound Transformers**
  - Wiring connection will always cause a phase shift
  - Positive Sequence rotates by a +30 degrees from HV to LV side
  - Negative Sequence rotates by a -30 degrees from HV to LV side
  - Zero Sequence does not rotate

Zero Sequence Circuit of Three Phase Transformer



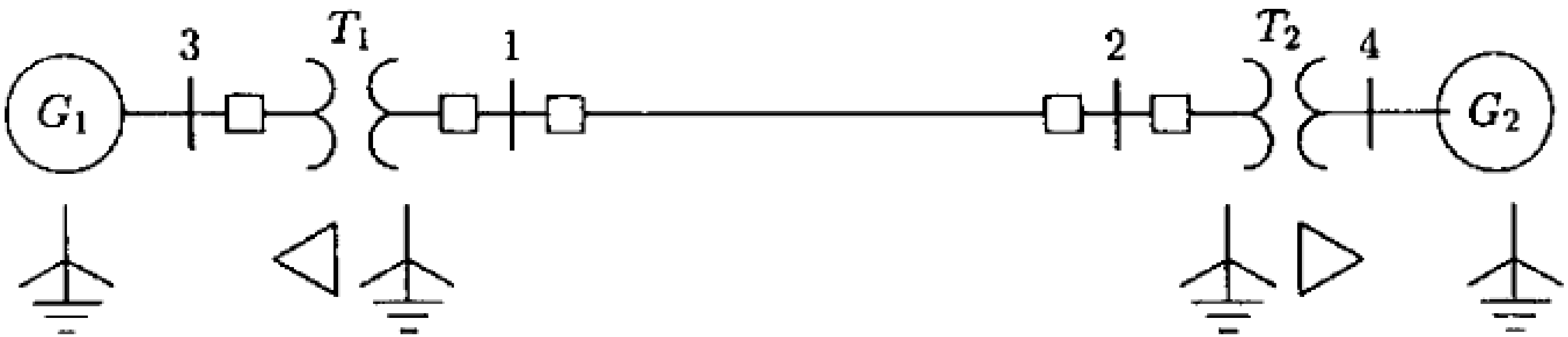
Example





Zero Sequence Network

Example:



Item	X^1	X^2	X^0
G_1	0.10	0.10	0.05
G_2	0.10	0.10	0.05
T_1	0.25	0.25	0.25
T_2	0.25	0.25	0.25
Line 1–2	0.30	0.30	0.50