Robust Controller Design Using H_∞ Loop-Shaping and Method of Inequalities

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Abstract— A **new approach to robust controller design is proposed. By using plant-weighting functions as the design parameters, the approach combines the method of inequalities with a loop shaping design procedure using H-∞ synthesis, that the closed loop performance satisfy certain prescribed constrains**.

I-INTRODUCTION:

SPECIFICATIONS for the performance of feedback control systems are often expressed in terms of inequalities which need to be satisfied. This fact resulted in the development of the method of inequalities [1], a design method where the design objectives are expressed explicitly as a set of algebraic inequalities representing desired bounds on a set of performance indices. For a successful design, the inequalities must be satisfied. A separate development has been the use of $H_{-\infty}$-optimization in a variety of approaches to design robust control systems. One such approach is the loop-shaping design procedure (LSDP) [2], [3]. This approach involves the robust stabilization to additive perturbation in the sense of H-∞ norm of normalized coprime factors of a weighted plant. The weighted-plant singular values are shaped by adjusting the weighting functions to give a desired open-loop shape which gives good closed-loop performance with stability robustness. Certain aspects of the LSDP make it suitable to combine this approach with the method of inequalities to design directly for both closed-loop performance and stability robustness. The MOI also can be used to enhance the closed loop performance after the robustification step as shown in section (IV).This brief paper describes this new approach and applies the proposed method to the design of robust controller for a synchronous generator

II- II-NORMALIZED COPRIME FACTORIZATION

The plant model $G = \tilde{M}^{-1}.\ \tilde{N}$ is a normalized left coprime factorization (NLCF) of G if $\tilde{M}, \tilde{N} \in RH_\infty$, there exists $V, U \in RH_\infty$ such that $\tilde{M}V + \tilde{N}U = I$, and $\tilde{M}\tilde{M}^* + \tilde{N}\tilde{N}^* = I$ where for a real rational function of s, X^* denotes $X'(-s)$.

Using the notation

$$G(s) = D + C(sI - A)^{-1}B \overset{s}{=} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad (1)$$

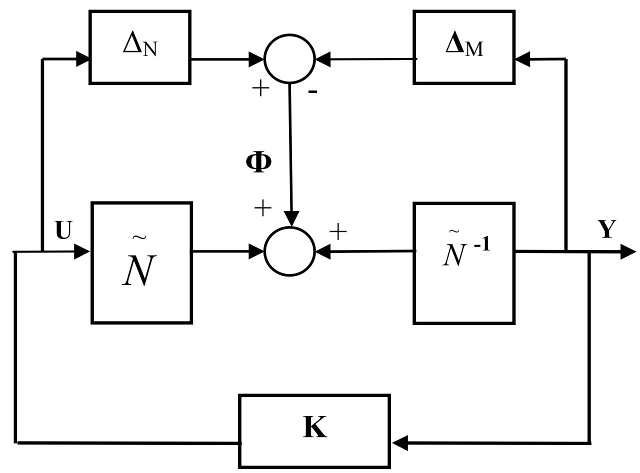


Fig. 1. Robust stabilization with respect to coprime factor uncertainty

Then as shown in [4]

$$\begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} \overset{s}{=} \left[\begin{array}{c|cc} A + HC & B + HD & H \\ \hline R^{-1/2}C & R^{-1/2}D & R^{-1/2} \end{array} \right] \qquad (2)$$

is a normalized coprime factorization of G, where

$H = -(BD' + ZC')R^{-1}$, $R = I + DD'$, and the matrix $Z > 0$ is the unique stabilizing solution to the algebraic Riccati equation (ARE).

$$(A - BS^{-1}D'C)Z + Z(A - BS^{-1}D'C)' - ZC'R^{-1}CZ + BS^{-1}B' = 0 \qquad (3)$$

$$G_\Delta = (\tilde{M} + \Delta M)^{-1}(\tilde{N} + \Delta N) \qquad (4)$$

where $\Delta M, \Delta N \in RH_\infty$. To maximize the class of perturbed models defined by (4) such that the configuration of Fig(1) is stable, we need to find the controller K which stabilizes the nominal closed-loop system and which minimizes γ where Note that from (4).

$$\gamma = \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - G.K)^{-1}.\tilde{M}^{-1} \right\|_\infty \qquad (5)$$

This is the problem of robust stabilization of normalized coprime factor plant descriptions as introduced in [3]. From the small gain theorem, the closed-loop system will remain stable if

$$\left\| \begin{bmatrix} \Delta N & \Delta M \end{bmatrix} \right\|_\infty < \gamma^{-1} \qquad (6)$$

γ is the H ∞ norm from Φ to [U Y] and $(I-G.K)^{-1}$ is the sensitivity function dor the positive feedback arrangement.

The H ∞ norm gives the worst case steady state gain for sinusoidal inputs of any frequency, it also equal to the worst case energy gain of the system.

The minimum value of the γ for all the stabilizing controller K is:

$$\gamma_o = \text{InfKstabilizing} \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - G.K)^{-1} . \tilde{M}^{-1} \right\|_\infty \qquad (7)$$

it is shown in [3] that :

$$\gamma_o = (1 + \lambda_{max}(z . x))^{1/2} \qquad (8)$$

where λ_{max} (.) represents the maximum eigenvalue , and $x \geq 0$, is the unique stabilizing solution of the ARE

$$(A - BS^{-1}D^TC)^T X + X(A - BS^{-1} D^T C) - XBS^{-1} B^T X + C^T R^{-1} C = 0 \qquad (9)$$

A controller which achieves γ_o is given in [4] by

$$K \stackrel{s}{=} \left[\frac{-L^T s + L^T (A + BF) + \gamma^2 ZC^T (C + DF)}{B^T X} \; \frac{\gamma^2 ZC^T}{-D^T} \right] \qquad (10)$$

where $F = -S^{-1} (D'C + B'X)$ and $Q = (1 - \gamma^2_o)I + XZ$.

From the above, the optimum controller is synthesized by the solution of two ARE's, unlike most $H\infty$ problems, which require an iterative search on γ to find the optimum.

In practice, to design control systems using normalized coprime factorizations, the plant needs to be weighted to meet closed-loop performance requirements. A design procedure has been developed [2], [3],[4] known as the LSDP, to choose the weights by studying the open-loop singular values of the plant and augmenting the plant with weights so that the weighted plant has an open-loop shape which will give good closed-loop performance.

The loop shaping is used to specify a closed loop objectives in terms of open loop singular value denoted σ (.) , by shaping the nominal plant singular value (using compensating weighting function), to give the desired open loop properties at frequencies of high & low loop gain, then the normalized coprime factor H-∞ is used to robustly stabilize this shaped plant, [5] by parameterizing such a controller to fulfill the main conditions for robustification according to the small gain therom [6],ensuring the necessary internal stability [6].

ε is a measure of both closed loop robust stability & the success of the design in meeting the loop shaping specifications

The nominal plant G is augmented with pre compensators and post compensators $W1$ and W_2 , respectively, so that the augmented plant G_s is $G_s = W_2GW_1$.The post & the pre-compensators weighting function can be combined into compensating weighting function W_p.

Using the procedure outlined earlier, an optimum feedback controller K_s is synthesized which robustly stabilizes the NLCF of G_s given by (N_s, M_s) where $G_s = \tilde{M}^{-1}_s \tilde{N}_s$. The final feedback controller K is then constructed by simply combining $K_{sub\ opt}$ with the weights to give

$$K_{opt} = W_1 .K_{sub\ opt}.W_2 \qquad (11)$$

Note that from [2].

$$\gamma_o = \text{Inf k stabilizing} \left\| \begin{bmatrix} W_1^{-1}.K \\ W_2 \end{bmatrix} (I - G.K)^{-1}. \begin{bmatrix} W_2^{-1} & G.W_1 \end{bmatrix} \right\|_\infty \qquad (12)$$

Essentially with the LSDP, the weights $W1$ and W_2 are the design parameters which are chosen both to give the augmented plant a "good" open-loop shape and to ensure that γ_o is not too large. γ_0 is a design indicator of the success of the loop-shaping as well as a measure of the robustness of the stability property.

III. Robust Design Using a Coprime Factor Plant Description with the Method of Inequalities

Two aspects of design using robust stabilization of normalized coprime factor descriptions of the weighted plant make it amenable to combine this approach with the MOI. First, unlike most H-∞optimization problems, the H-∞ optimal controller for the weighted plant can be synthesized from the solution of just two ARE's and does not require time-consuming γ-iteration. Second, in the LSDP described in Section II, the weighting functions are chosen by considering the open-loop response of the weighted plant, so effectively the weights W_1 and W_2 are the design parameters. This means that the design problem can be formulated as in the method of inequalities, with the weighting parameters used as the design parameters p to satisfy some set of closed-loop performance inequalities.

Such an approach to the MOI overcomes the limitations to the MOI described at the end of Section III. The designer does not have to choose the order or structure of the controller, but instead chooses the structure and order of the weighting functions. With low-order weighting functions, high-order controllers can be synthesized which often lead to significantly better performance or robustness than if simple low-order controllers were used, (The higher order controller can be reduced to the lower using typical reduction method) , Additionally, the problem of finding a stability point does not exist, because stability is guaranteed through the solution to the robust stabilization problem, provided that the weighting functions do not cause undesirable pole/zero cancellations

The design problem is now stated as follow:

Problem: For the system of fig (2), find a (W_p,K_p) such that:

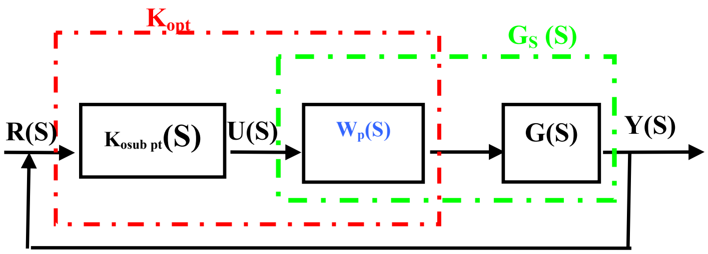
$$\gamma_o(w) \leq \varepsilon_\gamma \qquad (13)$$

and

$$\Phi_i(w,K_s) \leq \varepsilon_i \qquad (14)$$

for $i=1...n$

$\Phi_i(w,K_s)$ is a function of the closed loop system, ε_γ , ε_i are real numbers represents desired bounds on γ_o & Φ_i .



Fig(2) The standard H-∞ loop shaping closed loop system
------- The augmented plant
---------- The optimum controller K_{opt}
$W_p=W_1.W_2$

$$\gamma_o = \text{Inf k stabilizing} \left\| \begin{bmatrix} W_1^{-1}.K \\ W_2 \end{bmatrix} (I-G.K)^{-1}. \begin{bmatrix} W_2^{-1} & G.W_1 \end{bmatrix} \right\|_\infty \quad (15)$$

Essentially with the LSDP, the weights $W1$ and W_2 are the design parameters which are chosen both to give the augmented plant a "good" open-loop shape and to ensure that γ_o is not too large. γ_0 is a design indicator of the success of the loop-shaping as well as a measure of the robustness of the stability property

Design Procedure:

A design procedure to solve the above problem is:

1-Define the plant G, and define the functional Φ_i.

2-Define the values of ε_γ and ε_i.

3-Define the form and order of the weighting functions W_1 and W_2. Bounds should be placed on the values of w_i to ensure that W_1 and W_2 are stable and minimum phase to prevent undesirable pole/zero cancellations. the order of the weighting functions, and hence the value of q, should be small initially.

4-Define the form and order of the pre-compensator K_p. Bounds may be placed on the values of pi if desired. The order of the pre-compensator transfer function, and hence the value of r, should be small initially

5-Define initial values of w_i based on the open-loop frequency response of the plant. Define initial values ofP_i.

6-Implement the MBP in conjunction with (8) and (10) to find a W_p and K_{subopt} which satisfy inequalities (13) and (14) [By solving the algebraic ricatti equation],(if a solution is found the design is satisfactory , if no solution is found , either increase the order of the weighting function or , relax one or more of the desired bounds ε , or try again with different initial values of P_i.

7- The order of the K_{opt} can be reduced (e.g using the balanced truncate method) by:

- Scaling the system to enhance the numerical stability
- Eliminate uncontrolled or observable state.
- Eliminate the states that are weakly objecting the I/O, the small hanekl singular values indicate that the associate's states are weakly coupled.
- Compare (the step response & the bode response) between the original system & the reduced one.

8- With satisfactory weighting functions W_1 , W_2 the K_{opt} a satisfactory feedback controller is obtained from (11).,

IV.Example : the synchronous generator

The proposed approach is to design a control system for a synchronous generator described in [9], the following model is revelent:

$$G_p = \frac{K}{a.S^3 + b.S^2 + c.S + d}$$

$$G_p = \frac{9}{S^3 + 5.3S^2 + 11.4S + 4.5}$$

The design specification are to design a controller which garuntees for all [K,a,b,c,d] values:

1-Closed loop stability
2- Zero steady state.
3- Settling time Ts= 4 sec.
4- Rise time T_r = 2.5 sec.
5- Peak time T_p = 0 sec .
6- Percentage over shoot = 0 % .
7- The maximum contoller output = 1 .
8- The stability margin ϵ_{max} is not less than 0.5 (50% uncertainties).

The design attempt to use the MOI to satisfy the performance design specifications for the nominal plant G using the design criteria

$$\gamma_o(W) \le \varepsilon_\gamma \quad (16)$$

$$\Phi_i(G,W,K_p) \le \varepsilon i \;, \quad \text{for} \quad i = 1, 2, \cdots, 12 \quad (17)$$

where the prescribed bound for γ_o is not fixed, but for stability robustness, it should not be too large [9], and is here taken as

$$\varepsilon_\gamma = \mathbf{5.0} \quad \mathbf{(18)}$$

The variation in parameters is described as shown in table (1):

	G1	G2	G3	G4	G5
A	1.1	0.9	1.2	0.9	1.1
B	5.83	5.83	4.77	5.83	4.77
C	12.54	10.26	12.54	10.26	10.26
D	4.95	4.95	4.05	4.05	5
K	10.5	8.1	10.2	8.1	9.3

G6	G7	G8	G9	G10
0.9	1.2	1	1.1	0.95
4.4	4.77	4.4	4.77	4.77
10.26	10.26	10.26	12.54	10.26
4.05	4.05	4.05	4.95	4.75
9.5	10.3	8.5	9.7	8.7

After many trails the new approach failed to fulfill the 9 constrains, but , it was found that the best performance was done using a compensating weighting function(W_p) :

$W_p = \dfrac{0.164}{S}$, resulting a sub-optimum controller:

$$K_{subopt} = \frac{1.8621.S^3 + 9.923.S^2 + 21.4869.S + 8.8638}{S^4 + 6.8763.S^3 + 20.3936.S^2 + 25.94855.S + 14.8181}$$

$$K_{opt} = \frac{0.164}{S} * \frac{1.8621S^3 + 9.923S^2 + 21.4869S + 8.8638}{S^4 + 6.8763S^3 + 20.3936S^2 + 25.94855S + 14.8181}$$

And by Using the balanced truncate method:

$$K_{opt(\mathbf{red})} = \frac{0.15.S + 0.1}{S}$$

It was found that the performance specifications of the closed loop system, after applying a unit step function are follows:

T_p	T_r	T_s	M_p	P_o %	S.S error	G.M	P.M	ϵ_{max}
0 sec	9 sec	15 sec	0.49	0%	0	28.9 db	-180	0.5969

Comparing the closed loop response of the new approach with a H-∞ synthesis using a PID weighting function as follows:

$$G_c = \frac{0.89.S^3 + 4.717.S^2 + 10.146.S + 4.005}{S^3 + 6.S^2 + 12.S}$$

It was found that the performance specifications of the closed loop system, after applying a unit step function are follows:

T_p	T_r	T_s	M_p	P_o %	S.S error	G.M	P.M	ϵ_{max}
0 sec	**2.11 sec**	**3.4 sec**	0.89	0%	0	18.1 db	-180	0.5627

It was found that the performance specifications of the closed loop system using (the PID weighting function), is better and more successful, however how complex it is and how long it takes to construct it, but this is done in the first round only, (the M.O.I) still has another chance, that's applying it on the robust controller to enhance it's time response without destroying it's robustness,
finally the closed loop fulfilled using a controller after enhancing it's performance Using the method of in-equalities :

$$G_c(S) = \frac{0.66863.S + 0.337}{S}$$

It was found that the performance specifications of the closed loop system, after applying a unit step function are follows:

T_p	T_r	T_s	M_p	P_o %	S.S error	G.M	P.M	ϵ_{max}
0 sec	**1.96 sec**	**3.4 sec**	0.79	0%	0	15 db	- 180	0.555

It's obvious that the new approach has it's Owen economic features (the controller of the new approach is a first order one while the one done using a (PID) weighting function is a third order one), in spite the both have the same time response and robustification features , but under certain experimental circumstances applied later see fig(9),fig(10). The new approach was successful & better even it's first order.

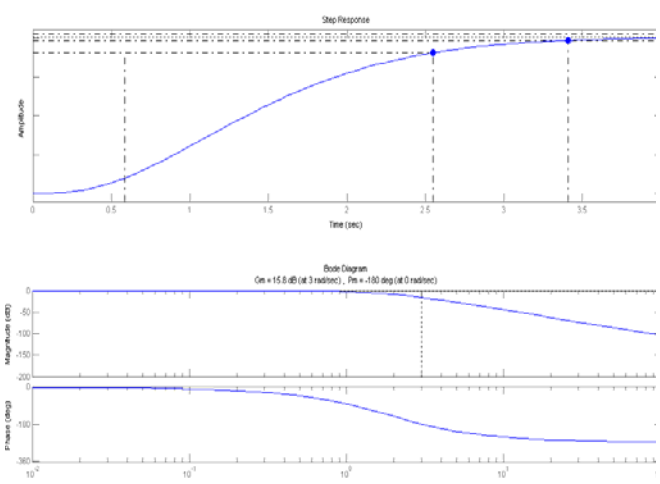


FIG (3) The system response due to step input in case of using the MOI weighting function for H- ∞ loop shaping

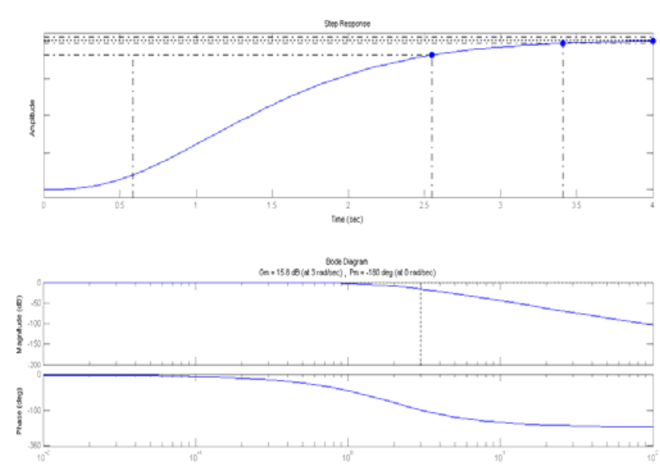
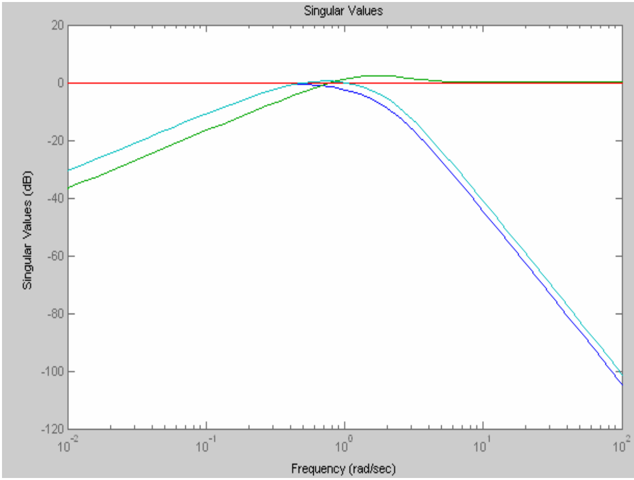
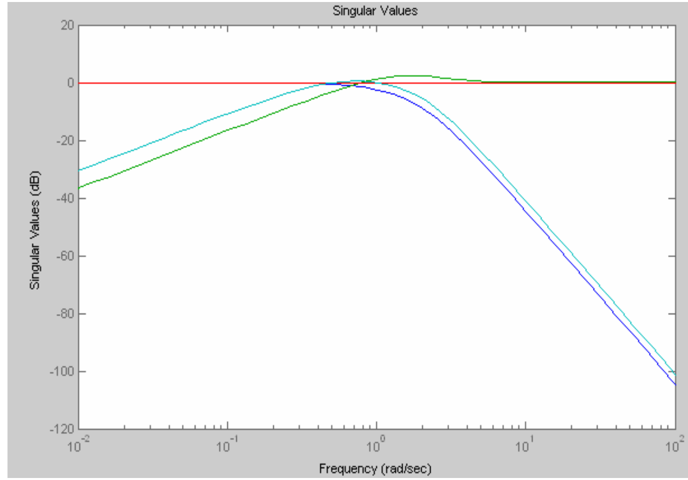


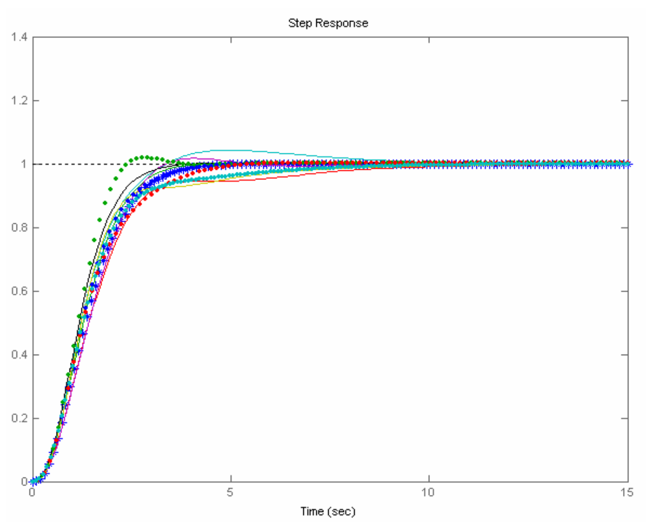
FIG (4) The system response due to step input in case of using the PID weighting function for H- ∞ synthes



Fig(5) The singular value of σ(T) the complementary sensitivity , σ(S) is the sensitivity, σ(Su)the control sensitivity, σ(So) the output sensitivity(Si) input disturbances sensitivity, σ(r)=σ(T+Si), in case of using MOI weighting function for H- ∞ loop shaping



Fig(6) The singular value of σ(T) the complementary sensitivity , σ(S) is the sensitivity, σ(Su)the control sensitivity, σ(So) the output sensitivity(Si) input disturbances sensitivity, σ(r)=σ(T+Si), in case of using a PID weighting function for H- ∞ synthesis



Fig(7) the system response due to Variation of parameters in case of using the MOI weighting function in the H- inf loop shaping

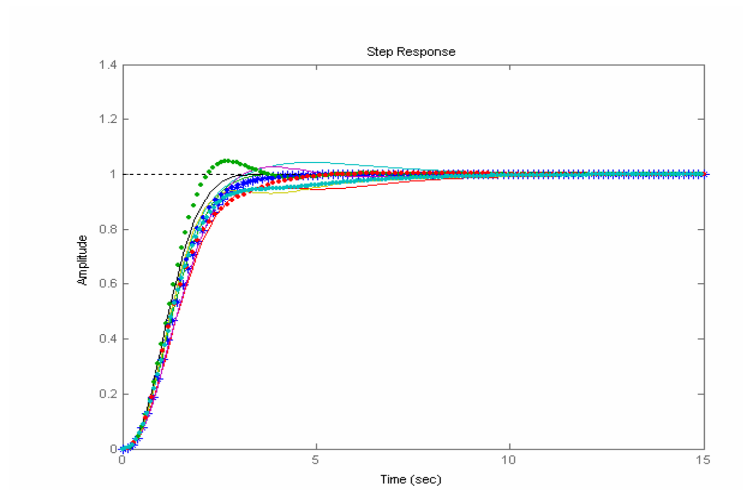


FIG (8) the system response due to Variation of parameters in case of using the PID weighting function in the H- inf synthesis

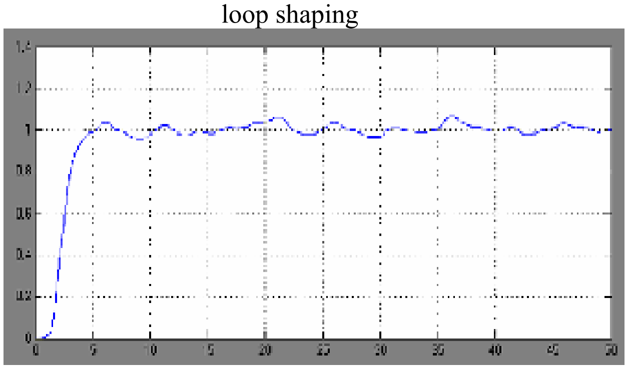


FIG (9) The system response due to a white noise of amplitude (0.01) and a sampling time (0.1) at the output and a square pulse of amplitude (0.5) for a period 0f (3 sec) and pulse width (50%), in case of using a MOI weighting function for the (H_∞ loop shaping)

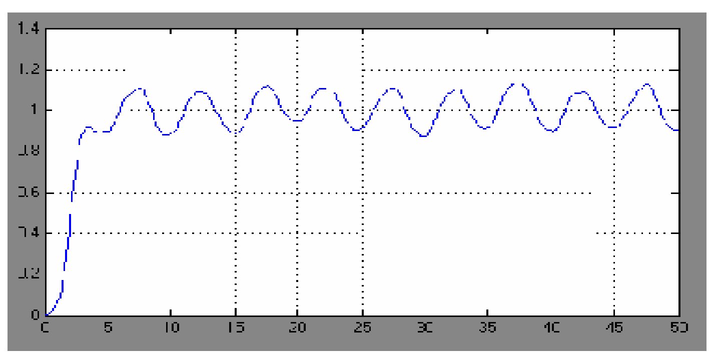


FIG (10) The system response due to a white noise of amplitude (0.01) and a sampling time (0.1) at the output and a square pulse of amplitude (0.5) for a period 0f (3 sec) and pulse width (50%), in case of using a PID weighting function for the (H_∞ synthesis)

V. Concluding Remarks :

The use of numerical methods to design the weights in an H-∞optimization problem appears to be new. The proposed method combines the flexibility of numerical optimization-type techniques with analytical optimization in an effective and practical manner, as demonstrated by the design of a controller for sync generator. The MOI is interactive, thus providing flexibility to the designer in formulating a realistic problem and in determining design trade-offs. Unlike the LSDP, closed-loop performance is explicitly considered in the formulation of the design problem and can include both time-and frequency-domain performance indices. It was found in practice, however, that the initial choice of weighting function parameters is very important in the subsequent progress of the MBP.

In the usual implementation, a search is conducted in a set of fixed-order controllers to find a feasible point.

The use of numerical methods to design the weighting functions is particularly suited to the NLCF approach because no γ-iteration is required. The MOI can be combined with $H\infty$-optimization methods which require γ-iteration [10] but the process is considerably slower.

The use of suboptimal controllers would speed up the process but the functional are infinitely discontinuous and so traditional search techniques cannot be used.

In this approach, the search is restricted to controllers which are already robustly stable, thus the problem of finding a stability region does not exist.

In this example, the performance was evaluated for a selection of extreme plant models chosen by the designer. The problem of efficiently determining the worst-case performance over the range of plants still exists, although more general measures of performance robustness could be included in the performance set. This would be of particular importance when the range of plant perturbations is less well known.

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