Journal of Coastal Research	24	1	273–277	West Palm Beach, Florida	January 2008

Application of Continuous Wavelet Analysis in Distinguishing Breaking and Nonbreaking Waves in the Wind–Wave Time Series

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ABSTRACT

ELSAYED, M.A.K., 2008. Application of continuous wavelet analysis in distinguishing breaking and nonbreaking waves in the wind–wave time series. *Journal of Coastal Research,* 24(1), 273–277. West Palm Beach (Florida), ISSN 0749-0208.

This study applied a new approach to distinguish between breaking waves and nonbreaking waves in the wind–wave time series through the use of wavelet analysis.

In this paper, the wavelet power spectrum is computed to identify the variations in the energy content of the wind–wave time series with time, and then it is integrated over frequency to provide the temporal variation of localized total energy.

The study shows that the fluctuations of the time series of wind–wave are highly intermittent, *i.e.,* the energy at different scales varies considerably with time. Furthermore, the local peaks of the energy densities correspond to the events of wave breaking in the wind–wave time series.

ADDITIONAL INDEX WORDS: *Morlet wavelet, wavelet power spectrum, intermittent.*

INTRODUCTION

Wave breaking is an important subject in the field of coastal engineering. Extensive research has been conducted to analyze wave breaking and its consequences. Breaking waves play an important role in the exchange of mass, momentum, and energy (AGRAWAL *et al.,* 1992) and gases (WALLACE and WIRICK, 1992) between the atmosphere and the ocean. Radar reflectivity is used extensively in both laboratory and field investigations of wave breaking (BANNER and PEREGRINE, 1993).

As stated by LIU (1993), "in the current stage of analyzing the wind-wave time series two methods are employed: zero-crossing method (ANONYMOUS, 1984) and spectrum analysis using Fourier transform (KINSMAN, 1984). These methods provide only general characteristics of waves in the measured time series of wind–wave and lack the ability to give local and time varying characteristics." The aims of this paper are summarized as follows:

- to use wavelet transform to analyze the variation of the energy content of the wind–wave time series with time.
- to use the technique developed by LIU, (1993, 1994) to analyze wave breaking in the measured wind–wave time series during the FETCH experiment.

RECENT STUDIES ON WAVE BREAKING

THORPE (1993) investigated the energy loss caused by wave breaking and found that breaking waves with wavelengths shorter than the dominant waves could contribute a considerable amount of energy to the total turbulent kinetic energy dissipation rate in the ocean surface mixed layer.

DAWSON, KRIEBEL, and WALLENDORF (1993) conducted a laboratory study of wave breaking in deep water with emphasis on the relative number of breaking waves observed at a certain section. The results indicated that the average

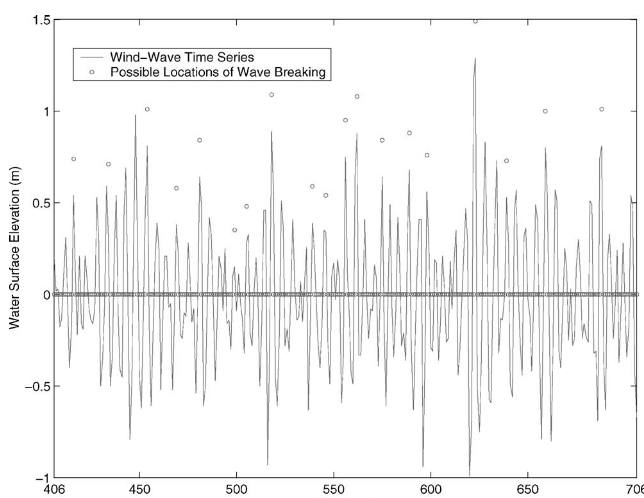

Figure 1. Wind–wave time series (segment 1).

DOI:10.2112/05-0443.1 received 18 February 2005; accepted in revision 1 June 2005.

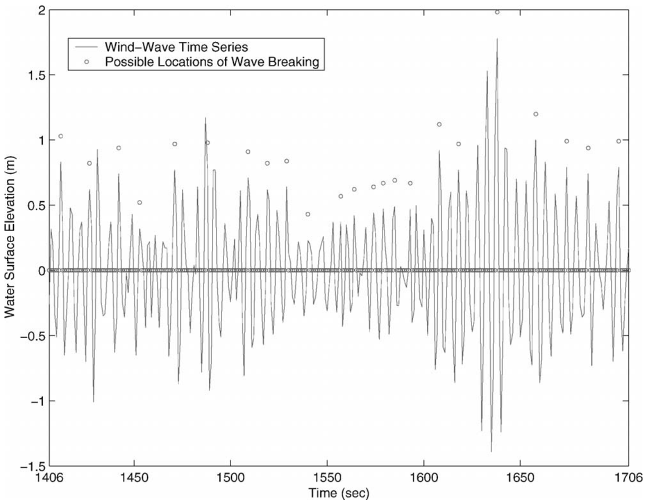

Figure 2. Wind–wave time series (segment 2).

downward crest acceleration of breaking waves is one-third the gravitational acceleration.

DING and FARMER (1994) analyzed the spatial and temporal statistics of breaking waves and found that wave breaking occurs at multiple scales.

GEMMRICH and FARMER (2004) observed that wave breaking caused high levels of turbulence. HORINOUCHI (2004) used a mesoscale meteorological model to study the breaking of waves and found that the disturbance of the initial instability developed into shear instability.

BRIDGES (2004) introduced a nonlinear theory that includes a mechanism for noisy wave breaking. MELSOM and SAETRA (2004) developed a theoretical model for the near-surface velocity profile in the presence of wave breaking and stated that momentum is accumulated by growing waves and is released upon wave breaking. ZHANG and YUAN (2004) established theoretical expressions for the effect of wave breaking on the air-sea fluxes of heat and moistures.

DATA

The data used in this work were collected during the FETCH experiment (DRENNAN *et al.,* 2003). An ASIS buoy was deployed on 18 March 1998 and remained until 10 April 1998. Two segments in the wind–wave time series are selected as follows:

(i) The first segment (Figure 1) corresponds approximately to the maximum wind speed observed during 6 hours of wave generation.

(ii) The second segment (Figure 2) contains the maximum observed wave height in the wind–wave time series.

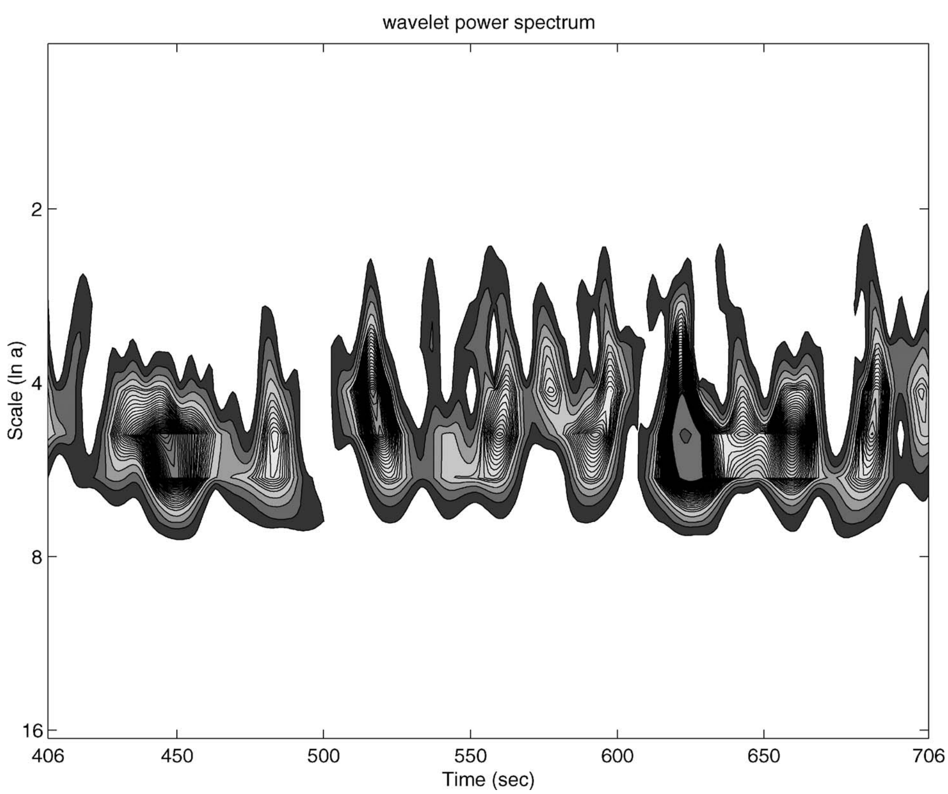

Figure 3. Wavelet power spectrum of segment 1. **For a color version of this figure, see page 137.**

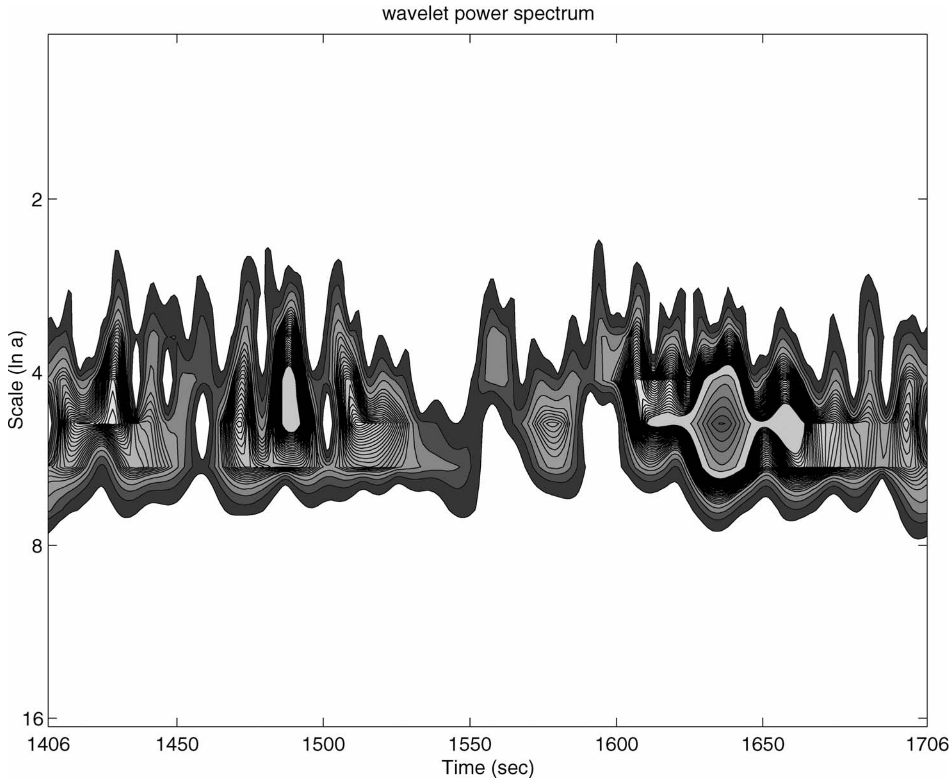

Figure 4. Wavelet power spectrum of segment 2. **For a color version of this figure, see page 137.**

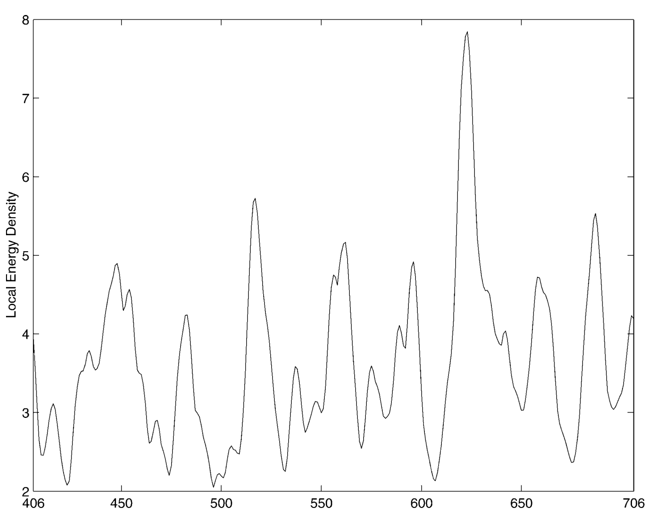

Figure 5. Temporal variations of localized energy density for segment 1.

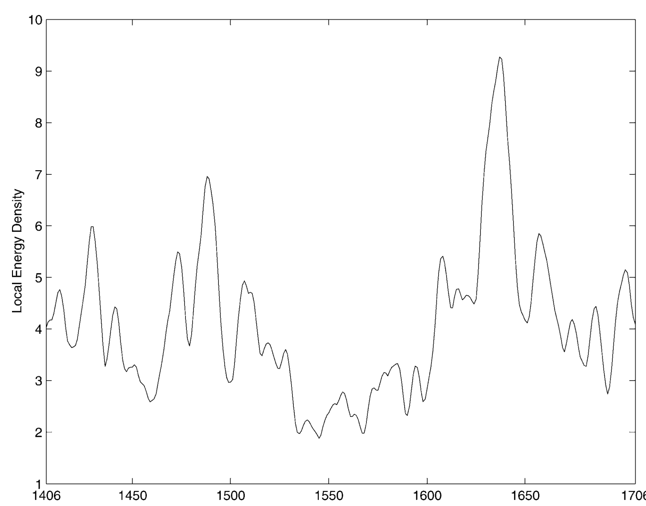

Figure 6. Temporal variations of localized energy density for segment 2.

ANALYSIS METHOD

Continuous Wavelet

The continuous wavelet transform $w(a, \tau)$ of a function $h(t)$, is defined as

$$w(a, \tau) = \int_{-\infty}^{+\infty} h(t)\, \psi^*_{a,\tau}(t)\, dt \tag{1}$$

where a and τ are scale and time variables respectively, $\psi_{a,\tau}$ represents the wavelet family generated by continuous translations and dilations of the mother wavelet $\psi(t)$, and * denotes the complex conjugate. These translations and dilations are obtained by

$$\psi_{a,\tau} = \frac{1}{\sqrt{a}}\psi\left(\frac{t-\tau}{a}\right). \tag{2}$$

Following ADDISON (2002) and TORRENCE and COMPO (1998), the complex Morlet wavelet (MORLET, 1981) to be implemented in this study is defined as

$$\psi(t) = \pi^{-1/4} e^{iw_o t} e^{-t^2/2}. \tag{3}$$

In this definition, w_o is chosen to be 6.0 to satisfy the wavelet admissibility condition (FARGE, 1992).

Local Total Energy

An integration of the wavelet transform over time gives energy spectrum, whereas integration of the wavelet transform over frequency provides a temporal variation of the localized total energy (LIU, 1993,1994).

The MATLAB code developed by TORRENCE and COMPO (1998) to compute the continuous wavelet transform of the time series of the signal is modified to compute the localized total energy.

RESULTS AND DISCUSSION

The wavelet power spectrum of the wind–wave time series of Figure 1 is given in Figure 3, and Figure 4 gives the wavelet power spectrum of the time series given in Figure 2. The wavelet power spectrum shows that wave energy fluctuates with time and that the influence of some low frequency components that are not visible in the classical energy spectrum can be seen. This conclusion was reached also by MASSEL (2001).

Wave groups are formed during wave generation and wave growth takes place within the group. Following LIU (1993, 1994), and assuming a linear dispersion relationship, the wave surface will break when its downward acceleration exceeds a fraction β of the acceleration of gravity g as follows:

$$A\eta^2 = \beta g \tag{4}$$

where A is local wave amplitude, η is local wave frequency, g is gravitational acceleration, and β is taken as equal to 0.50 in this work. The localized frequency spectrum is defined as

$$\psi_i(\zeta) = [w(\zeta, t)]_{t=t_i}, \tag{5}$$

where ψ is the local frequency spectrum and w is the wavelet transform of the signal.

Because the wave breaking is occurring in the high frequency range of the spectrum, ψ is defined as follows:

$$\eta_i = \left[\frac{\int_{\alpha\zeta_p}^{\zeta_n} \zeta^2 \psi_i(\zeta)\, d\zeta}{\int_{\alpha\zeta_p}^{\zeta_n} \psi_i(\zeta)\, d\zeta}\right], \tag{6}$$

where ζ_p is the localized frequency at the energy peak, ζ_n is the cut-off frequency, and α is a number greater than 1.0 (indicates the starting of a high frequency range).

LIU (1994) suggested the value of α to be 1.35; in this work we will use this value. The possible locations of the occurring of wave breaking are marked in both segment 1 and segment 2, as shown in Figures 1 and 2.

Figures 3 and 4 give the wavelet power spectra of the two selected segments of the wind–wave time series. The form of wave groupings is clearly indicated in these figures, which implies that the energy content of the wind–wave time series varies significantly with time.

The integration of wavelet transform over frequency gives the temporal variation of the localized energy, as shown in Figure 5 for the first segment and Figure 6 for the second segment. The peaks shown in these plots correspond to the events of wave breaking marked at the time series given in Figures 1 and 2.

SUMMARY AND CONCLUSIONS

In this study, a new technique in analyzing wave breaking is applied through the use of wavelet transform. The use of Fourier transform in the spectrum analysis has shortcomings in neglecting important temporal and localized characteristics of the signal.

The application of the wavelet power spectrum to the measured wind–wave time series shows that the energy content of the signal varies considerably with time. Furthermore, the influence of some low-frequency components could be seen that is not visible in the classical energy spectrum.

The integration of wavelet transform over frequency gives the temporal variations of the localized energy that are found to be correlated with the events of wave breaking in the wind–wave time series, and this agrees with LIU (1993, 1994).

ACKNOWLEDGMENTS

The author would like to thank Dr. William Drennan from the Rosenstiel School of Marine and Atmospheric Science, University of Miami, Florida, for providing the data necessary for conducting the present study. The constructive comments given by the two anonymous reviewers are highly appreciated.

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