

Extreme value distributions for peak pressure and load coefficients

Henry W. Tieleman*, Mohamed A.K. Elsayed,
Zhongfu Ge, Muhammad R. Hajj

*Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University,
Blacksburg, VA 24061-0219, USA*

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Abstract

The distributions of peak suction forces in separation regions on a surface-mounted prism are well represented by the Extreme Value Type I (Gumbel) distribution. The distributions of the peak coefficients representing these forces from either laboratory or field experiments are obtained either with the method of moment estimators or with the analytical Sadek–Simiu analysis. The laboratory-based results are mostly independent of the method selected to obtain the distributions. On the other hand, those of the field data obtained with the method of moment estimators are dependent on the scatter of the individual peak coefficients and their distributions are based on peaks obtained over the duration of the number of successive records, while the Sadek–Simiu method is based on the duration of just a single record.

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1. Introduction

Wind load data presented in the *ASCE 7 Standard (2002)* are based on the compression of large amount of observations into a few numbers, without specification of the fractile of the listed coefficients. The single-valued pressure coefficient to be applied at a given

*Corresponding author. Tel.: +1 540 231 6891; fax: +1 540 231 4574.
E-mail address: Tieleman@vt.edu (H.W. Tieleman).

location on a structure assumes these coefficients to be independent of incident turbulence intensity. However, wind tunnel results presented by Tieleman et al. (2003) reveal unambiguously that mean, rms, and peak pressure coefficients, and, therefore, the pressures themselves, vary indeed with turbulence intensity. It is generally accepted that sample records from wind tunnel experiments of fluctuating pressures and loads should be equivalent to 1 h in full scale or 1–2 min in the laboratory. Single peaks obtained from each of these short records may vary in magnitude from one sample record to another. Therefore, a stable estimate of extreme pressures and loads, at a certain level of non-exceedence, can only be derived from their distribution requiring a set of peaks each obtained from a single sample record that is one of a large set of sample records. For field observations, this requirement is a problem as wind conditions are seldom stationary for periods of 1 h or longer. Although the condition of stationarity in the wind tunnel is no problem, data acquisition and data handling for a large number of sample records requires excessive wind tunnel operating time including data analysis. In order to overcome these problems, a new procedure, developed by Sadek and Simiu (2002) from the National Institute of Standards and Technology (NIST), is employed to obtain from a single sample record wind load provisions at any level of non-exceedence. The method utilizes the full extent of the pressure and load time histories obtained from either field or laboratory experiments, permitting the future design of wind load sensitive parts of low-rise structures to be based on more accurate information.

2. Experimental

Surface pressure data analyzed in this work were obtained from the experimental building (9.1 m × 13.7 m × 4.0 m) at the wind engineering research field laboratory (WERFL) at Texas Tech University. The data consist of records of 15-min duration that were filtered at 10 Hz and sampled at 40 Hz. The laboratory data analyzed in this work are based on wind tunnel experiments of the 1:50 scale model of the WERFL building. The model was exposed to turbulent flows generated in the Clemson boundary-layer wind tunnel over seven different floor-roughness configurations (Tieleman et al., 2001). The turbulence intensities for the different configurations at roof height varied between 7.1% and 19.3% representing open terrain (category C) with a roughness-length range between 0.005 and 2.75 cm, respectively. Time histories from two sets of eight pressure taps provided the input for the analysis. Set A is located along a line normal to the leading edge at locations corresponding to the WERFL pressure taps 50900–50909 (Levitan and Mehta, 1992). Observations from this set were made for a zero azimuth angle, with the flow normal to the leading edge. Set B consists of eight pressure taps along a straight line originating from the roof corner and making an angle of 15° with the leading edge (Fig. 1). The amplified output signals from the pressure transducers were passed through tuned equalizers, filtered at 200 Hz, and sampled at 2000 Hz. The duration of the time histories obtained from the wind tunnel model is 2 min. For each flow condition and each azimuth angle, a total of 16 repeat records were acquired and analyzed. In order to obtain a more stable peak distribution, each of the 2-min sample records was divided into four 30-s sample records to obtain a total of 64 peaks. Tieleman et al. (2003) and Levitan and Mehta (1992) provide the details of the experimental setup, data acquisition and analysis of both laboratory and field experiments, respectively.

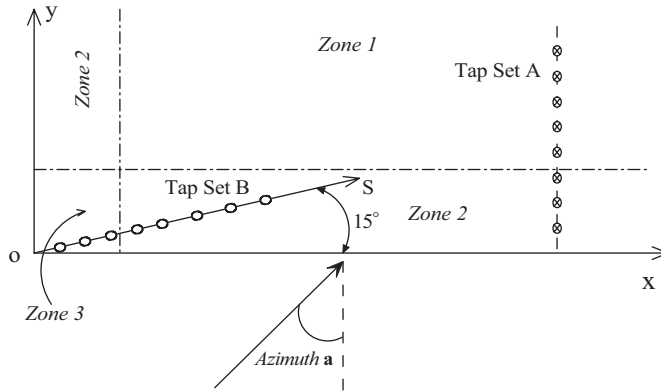


Fig. 1. Diagram of the pressure tap locations.

The analysis of the observed time histories were based on the output of individual pressure taps, but also on the pressure loads associated with set A or set B. For this purpose, simultaneous samples of each tap in the set were multiplied by a specific weighting coefficient associated with its respective tributary area and then added to form a time history of the load coefficients.

3. Estimation of peaks

Recent research has revealed that the time-varying pressures and loads in zones of separation are generally non-Gaussian (Sadek and Simiu, 2002; Tieleman et al., 2003). With wind tunnel experiments using multiple records, it is possible to obtain reliable peak distributions that can best be represented by the Gumbel distribution. The Type I Extreme Value distribution exhibits no curvature and appears as a straight line on Gumbel probability paper (Fig. 2), and is therefore a two-parameter distribution. Only for an extremely large number of sample records, requiring many hours of wind tunnel time, can the three parameters of the Weibull (Extreme Value Type III) distribution be determined with acceptable accuracy (Holmes and Cochran, 2003).

With the relative small sample size of 64 peak coefficients, each obtained from 64 independent records, the fitting of a Type I distribution seems to be most appropriate. For this distribution, the cumulative distribution function (CDF) takes the following form:

$$F(x) = \exp \left\{ - \exp - \frac{(x - \mu)}{\beta} \right\}, \tag{1}$$

where μ is the location parameter and β is the scale parameter. On Gumbel paper, this expression plots as

$$-(\ln(-\ln F(x))) = \frac{x}{\beta} - \frac{\mu}{\beta}. \tag{2}$$

With the CDF of the peaks plotted on Gumbel paper, μ and β can be obtained graphically. An alternate method to obtain these parameters is the use of the method of moments

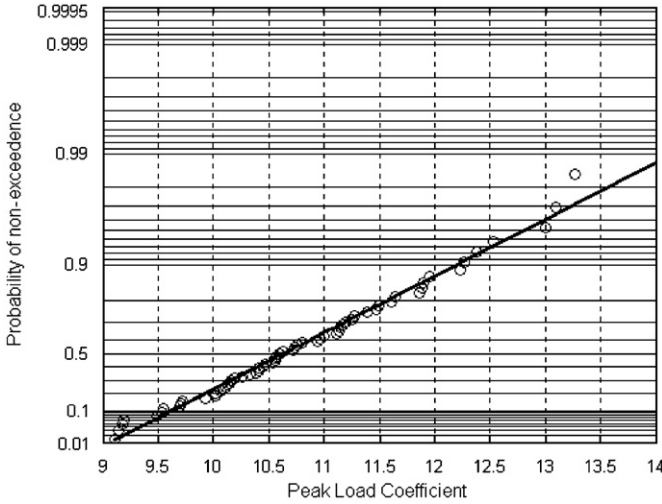


Fig. 2. Extreme value distribution of 64 peak pressure coefficients for tap 2, set B, configuration 8.

(NIST/SEMATEch e-Handbook of Statistical Methods):

$$\beta = \frac{p' \sqrt{6}}{\pi} \quad \text{and} \quad \mu = P - 0.5772 \beta, \tag{3}$$

where P and p' are the mean and the standard deviation of the sample of peaks, respectively. Comparison of the estimates of peak load coefficients based on these two methods shows that for a probability of 95% of non-exceedence, the peaks obtained with expression (3) exceed on average those obtained with the graphical method by 8.6%. In the case when only a single record is available, one must resort to the theoretical procedure developed for non-Gaussian time histories. This procedure was introduced by Sadek and Simiu (2002) for a data-base-assisted design to estimate the distribution of peaks of the internal forces induced by strong winds in low-rise building frames. This method is now applied to sample records of surface pressures and load coefficients for both laboratory and field data to obtain the distributions of their peak coefficients.

The preliminary step for this procedure is to identify the appropriate marginal distribution of the parent time history. Experimental results have revealed that the three-parameter gamma distribution fits the non-Gaussian pressure and load time histories quite well, and that good estimates for the three parameters can be obtained with the method of moment estimation (Sadek and Simiu, 2002). The general formula for the probability density function (PDF) of the gamma distribution is

$$f(x) = \left\{ \frac{(x - \mu)}{\beta} \right\}^{\gamma-1} \exp \frac{\{-(x - \mu)/\beta\}}{(\beta \Gamma(\gamma))} \quad \text{for } x > \mu, \tag{4}$$

where β , γ and μ are the scale, shape and location parameters, respectively, and $\Gamma(\cdot)$ is the gamma function. The condition that $x > \mu$ requires that for the negative suction pressures the time histories be multiplied by -1 .

The routine estimation of the three parameters involves knowledge of the skewness, S , the standard deviation, x' , and the mean, X , of the parent time history. For the gamma

distribution estimates for the three parameters based on the method of moments can be obtained from the following expressions:

$$\text{Shape parameter : } \gamma = \left(\frac{2}{S}\right)^2, \quad (5)$$

$$\text{Scale parameter : } \beta = \frac{x'S}{2}, \quad (6)$$

$$\text{Location parameter : } \mu = X - \frac{2x'}{S}. \quad (7)$$

It should be noted that for near-normal distributions, the skewness is low and, therefore, the value of γ becomes large and $\Gamma(\gamma)$ approaches infinity. Only for pressure records acquired with the low turbulence incident flows, whose skewness is frequently less than 0.35, is the normal distribution more appropriate. Once the optimum marginal probability distribution and its defining parameters of the parent time history are established, the CDF of the peaks can be obtained with the standard translation process as described in detail by Sadek and Simiu (2002). From the obtained CDF, one can derive by differentiation the PDF from which the mean (P) and the rms (p') follows. Assuming that the Gumbel distribution is appropriate for the peaks, its two parameters can then be obtained with the method of moments (3). Finally, estimates of peak values at any level of non-exceedence can be obtained with the expression for the Gumbel distribution.

4. Results

When peaks are available from multiple independent pressure records from the same tap obtained under identical flow conditions, two methods can be employed to obtain the distribution of the peaks. Analysis of the 64 peaks from the 30-s records has revealed that their cumulative distributions are well represented with the Extreme Value Type I (Gumbel) distribution (Fig. 2). For this distribution, the scale and the location parameters can be obtained from the slope and intercept of the Gumbel plot of the peaks (2). In case the number of peaks is not large enough, these parameters can be obtained from the moment estimators (3). Previously, it was found that the estimation of the peaks at a level of 95% non-exceedence based on the moment estimators from observed wind tunnel peaks systematically exceeded those obtained graphically by 8% or slightly higher (Tieleman et al., 2004).

The availability of field records for this article is limited to eight 15-min records for tap 50901 and only four records for the corresponding loads. Since more peak pressure coefficients are available for tap 50901 and tap 50501, the suitability of the Gumbel distribution for field data is tested by using these peak coefficients with the method of moment estimators (3). Figs. 3 and 4, representing the Gumbel plots of these peak pressure coefficients reveal that the Gumbel distribution for these field data is inadequate. Obviously, the field data are affected by non-stationary atmospheric conditions that can influence the peak pressure coefficients (Tieleman et al., 2001). The pressure coefficients for tap 50501 are also appreciably affected by the variation with azimuth angle that for the normal flow for tap 50901 is much less. Indeed, the results of tap 50501 exhibit more deviation from the expected Gumbel distribution that requires linearity when plotted on

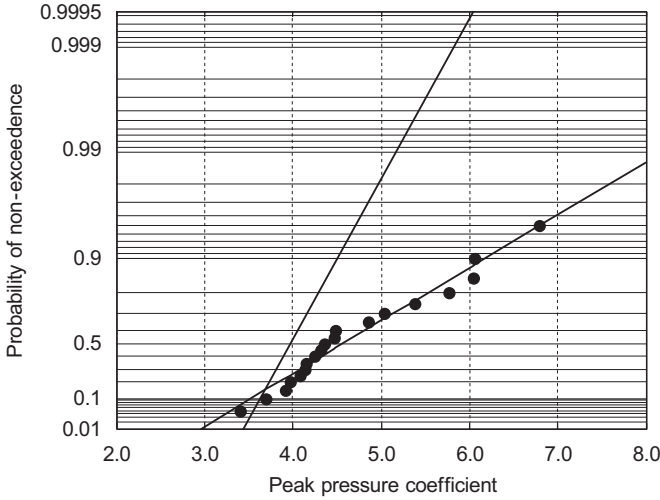


Fig. 3. Extreme value distribution of individually observed peak pressure coefficients and that obtained from 15-min WERFL records, tap 50901, azimuth 0°.

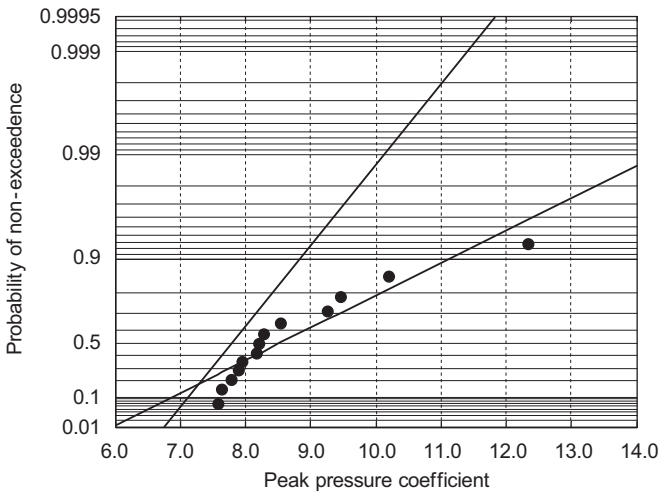


Fig. 4. Extreme value distribution of individually observed peak pressure coefficients and that obtained from 15-min WERFL records, tap 50501, azimuth 25°.

Gumbel paper. The parameters of the Gumbel distribution based on the graphical and moment-estimator methods in addition to those obtained with the Sadek–Simiu procedure for both taps are presented in Table 1. Although results for the same taps were not available from the 16 repeat wind tunnel records, scale and location parameters obtained with the moment estimator and Sadek–Simiu methods for tap 50900 (set A) and the first tap (S1) for set B are included in Table 1 for comparison purpose. The WERFL results reveal a much smaller scale parameter when individual records are analyzed with the

Table 1
Gumbel parameters

Tap 50901			Tap 50501			Method	Source
Scale	Location	#	Scale	Location	#		
0.861	4.242	20	1.378	8.013	14	Graphical	WERFL
0.793	4.354	20	1.253	8.272	14	M.E.	WERFL
0.298	3.739	8	0.553	7.540	3	S-S	WERFL
Tap 50900			S1				
Scale	Location	#	Scale	Location	#		
0.148	2.44	16	0.214	9.74	16	M.E.	Conf. 1
0.050	1.90	16	0.275	10.17	16	S-S	Conf. 1
0.441	3.41	16	0.778	14.67	16	M.E.	Conf. 2
0.202	3.30	16	0.921	14.61	16	S-S	Conf. 2
0.269	3.81	16	0.710	12.09	16	M.E.	Conf. 6
0.210	3.50	16	0.765	12.63	16	S-S	Conf. 6
0.424	4.59	16	0.696	14.28	16	M.E.	Conf. 7
0.253	4.26	16	0.774	13.83	16	S-S	Conf. 7
0.226	4.09	16	0.955	16.38	16	M.E.	Conf. 3
0.266	4.16	16	1.085	16.43	16	S-S	Conf. 3
0.430	5.05	16	1.062	17.38	16	M.E.	Conf. 4
0.344	5.09	16	1.366	18.29	16	S-S	Conf. 4
0.605	4.69	16	1.491	18.05	16	M.E.	Conf. 8
0.313	4.74	16	1.298	18.03	16	S-S	Conf. 8

Note: M.E.: moment estimators; S-S: Sadek–Simiu.

Sadek–Simiu procedure in comparison to the scale parameters obtained from observed peak pressure coefficients with either the graphical or moment-estimator methods. In general, large values for the standard deviation of the sample of observed peaks give rise to large values for the scale parameters and consequently small values for the slope of the Gumbel plot (2). This condition indicates a large dispersion of the peak coefficients. In Figs. 3 and 4, the distribution of the lower valued peaks indicate a much steeper slope in agreement with the lower magnitude of the scale parameter and, therefore, matching the steeper slope obtained with the Sadek–Simiu analysis. Included in these figures is the distribution of the peak pressure coefficients from the WERFL records using their average scale and location parameters obtained with the Sadek–Simiu analysis (Table 1). The average scale parameters for the WERFL records are much lower than those based on the individually observed peak coefficients and, therefore, exhibit a much steeper slope.

The extreme value plots based on eight 15-min records of tap 50901 using the Sadek–Simiu analysis are presented in Fig. 5 together with the extreme value distribution based on the moment estimator of the corresponding eight observed peak pressure coefficients. The larger dispersion of the observed peaks results in a high value of the standard deviation and, therefore, a higher scale parameter in comparison to those obtained with the Sadek–Simiu procedure. The latter are based on individual 15-min records while the method of moments utilizes eight observed peaks over a 2-h period.

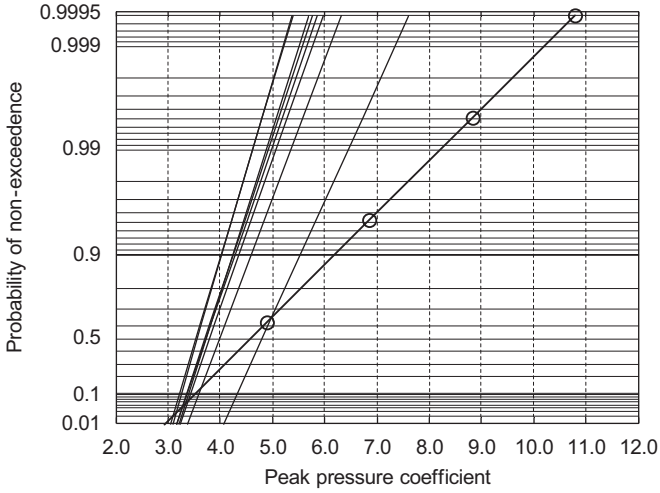


Fig. 5. Extreme value distributions of WERFL peak pressure coefficients for tap 50901 obtained from 8 records with the Sadek–Simiu procedure and the method of moments of the corresponding observed peaks (heavy line with symbols).

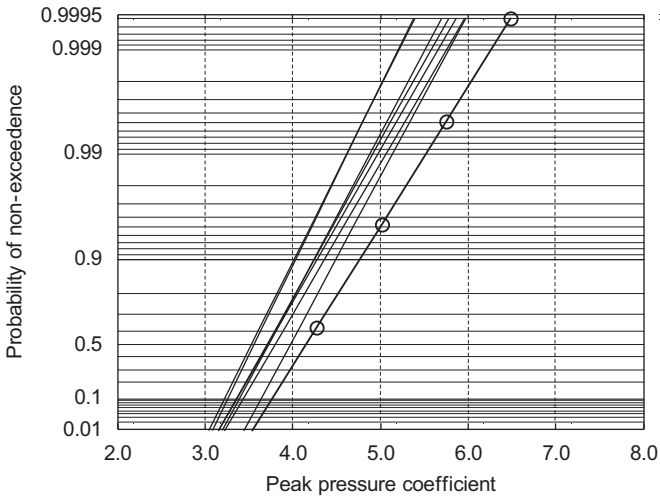


Fig. 6. Extreme value distributions of WERFL peak pressure coefficients for tap 50901 obtained from seven records with the Sadek–Simiu procedure and the method of moments of the corresponding observed peaks (heavy line with symbols).

The eight records were obtained on four different dates and different times of the day. When one of the records with a much larger observed peak coefficient is eliminated from the sample, the resulting scale parameter based on the seven remaining observed peaks fall in line with those obtained with the Sadek–Simiu analysis (Fig. 6).

The Gumbel plots of the peak pressure coefficients for taps 50901 and 50501 (S5) based on the moment estimators of the 16 peaks from the wind tunnel experiments are presented in Figs. 7 and 8. The distribution of the WERFL peak pressure coefficients based on 20

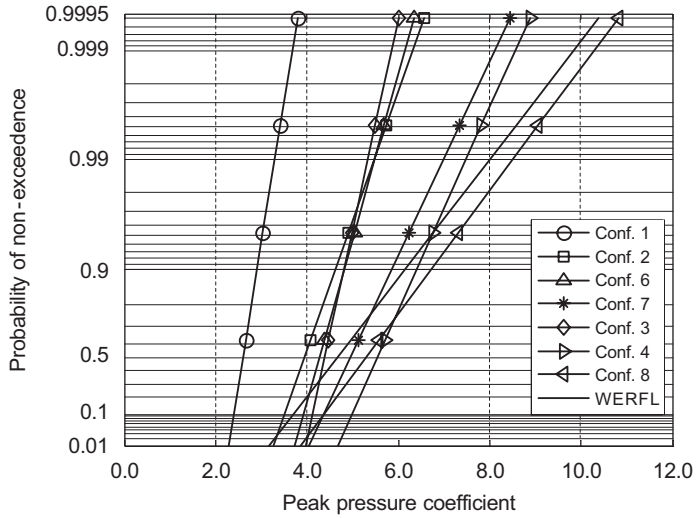


Fig. 7. Extreme value distributions of peak pressure coefficients of tap 50901 for the seven configurations and the available WERFL field data, derived with the method of moments.

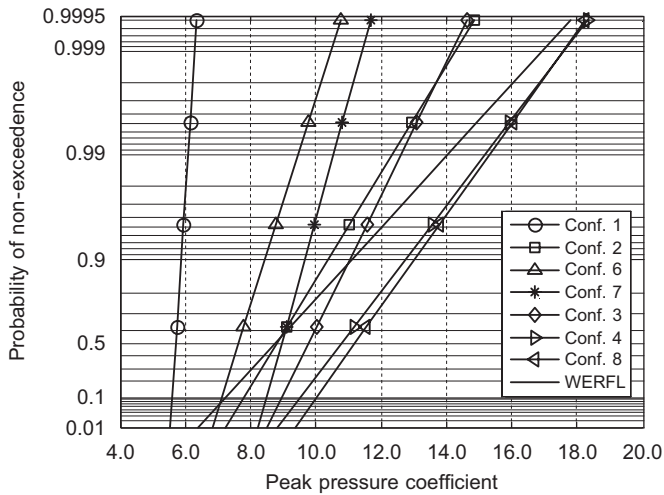


Fig. 8. Extreme value distributions of peak pressure coefficients of tap 50501 (S5) for the seven configurations and the available WERFL field data, derived with the method of moments.

observations for tap 50901 and 16 for tap 50501 are included for comparison. The order of the configurations in the legend of these figures is associated with increasing turbulence intensity of the incident flow. The distributions based on the wind tunnel results generally reveal increase in magnitude and in dispersion with the higher turbulence intensities. The distribution of the WERFL observations for tap 50901 is closely matched with the distribution for configuration 8. For tap 50501, the WERFL distribution exhibits a higher

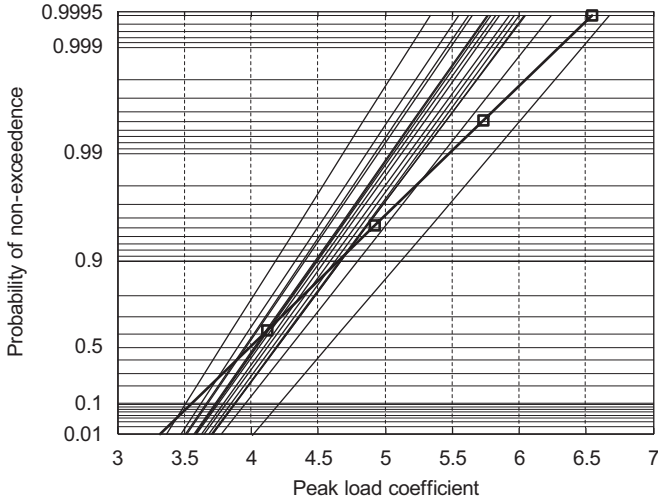


Fig. 9. Extreme value distributions of the peak load coefficients obtained from the 16 individual repeat records (Sadek–Simiu procedure) compared with the distribution obtained from the 16 observed peak load coefficients (method of moments), set A, configuration 4.

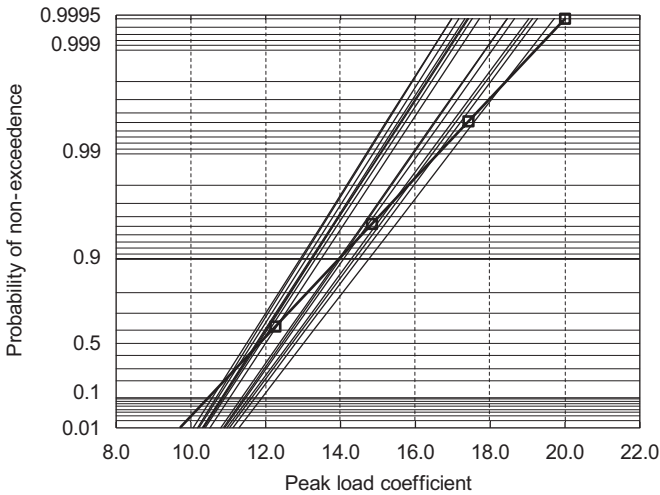


Fig. 10. Extreme value distributions of the peak load coefficients obtained from the 16 individual repeat records (Sadek–Simiu procedure) compared with the distribution obtained from the 16 observed peak load coefficients (method of moments), set B, configuration 8.

dispersion than the wind tunnel results, as is evident from its slope being less than those of the wind tunnel distributions.

The distributions for the peak load coefficients of the 16 repeat records obtained with the Sadek–Simiu analysis are presented in Figs. 9 and 10 for set A, configuration 4 and set B, configuration 8. The distributions exhibit an appreciable scatter at higher levels of non-exceedence. Also shown are the distributions based on the moment estimators obtained

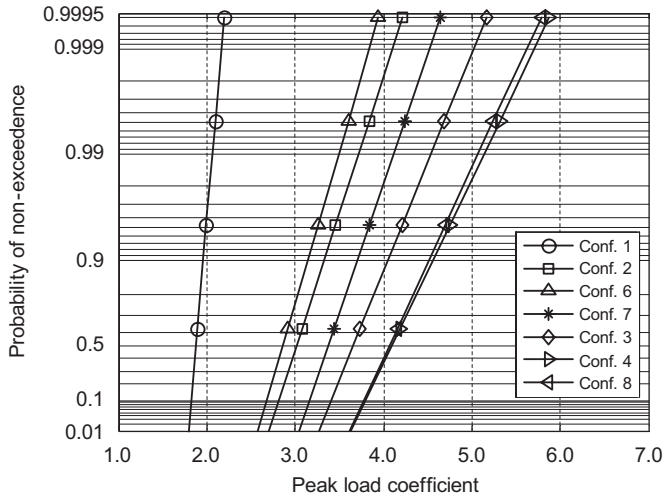


Fig. 11. Extreme value distributions of peak load coefficients for set A using the average scale and location parameters obtained from the 16 repeat records with the Sadek–Simiu procedure.

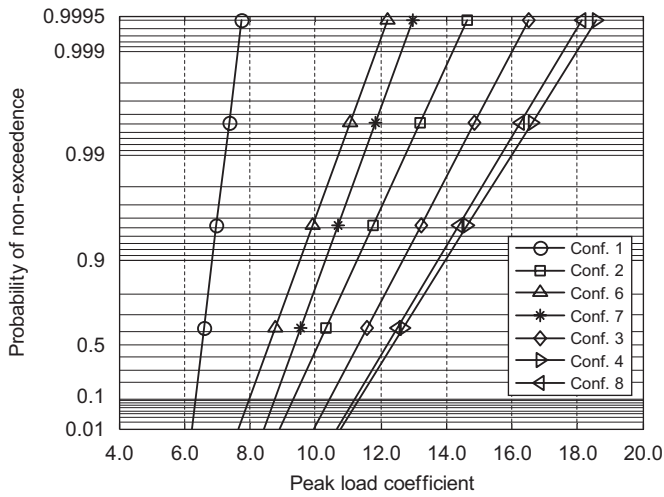


Fig. 12. Extreme value distributions of peak load coefficients for set B using the average scale and location parameters obtained from the 16 repeat records with the Sadek–Simiu procedure.

from the average and standard deviation of the 16 observed peaks for each 2-min record. These distributions reveal an increased level of dispersion as is evident from the lower slopes in comparison with those of the 2-min records obtained with Sadek–Simiu analysis. The distributions of the peak load coefficients based on the averages of the scale and location parameters of the sixteen 2-min records for each configuration and for the two sets of pressure taps are presented in Figs. 11 and 12, respectively. One can observe the increase in dispersion and increase in magnitudes with increasing turbulence intensity.

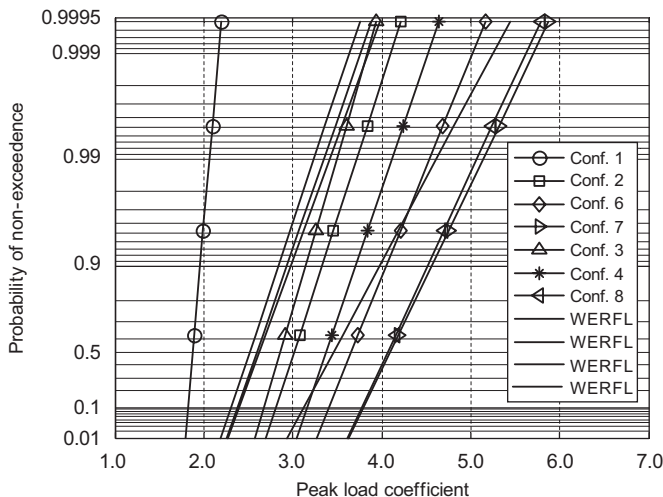


Fig. 13. Extreme value distributions of peak load coefficients obtained with the Sadek–Simiu procedure for set A using the average of 16 scale and location parameters compared with those from four field records.

Fig. 13 is identical to Fig. 11 (set A) except the distributions of the load coefficients from four WERFL 15-min records are added for comparison. Three of the WERFL distributions are nearly identical and match the distribution of configuration 6, while one WERFL distribution is closely matched with configuration 3 but with somewhat greater dispersion.

5. Conclusions

A new procedure is employed for estimating peaks of individual non-Gaussian time histories representing pressure forces under separated shear layers on the upper surface of a surface-mounted prism in a variety of turbulent boundary layers.

The distribution of these time histories, required for the input of the procedure, is well represented by the Gamma distribution. The two parameters for the Gumbel distribution of the suction peaks can then be estimated with the use of a standard translation processes approach. Results from the application of the procedure to wind tunnel and field data are presented and compared with the distribution obtained from individual peaks of pressure and load coefficients with the method of moment estimation.

The Gumbel distributions of the peak pressure and load coefficients obtained with the Sadek–Simiu procedure from sixteen 2-min wind tunnel records under identical flow conditions vary significantly at the higher levels of non-exceedence. The distributions obtained with the method of moments based on the 16 observed peaks from each 2-min record, exhibit a slightly higher dispersion than those obtained with the Sadek–Simiu procedure. Field and wind tunnel distributions both derived with the Sadek–Simiu procedure vary in similar fashion without appreciable differences.

In general, it was observed that discrepancies between the distributions obtained with the two methods are due to a higher standard deviation of the observed pressure coefficients. This leads to higher values of the scale parameter and to milder slopes of the

distribution when obtained with the method of moments and a higher dispersion of the peaks. This phenomena was particularly observed for the field data where the distributions from individual 15-min records with the Sadek–Simiu analysis are compared with distributions obtained with a number of single peaks obtained from the same number of 15-min records frequently observed at different times with varying non-stationary atmospheric conditions.

Wind engineers must realize that wind tunnel experiments of the atmospheric flows never duplicate the non-stationary conditions that occur in the atmosphere. Consequently, it should not come as a surprise that peak distributions obtained over a long period with the method of moments exhibit much greater dispersion of the peaks than the distributions obtained over a much shorter period with the Sadek–Simiu procedure.

Acknowledgments

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