Journal of Coastal Research	26	3	535–540	West Palm Beach, Florida	May 2010

An Overview of Wavelet Analysis and Its Application to Ocean Wind Waves

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ABSTRACT

ELSAYED, M., 2010. An overview of wavelet analysis and its application to ocean wind waves. *Journal of Coastal Research*, 26(3), 535–540. West Palm Beach (Florida), ISSN 0749-0208.

Wavelet analysis is considered a state-of-the-art technique in signal processing; it transforms the signal into a scale-time representation (scalogram) with high resolution, preserving the temporal characteristics of the signal. In the present study, an overview of Fourier analysis, short-time Fourier analysis, and wavelet analysis are given, and the differences between them are addressed. To demonstrate the importance of wavelet analysis in the field of coastal engineering and other related fields, a summary of the recent literature concerning its applications is given first, and then wavelet analysis is used to analyze the complex phenomenon of wave growth due to a sudden change in the wind conditions. This study shows that the temporal characteristics of the time series of wind speed and significant wave height can be explored qualitatively by using the complex Morlet wavelet analysis. The results reveal that the waves are responding immediately to the sudden change in wind conditions.

ADDITIONAL INDEX WORDS: *Morlet wavelet, Fourier analysis, short-time Fourier analysis, wave growth.*

INTRODUCTION

Wavelet analysis has proved to be a useful tool in numerous applications in the engineering and science fields. The basic advantage of wavelet-based analysis is its ability to unfold signals into both time and scale and thus overcome the shortcomings of applying Fourier-based analysis to nonstationary signals.

Compared with short-time Fourier transformations (STFT), which is also thought better than the Fourier analysis in processing nonstationary signals, wavelet analysis ensures better performance because it yields better time-frequency resolution through the use of more-flexible window widths (Mallat, 1999).

The aims of this article are as follows:

- To give an overview of wavelet analysis and the advantages of applying it over the classical Fourier analysis and STFT methods
- To summarize the recent literature about wavelet applications in the field of coastal engineering and other related disciplines
- To apply wavelet analysis in studying the complex phenomenon of wave growth due to a sudden change in wind conditions in order to better understand this complicated phenomenon

DOI: 10.2112/04-0274.1 received 24 June 2004; accepted in revision 12 February 2010.

FOURIER ANALYSIS

Fourier analysis is one of the most popular methods of signal processing. The Fourier transform of a piecewise continuous function $h(t)$ is defined as:

$$\hat{h}(f) = F[h(t)] = \int_{-\infty}^{+\infty} h(t)e^{-i2\pi ft}dt \tag{1}$$

where f is the frequency variable in the frequency domain and $i = \sqrt{-1}$.

Therefore, Fourier analysis can be regarded as a mathematical technique for transforming the signal from time-based to frequency-based. However, Fourier analysis has a shortcoming in transforming from time domain to frequency domain because the frequency content of the signal does not change in time, assuming that the signal is stationary.

SHORT-TIME FOURIER ANALYSIS

To overcome the deficiency of Fourier transform in preserving the time-varying characteristics of any signal, Gabor transform, or short-time Fourier transform, can be applied to the signal. In Gabor transform, only a small section of the signal is analyzed by using windowing technique. This technique maps a signal into a two-dimensional function of time and frequency, giving information about both when and at what frequencies a signal event occurs.

However, in STFT, the size of the window is fixed for all frequencies, and the time resolution is controlled by the size of the selected window. Therefore, in order to have a better resolution in time, a narrow window should be used; however, narrowing the window causes the frequency resolution to become poorer.

CONTINUOUS WAVELET ANALYSIS

While frequency analysis is performed by projecting a signal onto a number of sinusoids which are infinite in extent, wavelet analysis is performed by projecting the signal onto a set of highly localized basis functions. These basis functions are called wavelets and are obtained from a single "mother" wavelet by dilations and translations. Thus, in wavelet analysis, the notion of a scale replaces that of frequency, which leads to a scale-time representation (scalogram) with a high resolution. Since it is localized in time, a scale representation is more suitable than a frequency representation for examining temporal characteristics of the signal.

The continuous wavelet transform $w(a,\tau)$ of a function $h(t)$ is defined as:

$$w(a,\tau) = \int_{-\infty}^{+\infty} h(t)\psi^{*}{}_{a,\tau}(t)dt \tag{2}$$

where a and τ are scale and time variables, respectively, and $\psi_{a,\tau}$ represents the wavelet family generated by continuous translations and dilations of the mother wavelet $\psi(t)$. These translations and dilations are obtained by:

$$\psi_{a,\tau} = \frac{1}{\sqrt{a}}\psi\left(\frac{t-\tau}{a}\right) \tag{3}$$

The Morlet wavelet (Addison, 2002; Torrence and Compo, 1998) to be implemented in this study is defined as:

$$\psi(t) = \Pi^{-1/4}e^{iw_{o}t}e\frac{-t^{2}}{2} \tag{4}$$

In this definition, w_o is chosen to be 6.0 to approximately satisfy the wavelet admissibility condition.

Figure 1 gives a comparison between a Fourier spectrum and a wavelet spectrum using a typical 10-minute time series datum of surface wave fluctuations (Liu, 2000a). The energy frequency spectrum is given by the blue line in Figure 1b, the wavelet spectrum is given in Figure 1c, and Figure 1d gives the Fourier spectrum of the signal. It can be deduced from this figure that wavelet analysis gives a detailed picture of the variation of the energy spectrum with time, whereas Fourier analysis does not detect this variation.

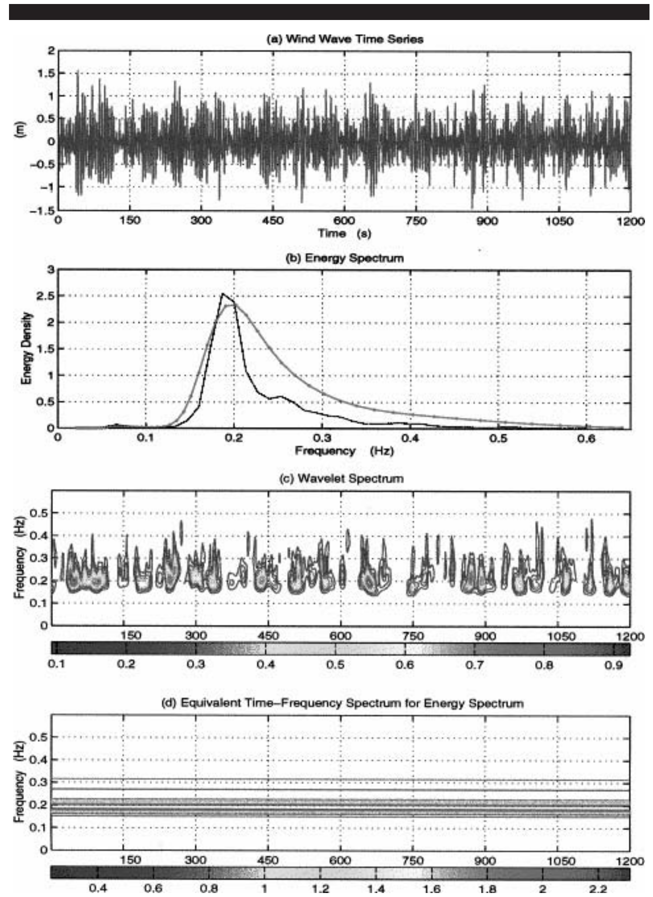

Figure 1. Comparison of a wind-wave energy spectrum and a wind-wave wavelet spectrum. a) Wind-wave time series, b) Fourier spectrum, c) Wavelet spectrum, d) Equivalent Wavelet spectrum for Fourier spectrum. (Adapted from Liu, 2000a, Figure 1).

APPLICATIONS OF WAVELET ANALYSIS: A REVIEW

In their effort to investigate coastal morphological evolution using artificial neural network (ANN) models, Bazartseren and Holz (2002) used a wavelet transform and principal component analysis (PCA) for down sampling the data of the cross-shore bathymetry. They concluded that the ANNs adapted the wavelet decomposition coefficients better than PCA; hence the wavelet transform might be a good alternative for profile data sampling. Little (1994) showed that wavelet analysis was a useful tool for decomposing the texture of the seafloor, which consequently helps in understanding the processes that occur at many different scales.

Meyers, Kelly, and O'brien (1993) applied wavelet analysis to their study of the dispersion of Yanai waves; they found that the narrowing of the range of wavelengths supports the hypothesis that the narrow range of frequencies observed in western equatorial oceans is a consequence of Yanai wave dispersion, and that these results would not have been achieved using classical Fourier transformation. Aksoy, Akar, and Unal (2004) applied wavelet analysis to an annual mean suspended sediment discharge series and compared it to a moving-average process fitted to the data set. The results of their study showed that the wavelet-based approach is an alternative to the traditional autoregressive moving-average models.

Gurley and Kareem (1999) stated that analysis and simulation of nonstationary processes involving wind, wave, and earthquake engineering applications were accomplished by decomposition into localized basis functions *via* the discrete or continuous wavelet transform. Jordan, Hajj, and Tieleman (1997) applied the Morlet wavelet to determine the time variation of the energy of different turbulence scales. By using the continuous Morlet wavelet, it was possible to characterize turbulence scales of the velocity components of wind in the atmospheric surface layer. Jordan, Miksad, and Powers (1997)

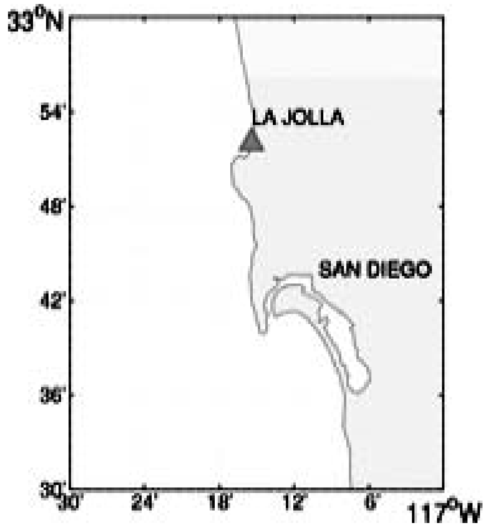

Figure 2. Location map showing the recording station of ocean data at La Jolla, California, United States.

used the complex Morlet wavelet to analyze the velocity fluctuations measured in a subsonic wake undergoing transition to turbulence. The results of their study reinforced the fact that the temporal characteristics of the measured signal of velocity could not be obtained from spectral analysis.

Stamos and Hajj (2001) used continuous wavelet analysis to separate the incident and reflected wave components in their experimental work to compare the reflection and transmission characteristics of a water-filled flexible breakwater model to those of a rigid model with the same dimensions. Willemsen (1995) applied the discrete wavelet packet transform in his study of wave response to wind activity. The use of wavelet analysis was suitable for detecting the cut examples of sea response to a sudden change in wind activity.

Gourdeau (1998) used wavelet analysis in a study of internal tides in the ocean. In a study of air-sea interaction (Savtchenko *et al.*, 1999), the application of wavelet analysis to surface wave data revealed uniformly distributed instances of bursts and sweeps over a dominant wind-wave profile. Shen, Wang, and Mei (1994) applied the Morlet wavelet in his study of the fine structure of wind waves and showed that the conservation law is not suitable for describing the dynamics of short wind-wave components in the equilibrium range when the fetch is not too short. In their study of subtidal coastal sea level fluctuations, Percival and Mofjeld (1997) showed that wavelet analysis guarantees additivity to both multiresolution and variance analyses.

Liu (1994, 2000a, 2000b, 2002) carried out a wavelet spectrum analysis to study wave grouping effects, wave growth, and wave breaking. It was deduced that the temporal characteristics of these phenomena were identified by using wavelet analysis.

Massel (2001) analyzed the time series of wind-induced deep water waves, waves breaking on the tropical coral reefs, and mechanically generated waves in the wave flume using wavelet analysis. The study showed that wavelet analysis was capable of detecting the complex variability of these signals in the time-frequency domain.

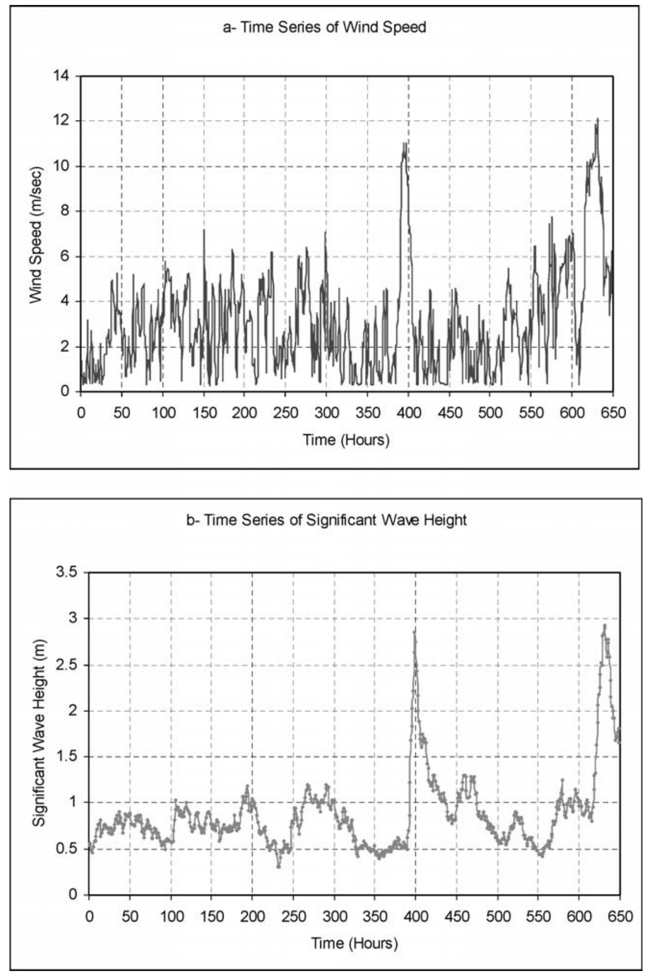

Figure 3. Time series of wind speed and significant wave height measured at Scripps Pier, La Jolla, California (February, 1997).

Haung (2004) conducted a preliminary study to identify wave parameters and functions in wavelet analysis and introduced a new parameter which is highly correlated to significant wave height and significant wave period and hence can be used to study wave growth and decaying phenomena.

APPLICATION OF WAVELET ANALYSIS TO OCEAN WAVES

The Collected Ocean Data

The data used in the present study were recorded at Scripps Pier, La Jolla, California (32°52.000′ N, 117°15.400′ W) with a water depth of 6.80 m. According to National Oceanic and Atmospheric Administration documentation, the pier deck is 10.25 m above mean low low water, which is the common zero tide mark referenced. This station was selected because the wind and waves were measured simultaneously on an hourly basis with a frequency of 1 Hz, and this type of detailed

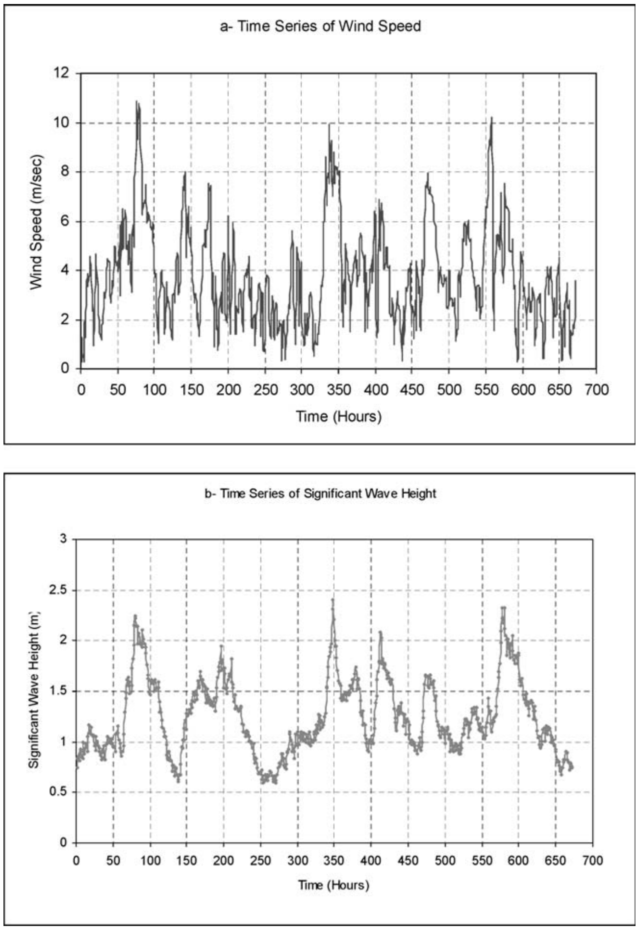

Figure 4. Time series of wind speed and significant wave height measured at Scripps Pier, La Jolla, California (February, 1998).

information was not available at other stations. Figure 2 gives the location map of the wind-wave recording station.

The time series of wind speed and significant wave height (the average height of the highest one-third of the waves in the record) for February 1997 (Figure 3) and February 1998 (Figure 4) were selected because these time series have the feature of a sudden change in wind conditions, and hence they are suitable for studying the wave growth due to a sudden change in wind conditions.

Wave Growth Due to a Sudden Change in Wind Conditions

When the wind blows over the surface of the ocean it generates waves. The size of the waves depends on the amount of energy supplied by the wind. Wavelet analysis provides a powerful tool for analyzing the temporal characteristics of the wind speed and the significant wave height signals. The wavelet transform contours for the time series of wind speed and significant wave height for February 1997 are given in Figures 5 and 6, respectively, while the wavelet transform contours for the time series of wind speed and significant wave height for February 1998 are given in Figures 7 and 8, respectively. The contours ascend from the outside contour to the inside contour.

The wavelet transform contours show that the high-energy contours appearing in wavelet transform contours of wind speed and significant wave height represent "pulses" of energy of a certain scale over a certain time, and these high-energy contours correspond to the times of a sudden increase in wind speed. It can be deduced that the wind and waves are

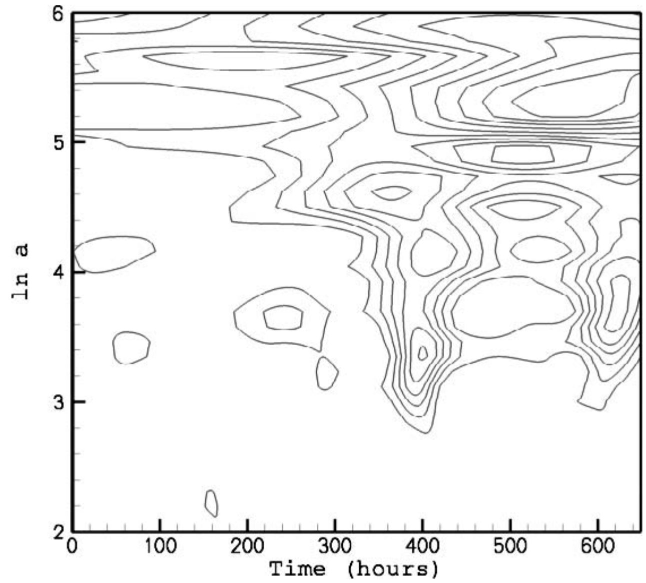

Figure 5. Wavelet transform contour for the time series of wind speed given in Figure 3.

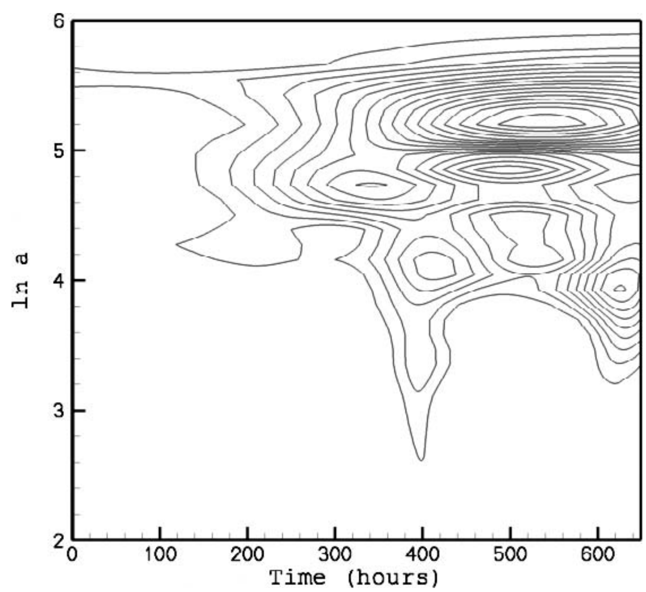

Figure 6. Wavelet transform contour for the time series of significant wave height given in Figure 3.

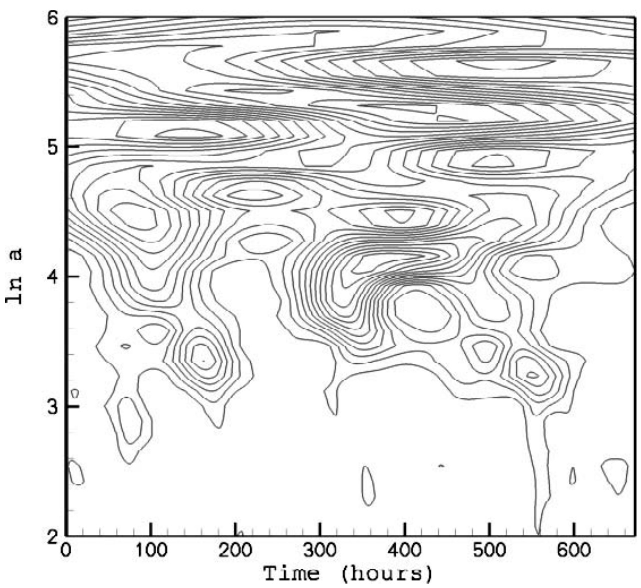

Figure 7. Wavelet transform contour for the time series of wind speed given in Figure 4.

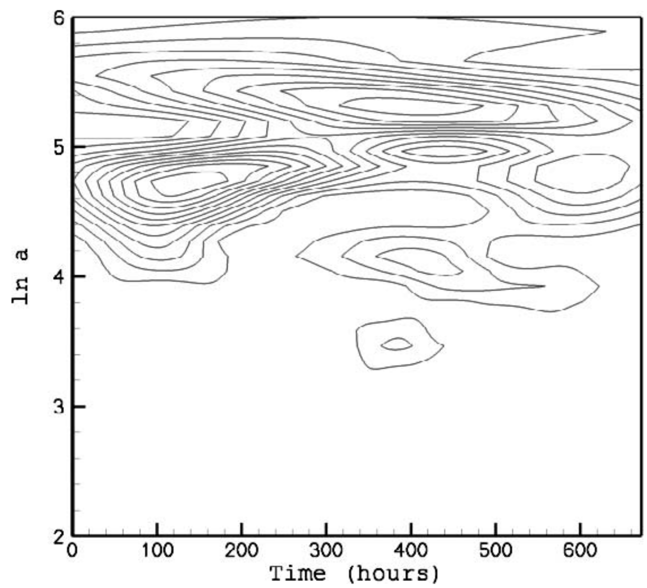

Figure 8. Wavelet transform contour for the time series of significant wave height given in Figure 4.

interacting simultaneously during wave growth, which is consistent with the findings of Liu (1994).

SUMMARY AND CONCLUSIONS

For analyzing nonstationary signals, wavelet analysis overcomes the shortcomings of classical Fourier analysis and the limitations on the window width of STFT. This study shows that wavelet analysis is a powerful tool for studying the temporal characteristics of signals, and hence it has many useful applications in the field of coastal engineering, as well as other fields. The application of wavelet analysis to study wave growth due to a sudden change in wind conditions shows that high-energy contours occur in the wavelet transform contours for both wind speed and significant wave height over the same scale ranges. It can be deduced that the waves are responding immediately to any change in wind conditions, which is consistent with the results of Liu (1994). Further research on other wind-wave time series should be conducted using cross wavelet spectrum analysis, because cross wavelet spectrum analysis will allow for exploring more details of wind-wave interaction. It can be concluded that the results of wavelet analysis are promising and constitute a basis for future studies of wind-wave interaction.

ACKNOWLEDGMENT

The author thanks the Scripps Institution of Oceanography Coastal Data Information Program for making data available.

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