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ABSTRACT

Mathematical formulation for Magnetohydrodynamic (MHD) stability of a streaming cylindrical model penetrated by varying transverse magnetic field is presented. Eigen value relation is derived and discussed analytically. In the current paper, Artificial Neural Network (ANN) model, one of the artificial intelligence techniques, is developed to simulate the stability of streaming jet penetrated by magnetic field. The ANN results presented in the current study showed that ANN technique, with less effort and time, is very efficiently capable of simulating and predicting the effect of magnetic field variation and axial exterior field on the stability of the streaming jet. The influence of magnetic field has a stabilizing effect for all short and long wavelengths. However the streaming is strongly destabilizing.

Keywords: Magnetohydrodynamic, Streaming jet, Magnetic field, Numerical simulation, Artificial neural network.

1. INTRODUCTION

There are many applications of Magnetohydrodynamic stability in several fields of science such as geophysics, astrophysics and engineering. In geophysics, the fluid of the core of the Earth and other theorized to be a huge MHD dynamo that generates the Earth's magnetic field due to the motion of the liquid iron. MHD applies quite well to astrophysics since 99% of baryonic matter content of the universe is made of plasma, including stars, the interplanetary medium, nebulae and jets, stability of spiral arm of galaxy, *etc*. Many astrophysical systems are not in local thermal equilibrium, and therefore require an additional kinematic treatment to describe all phenomena within the system. In engineering, there are many forms include oil and gas extraction process if it surrounded by electric field or magnetic field, gas and steam turbines, MHD power generation systems and magneto-flow meters, *etc*.

The classical stability analysis of a full fluid jet has been extensively studied by Rayleigh [1], Chandrasekhar [2], Cheng [3], Kendall [4], and Drazin and Reid [5] and documented by Radwan [6]. The latter author studied the Hydromagnetic stability of a fluid jet pervaded by uniform magnetic field for axisymmetric perturbation. Radwan [7] developed the magnetohydrodynamic stability of that model for all axisymmetric and non-axisymmetric modes subject to electromagnetic forces generated due to constant magnetic field. The stability of different models under the action of selfgravitating force in addition to other forces has been elaborated by Radwan and Hasan [8] and [9]. Hasan [10] has discussed the stability of oscillating streaming fluid cylinder subject to combined effect of the capillary, selfgravitating and electrodynamic forces for all axisymmetric and non-axisymmetric perturbation modes. Artificial intelligence has proven its capability in simulating and predicting the behavior of the different physical phenomena in most of the engineering fields. Artificial Neural Network (ANN) is one of the artificial intelligence techniques that have been incorporated in various scientific disciplines. Kheireldin [11] presented a study to model the hydraulic characteristics of severe contractions in open channels using ANN technique. The successful results of his study showed the applicability of using the ANN approach in determining relationship between different parameters with multiple input/output problems.

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input/output problems. Allam [12] used the artificial intelligence technique to predict the effect of tunnel construction on nearby buildings which is the main factor in choosing the tunnel route. Allam, in her thesis, predicted the maximum and minimum differential settlement necessary precautionary measures. Mohamed [13] proposed an artificial neural network for the selection of optimal lateral load-resisting system for multi-story steel frames. Mohamed, in her master thesis, proposed the neural network to reduce the computing time consumed in the design iterations. Abdeen [14] utilized ANN technique for the development of various models to simulate the impacts of different submerged weeds' densities, different flow discharges, and different distributaries operation scheduling on the water surface profile in an experimental main open channel that supplies water to different distributaries. Abdeen and Hodhod [15] introduced the (ANN) technique to investigate the effect of light local weight aggregate on the performance of the produced light weight concrete. Gaafar *et al*. [16] introduced the (ANN) technique to simulate and predict the acoustic properties of some tellurite glasses.

In the present paper, the magnetohydrodynamic stability of a fluid jet pervaded by transverse varying magnetic field while its surrounding tenuous medium is penetrated by uniform magnetic field is investigated analytically. ANN model is developed to understand, simulate and then predict the stability of the fluid jet.

2. ANALYTIC FORMULATION

We consider an incompressible, inviscid and conducting fluid column of radius R_o in the basic state.

A transverse varying magnetic field is assumed to be pervaded interior the fluid, viz.,

$$\underline{H}_o = \left(0, \frac{H_o r}{R_o}, 0\right) \tag{1}$$

The surrounding tenuous medium of the fluid jet is penetrated by the axial homogenous magnetic field

$$\underline{H}_o^{tn} = (0, 0, \alpha H_o) \tag{2}$$

where α is parameter while H_o is the intensity of the transverse magnetic field across the fluid surface at $r = R_o$.

The fluid streams in the initial state with velocity

$$\underline{u} = (0, 0, U_o) \tag{3}$$

where U_o is (uniform) the amplitude of the velocity $\underline{u}$. The components of $\underline{H}_o$ and $\underline{H}_o^{tn}$ are considered along the utilizing cylindrical coordinates (r, φ, z) with the z-axis coinciding with the axis of the cylinder (see Fig. 1)

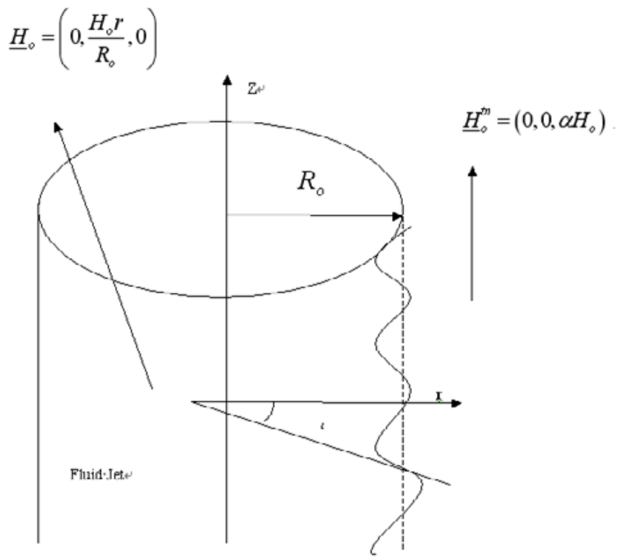

Fig. 1 Sketch for MHD fluid jet

The basic magnetodynamic equations appropriate for studying such a problem may be formulated as follows:

Interior the fluid cylinder

$$\rho \frac{d\underline{u}}{dt} = -\nabla P + \mu(\nabla \wedge \underline{H}) \wedge \underline{H} \tag{4}$$

$$\nabla \cdot \underline{u} = 0 \tag{5}$$

$$\frac{\partial \underline{H}}{\partial t} = \nabla \wedge (\underline{u} \wedge \underline{H}) \tag{6}$$

$$\nabla \cdot \underline{H} = 0 \tag{7}$$

$$P_s = T\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = T\left(\nabla \cdot \hat{\underline{N}}_s\right) \tag{8}$$

where r_1 and r_2 are the principle radii of curvature and T is the surface tension coefficient. $\hat{\underline{N}}_s$ is the unit outward vector normal to the surface, given by

$$\hat{\underline{N}}_s = \nabla f(r, \varphi, z; t) / \left|\nabla f(r, \varphi, z; t)\right| \tag{9}$$

The space ∇ and the time d/dt operators are given by

$$\nabla \equiv \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \varphi}, \frac{\partial}{\partial z}\right), \quad \frac{d}{dt} = \frac{\partial}{\partial t} + (\underline{u} \cdot \nabla) \tag{10}$$

Here $\rho, \underline{u}$ and P are the fluid mass density, velocity vector and kinetic pressure, $\underline{H}$ and $\underline{H}^{tn}$ are the magnetic field intensity of fluid and of tenuous medium surrounding the fluid jet. μ is the magnetic field permeability coefficient.

In the surrounding tenuous medium outside the fluid jet, the basic equations are

$$\nabla \cdot \underline{H}^{tn} = 0 \tag{11}$$

$$\nabla \wedge \underline{H}^{tn} = 0, \quad \text{there is no current} \tag{12}$$

For small departures from the initial state, every physical quantity could be expressed as

$$Q(r, \varphi, z; t) = Q_o(r) + \varepsilon(t)\, Q_1(r, \varphi, z) \tag{13}$$

where Q stands for $\underline{u}, P, \underline{H}$ and $\underline{H}^{tn}$ while, $\varepsilon(t)$ is the amplitude of the perturbation, (Chandrasekhar [2]) being

$$\varepsilon(t) = \varepsilon_o \exp(\sigma t) \tag{14}$$

Here σ is the growth rate at time t and $\varepsilon_o(= \varepsilon$ at $t = 0)$ is the initial amplitude.

Based on the expansion (13), the perturbed fluid-tenuous interface may be expressed as

$$r = R_o + R_1(\varphi, z, t) \tag{15}$$

with

$$R_1 = \varepsilon_o \exp\left(i(kz + m\varphi) + \sigma t\right) \tag{16}$$

Here R_1 is the elevation of the surface wave measured from the unperturbed position, where k is the longitudinal wavenumber and m is the azimuthal wavenumber. By an appeal to the perturbation technique, we may write

$$Q_1(r, \varphi, z) = Q_1^*(r) \exp\left(i(kz + m\varphi)\right) \tag{17}$$

Consequently, the linearized system of equations are solved and the required boundary conditions are applied, we finally obtained the dispersion relation

$$(\sigma + i\,kU_o)^2 = \frac{\mu H_o^2}{\rho R_o^2}\left[-m^2 + \alpha^2 x^2 \frac{K_m(x) I_m'(x)}{K_m'(x) I_m(x)}\right] + \left(\frac{T}{\rho R_o^3}\right)\left(\frac{x\, I_m'(x)}{I_m(x)}\right)(1 - m^2 - x^2) \tag{18}$$

where $I_m(kr)$ and $K_m(kr)$ are the modified Bessel functions, respectively, of the first and second kind of order m while $x(= k\ R_o)$ is the non-dimension longitudinal wavenumber.

The numerical analysis has been carried out in order to identify and examine the magnetic field influence and surface tension and also the effect of the streaming on stability of the model. In addition to that the oscillation states and the transition points from these states to those of instability may be also determined for given values of the magnetic field intensity.

This has been elaborated by computing the non-dimension dispersion relation

$$\sigma^* = \sqrt{h\left(m^2 + \alpha^2 x^2 \frac{K_m(x) I_m'(x)}{K_m'(x) I_m(x)}\right) + \left(\frac{x\, I_m'(x)}{I_m(x)}\right)(1 - m^2 - x^2)} \ + U \tag{19}$$

where

$$\sigma^* = \left(\sigma / \sqrt{\frac{T}{\sigma R_o^3}}\right), \quad h = \left(\frac{H_o}{H_s}\right), \quad U = \left(-ikU_o / \sqrt{\frac{T}{\sigma R_o^3}}\right) \tag{20}$$

In the computer simulation for the most dangerous sausage mode $m = 0$ for the different values of (α) and h, where $H_s\left(= (T / \mu R_o)\right)$ has the dimension of magnetic field while $\sigma(= i\omega)$ is the growth rate and ω is the oscillation frequency.

The numerical data associated with $\omega / (T / \rho R_o^3)^{1/2}$ corresponding to the stable states and those associated with $\sigma / (T / \rho R_o^3)^{1/2}$ corresponding to the unstable states are collected, tabulated and presented graphically taking into account $\alpha = 1.0$.

3. ANN, ARTIFICIAL INTELLIGENCE TECHNIQUE

Neural networks are models of biological neural structures. Briefly, the starting point for most networks is a model neuron as shown in Fig. 2. This neuron is connected to multiple inputs and produces a single output. Each input is modified by a weighting value (w). The neuron will combine these weighted inputs with reference to a threshold value and an activation function, will determine its output. This behavior follows closely the real neurons work of the human's brain. In the network structure, the input layer is considered a distributor of the signals from the external world while hidden layers are considered to be feature detectors of such signals. On the other hand, the output layer is considered as a collector of the features detected and the producer of the response.

The resulting output, Y, is an input to the next layer or it is a response of the neural network if it is the last layer. In applying the Neural Network technique, in this study, Neuralyst Software, Shin [17], was used.

To fully investigate the effect of longitudinal wave number, magnetic field and fluid stream velocity on the growth rate an artificial intelligence technique, using ANN approach, is developed in this study. The developed simulation model used one solution output obtained from the analytic approach to design the ANN model.

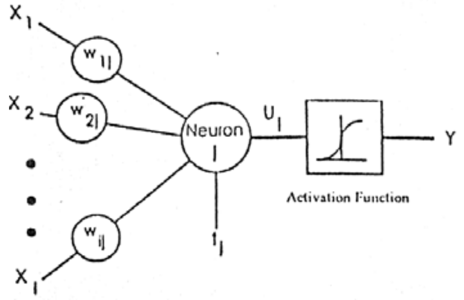

Fig. 2 Typical Picture of a Model Neuron

3.1 Neural Network Design

To develop neural network (NN) model to simulate the growth rate, first input and output variables have to be determined. Input variables are chosen according to the nature of the problem and the type of data that would be collected. To clearly specify the key input variables for the neural network model and their associated outputs, Table 1 are designed to summarize all neural network key input and output variables

Several neural network architectures are designed and tested for the numerical model investigated in this study to finally determine the best network model to simulate, very accurately, the effect of wave number, magnetic field and fluid stream velocity on the growth rate based on minimizing the Root Mean Square Error (RMS-Error).

The training procedure for the developed NN model, in the current study, uses one raw of the data from the analytic results to let the ANN understands the behavior. After sitting finally the NN model, this model is used to predict the growth rate for different values of x, h and U.

Table 2 shows the final neural network models for the three boundary conditions and their associate number of neurons. The input and output layers represent the key input and output variables described previously. It is important to mention here that the structure of the three models are chosen to be the same but the difference between them will be in RMS-Error and the number of trials to achieve accepted accuracy represented by maximum percentage relative error.

The parameters of the various network models developed in the current study are presented in Table 3, where these parameters can be described with their tasks as follows:

Table 1 Key Input and Output Variables for Neural Network Model

ANN Model	Input Variables			Output
MHD Model	x Wave Number	h Magnetic Field	U Fluid Stream Velocity	σ^* Growth Rate

Table 2 The Developed Neural Network Models

ANN Model	No. of layers	No. of Neurons in each layer			
		Input Layer	First Hidden	Second Hidden	Output Layer
MHD Model	4	3	5	3	1

Table 3 Parameters used in the Developed Neural Network Models

ANN Model	Simulation Parameters				
	TRT	TST	Training Epochs	RMS-Error	MPRE
MHD Model	0.01	0.03	239008	0.009	4.89

Training Tolerance (TRT): Defines the percentage error allowed in comparing the neural network output to the target value to be scored as "Right" during the training process = 0.001 in the current study.

Testing Tolerance (TST): It is similar to Training Tolerance, but it is applied to the neural network outputs and the target values only for the test data = 0.003 in the current study.

Training Epochs: Number of trails to achieve the present accuracy.

Percentage Relative Error (PRR): Percentage relative error between the numerical results and actual measured value for and is computed according to Eq. (6) as follows:

PRE = (Absolute Value (ANN_PR - AMV)/AMV) × 100

where

ANN_PR : Predicted results using the developed ANN model
AMV : Actual Measured Value
MPRE : Maximum percentage relative error during the model results for the training step (%)

4. RESULTS AND DISCUSSION

4.1 Stability Discussion

Equation (18) is the desired capillary dispersion relation of a streaming fluid column pervaded by varying transverse magnetic field and surrounded by uniform magnetic field. This relation is valid for all axisymmetric and non-axisymmetric perturbation modes. It relates the growth rate σ with the wave numbers k and m, the problem parameters μ, H_o, ρ, R_o and the speed U_o of the streaming fluid, and with the modified Bessel functions I_m and K_m and their derivatives.

For axisymmetric perturbation (m = 0), the general dispersion (18) reduces to

$$(\sigma + i\,kU_o)^2 = \left(\frac{-\mu H_o^2}{\rho R_o^2}\right)(\alpha^2 x^2)\frac{K_0(x)I_1(x)}{K_1(x)I_0(x)} + \left(\frac{T}{\rho R_o^3}\right)\left(\frac{x\,I_1(x)}{I_0(x)}\right)(1-x^2) \qquad (21)$$

For the lowest non-symmetric perturbation mode (m = 1), the dispersion relation (18) yields

$$(\sigma + i\,kU_o)^2 = \frac{\mu H_o^2}{\rho R_o^2}\left[-1+\alpha^2 x^2 \frac{K_1(x)I_1'(x)}{K_1'(x)I_1(x)}\right] - \left(\frac{T}{\rho R_o^3}\right)\left(\frac{x^3\,I_1'(x)}{I_1(x)}\right) \qquad (22)$$

For the higher non-axisymmetric perturbation $m \geq 2$, the eigen value relation of such case may be easily obtained from Eq. (18) as $m \geq 2$.

By the use of the recurrence relations (Abramowitz and Stegun [18])

$$2I'_m(x) = -I_{m+1}(x) + I_{m-1}(x) \qquad (23)$$

$$2K'_m(x) = -K_{m-1}(x) - K_{m+1}(x) \qquad (24)$$

and that $I_m(x)$ is monotonically increasing and positive definite for every non-zero value of x, we have

$$I_m(x) > 0 \qquad (25)$$

The function $K_m(x)$ is monotonically decreasing but never negative for since $x \neq 0$ values

$$K_m(x) > 0 \qquad (26)$$

Upon using (25) and (26) for the relations (23) and (24), we get

$$I'_m(x) > 0 \ , \quad K'_m(x) < 0 \qquad (27)$$

for all axisymmetric and non-axisymmetric perturbation modes $m \geq 0$. By an appeal to the recurrence relations (23) and (24) and the inequalities (27) for the stability criterion (18), we deduce the following.

1. The toroidal varying magnetic field interior the fluid has no influence at all on the stability of the fluid column.
2. The uniform exterior magnetic field is stabilizing for all short and long wavelengths in all kinds of perturbation $m \geq 0$.
3. The streaming is strongly destabilizing.

Therefore, the magnetohydrodynamic streaming fluid column is not completely stable in the axisymmetric mode $m = 0$, but there will be exist some unstable domains.

The stability discussions for the case of the lowest non-axisymmetric mode ($m = 1$), may be carried out by utilizing the relation (22). It is found that both the interior toroidal and exterior axial magnetic fields are stabilizing. Note that the streaming has a strong destabilizing effect.

Therefore, the present model is (MHD) unstable in the lowest non-axisymmetric mode ($m = 1$) of perturbation.

In a similar way, one may show that the fluid column is MHD unstable for any higher non-axisymmetric mode $m \geq 2$ of perturbation.

Physically, the stabilizing effect of the exterior magnetic field is expected because it has been assumed that the pervading magnetic field is uniform.

Moreover, the stabilizing effect of the toroidal magnetic field in the fluid region is due to the influence of Lorentz force that comes out from the interaction of the magnetic induction and the electric current produced due to the pervading magnetic field.

The magnetic pressure $(\mu/2)(\underline{H} \cdot \underline{H})$ per unit area acting in all directions of the fluid (resistivity is neglected here) and an equal magnetic tension $(\mu/2)(\underline{H} \cdot \underline{H})$ per unit area acting along the magnetic lines of force. Due to these stresses the lines of force are able to endow the fluid with a sort of rigidity. The magnetic fields exert strong influence not only to the axisymmetric mode ($m = 0$) that causes only the bending of the magnetic lines of force but also to non-axisymmetric modes that lead to twisting of the lines of force.

The effect of magnetic field and streaming jet velocity on the surface tension are presented analytically and numerically in Figs. 3 and 4. From Fig. (3) one can see that, for a certain value of magnetic field ($h = 0.7$), by increasing the streaming jet velocity the domains of instability increases. It is clear from figure (4) that, for a certain value of streaming jet velocity ($U = 0.7$), the instability domains are decreasing with increasing (H_o/H_s).

4.2 ANN Discussion

Numerical results using ANN technique will be discussed in this section for the neural network (NN) model to show the simulation and prediction powers of ANN technique studying the effect of wave number, magnetic field and fluid stream velocity on the growth rate.

Figures 3 and 4 show some of the ANN results with analytic ones. It is very clear, from these figures, that the developed neural network model is very efficiently capable of simulating and predicting the growth rate for any variation of wave number, magnetic field and fluid stream velocity. It is very important to mention here, from these figures, that the ANN model succeeded to simulate and predict the stability domain and its variation according to magnetic field and fluid stream velocity.

Figures 5 ~ 7 show the predicted results from the developed MHD model, using ANN technique, to understand the stable and unstable domains. It could be noticed from these figures that, the unstable domain increases by increasing the stream velocity. In the stable domain, by increasing the magnetic field, the growth rate increases.

It is important to concentrate here on one of the advantages of ANN, which is all of the results presented in Figs. 3 ~ 7 are obtained from one run of the developed model and the model can produce more than that rather than the analytic solution which produces one curve from these figures every cycle of calculation.

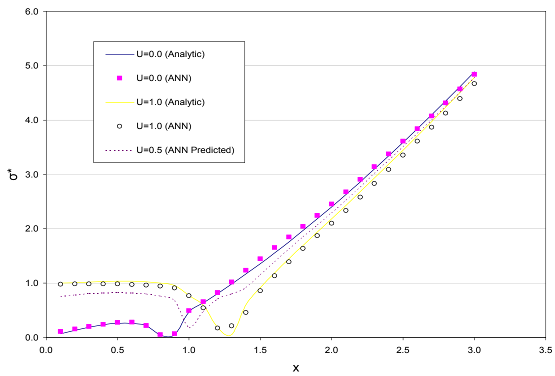

Fig. 3 MHD Stable and Unstable Domains (Analytic and ANN for $h = 0.7$)

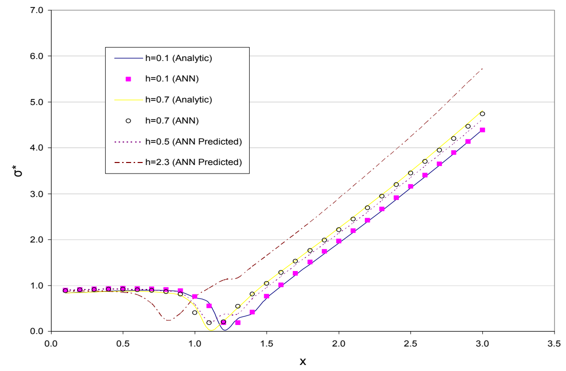

Fig. 4 MHD Stable and Unstable Domains (Analytic and ANN for $U = 0.7$)

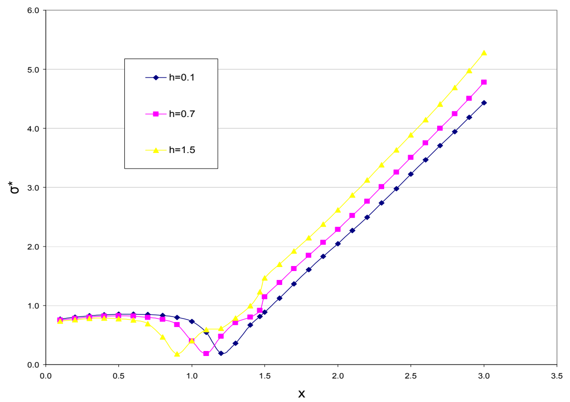

Fig. 5 MHD Stable and Unstable Domains (ANN for $U = 0.5$)

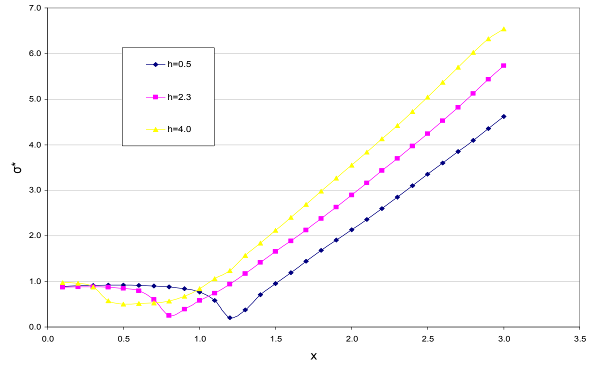

Fig. 6 MHD Stable and Unstable Domains (ANN for $U = 0.7$)

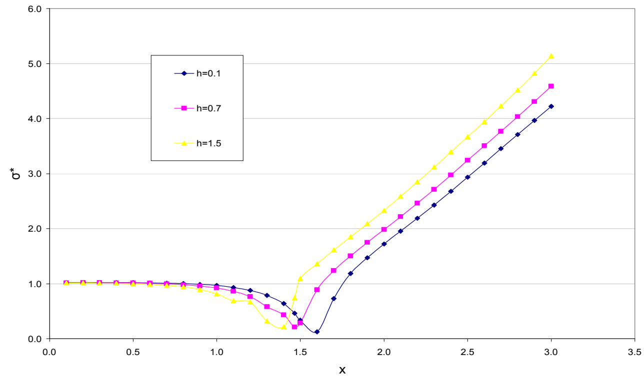

Fig. 7 MHD Stable and Unstable Domains (ANN for $U = 1.3$)

5. CONCLUSIONS

From the foregoing figures we may deduce the following. For the same value of U, it is found that the unstable domains are decreasing with increasing h values. This means that the influence of magnetic field has a stabilizing effect for all short and long wavelengths.

For the same value of h, it is found that the unstable domains are increasing with increasing U values. This means that the streaming is strongly destabilizing.

Based on the results of implementing the ANN technique in this study, the following can be concluded:

1. The developed ANN model presented in this study is very successful and smarting in simulating the effect of longitudinal wave number, magnetic field and fluid stream velocity on the growth rate.
2. The presented ANN model is very efficiently capable of direct predicting the stable and unstable domains for the growth rate for different parameters.
3. Using single set of output results from the analytic solution, The ANN model succeeded to understand the stability behavior of streaming jet field and became ready to give the growth rate for different parameters without solving such kind of problem again using analytic or any other approach.
4. The ANN model can produce a huge number of results for any variation of magnetic field and fluid stream velocity from one run which takes a few minutes and accepted error.

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