OPTIMUM LOCATION OF EMERGENCY FACILITIES RESPONDING TO DEMANDS RANDOMLY INITIATED IN SPACE AND TIME

Ahmed Farouk Abdul Moneim[1], Khaled S. El-Kilany[2], and Ahmed G. El-Shentenawy[3]

Department of Industrial and Management Engineering
College of Engineering and Technology
Arab Academy for Science, Technology, and Maritime Transport
AbuKir Campus, P.O. Box 1029, Alexandria, Egypt
[1]mail@ahmedfarouk.net, [2]kkilany@aast.edu, [3]ahmedelshentenawy@gmail.com

ABSTRACT

Selecting optimum locations for emergency facilities such as ambulances, fire fighting brigades and security forces capable to respond to demands stochastically initiated at random points in a specified demand space is one of the crucial issues requiring further investigations. This work emphasis that the uncertainty is not only as to when a demand (an accident) happens with a given probability distribution of time but also regarding to where it takes place with another given probability distribution of demand position. Additionally, a third element of uncertainty is considered, which the facility failure upon demand is. Optimum Robust Locations of Facilities aims at arranging the facilities in a way that minimizes the *coverage* Coefficient of Variation (COV) under the constraint of limited number of facilities; where, *coverage* of a system of facilities means the percentage of accidents handled by the system of facilities within two preset standards for the two components of the rescue time. The mathematical formulation of this problem along with a hybrid Genetic Algorithm – Monte Carlo simulation model is presented for the solution of the problem. Also, a case study of the optimum and robust distribution of limited number of ambulance depots along the Cairo – Alexandria highway in Egypt is presented. Treating demand centres as points randomly initiated in a Demand Space (the highway in the case study) is a new contribution to the formulation of Maximal Coverage Location problem.

KEYWORDS: Optimum robust locations, genetic algorithms, Monte Carlo simulation.

1. INTRODUCTION

The problem of location of emergency facilities has received a considerable attention in the past four decades. Several articles were devoted to present a survey of research works dealing with the problem. Earlier treatments related to this work dealt with deterministic facility location problem with discrete demand points by Toregas *et. al.* in 1971 with the objective of minimizing the number of ambulances needed to cover all demand points [5]. The maximal coverage location problem (MCLP) was introduced by Church and Revelle [1] in 1974. Daskin in [2] formulates the problem of ambulance location in a probabilistic model aiming at maximizing the expected coverage extended by the facilities. Revelle and Hogan [4] in 1989 used the chance constrained stochastic programming to maximize the covered demand with a given probability.

Traditionally, as treated in the indicated before works, the demand for an emergency facility arises from discrete demand points forming a demand space. However, problems with continuous demand space rather than discrete one are not covered. Having continuous demand space such as highways, there appears a new problem of treating the demand (accident) location as a random variable. Therefore, the demand will be uncertain not only in time of occurrence but also in space where it takes place. This will further require including a new modified definition of the coverage notion based on preset standard times as proposed in several works as the one presented by Luce Brotcorne *et. al.* [3].

This work addresses the problem of optimum and robust locations selection for a limited number of emergency facilities to respond to accidents (demands) happening randomly in time and position in a specified demand space. Problem statement is presented in the next section followed by the mathematical formulation of the problem. A hybrid genetic algorithm – Monte Carlo (GA-MC) Simulation model is then presented, which is proposed for solving this problem and aims at ensuring the robustness of the emergency facilities system by minimizing the coefficient of variation of coverage. Finally, application of the proposed model to the solution of a case study is presented; followed by the conclusions drawn from this work.

2. PROBLEM STATEMENT

Given a Demand Space (a road has a given length L). The demand space is divided into U different zones with different demand intensities (accidents rates). The number of emergency facilities (ambulance stations) should be less than or limited to M facilities. There exist N proposed locations for locating the facilities in an optimum and robust arrangement. Q hospitals with known locations can receive the injured persons in accidents happening in the demand space.

It is required to determine the optimum robust arrangement of the given emergency facilities within the set of proposed locations so as to minimize the Coefficient of Variation (*COV*) of the coverage extended by the facilities.

The accident (demand) times $T_{ACC}(j)$; (j=1,2,…U) in different zones of the demand space are considered random and distributed exponentially with given accident rates λ_j; (j=1,2,…U). The locations where accidents take place $X_{ACC}(j)$; (j=1,2,…U) are random and assumed uniformly distributed along the road zones. Accidents take place in the different zones independently. Other distributions for time and demand positions can also be accommodated.

A facility may exist in one of three different states: (Free, Busy and Failed). Transition from a state to another is taking place randomly. The Free/Busy states are defined by whether the available time of the facility i, $T_{Avail}(i)$; (i=1,2,…M), is less than or greater than an accident time calling for the facility $T_{ACC}(j)$ respectively. The Failed state is defined by a given availability A_i of facility i.

A rescue scenario is defined in two steps: First, as an accident happens in any one of the U zones, the *working* facility with the shortest *proximity time* is called for rescue. The time to approach the accident scene in one of the zones by the selected facility T_{App} is evaluated in terms of absolute distance between the assigned facility and accident locations and an assumed speed of the facility. Second, the injured person is transferred to the nearest hospital in a time T_{Trans} and the facility returns back to its assigned depot in time T_{Return} and; hence, will be available (if not failed) for a next rescue operation.

3. MATHEMATICAL FORMULATION

3.1. Nomenclature

A_i: Availability of facility i; (i=1, 2,…G)

$E(Coverage)$: Expected value of coverage

L_j: Length of zone j in Km (j=1, 2,...U)

l: Distance interval between proposed locations in km

G: Number of facilities utilized

$HP(k)$: Hospital k location from origin of demand space in km (k=1,2,…Q)

M: Upper limit of number of facilities to be utilized

N: Number of locations proposed to install the G facilities

Q: Number of hospitals

S_j: Distance from origin of demand space (road) to start of zone j in Km

$T_{Acc}(j)$: Time of occurrence of an accident in zone j

$T_{Avail}(i)$: Time at which facility i is available for next rescue operation

T_{App}: Approach time required to reach the scene of accident in hours

T_{Trans}: Transfer time required to reach to a selected hospital
T_{Return}: Time taken to return from a hospital back to its assigned location
U: Number of zones in the demand space
V: Average speed of an ambulance in Km/hr
$X_{Acc}(j)$: Location of an accident in zone *j*
λ_j: Rate of accidents in zone *j* in accidents/hour
μ_{App}: Membership function of approaching time
μ_{Trans}: Membership function of Transfer time
$\sigma^2(Coverage)$: Variance of coverage

3.2. Decision Variables

$$Y_k = \begin{cases} 1;\ if\ a\ facility\ is\ located\ in\ location\ k = (1,2,\dots N) \\ 0;\ \text{otherwise} \end{cases}$$

$Loctn(i) = (1,2,\dots or\ N)$; Location of facility *i (i*=1,2,...*G)*
$Pos(i) = l * Loctn(i)$; Position of facility *i* measured from start of the demand space (road) in km.
$G = \sum_{k=1}^{N} Y_k$; Number of facilities utilized

3.3. Membership Functions

The Coverage ratio, in the present work, is proposed to be the product of two membership functions: μ_{App} and μ_{Trans}; the Approach and Transfer membership functions defined later.

$$\mu_{App} = \begin{cases} 1;\ if\ T_{App} \le \tau_{norm\_App} \\ \frac{\tau_{max\_App} - T_{App}}{\tau_{max\_App} - \tau_{norm\_App}};\ if\ \tau_{norm\_App} \le T_{App} \le \tau_{max\_App} \\ 0;\ if\ T_{App} \ge \tau_{max\_App} \end{cases} \quad (1)$$

$$\mu_{Trans} = \begin{cases} 1;\ if\ T_{Trans} \le \tau_{norm\_Trans} \\ \frac{\tau_{max\_Trans} - T_{trans}}{\tau_{max\_Trans} - \tau_{norm\_Trans}};\ if\ \tau_{norm\_Trans} \le T_{Trans} \le \tau_{max\_Trans} \\ 0;\ if\ T_{Trans} \ge \tau_{max\_Trans} \end{cases} \quad (2)$$

τ_{max_App} and τ_{max_Trans} are preset times beyond which the life of an injured person is severely jeopardized in case of approaching the scene of accident and in case of transferring him to a hospital respectively.

τ_{norm_App} and τ_{norm_Trans} are preset times below which the injured person could be safely treated in case of approach by giving first aids and in case of transfer to hospital respectively and thereby the accident is 100% covered. The Coverage is then evaluated as follows:

$$\text{Coverage} = \mu_{App} * \mu_{Disp} \quad (3)$$

3.4. Objective Function

$$Minimize \left[\frac{\sigma(Coverage)}{E(Coverage)}\right]^2 \quad (4)$$

3.5. Constraints

$$G \leq M \quad (5)$$

4. GENETIC ALGORITHM

An initial population of 30 chromosomes is built to form the space of solutions. Each chromosome consists of N genes representing N proposed locations. Each gene carries a binary digit 1 or 0 depending on whether there is a facility located in this location or not respectively. Having a chromosome representing one of the arrangements of the facilities, a Monte Carlo simulation procedure is run to generate accidents and to perform the corresponding rescue. Collecting statistics from the simulation procedure, a Fitness function for the considered chromosome can be evaluated. This procedure is repeated for the other chromosomes and then the population is sorted using the fitness of the chromosomes.

Based on the mathematical formulation presented in the previous section, the following Fitness Function can be deduced:

$$Minimize\ \boldsymbol{Fit} = \left[\frac{\sigma(Coverage)}{E(Coverage)}\right]^2 + Max[(G - M), 0] \quad (6)$$

Here**,** G is the number of facilities randomly generated in a chromosome. The second term in the fitness function (Fit) in (6) is included in order to penalize infeasible solutions. The effective exploration of the space of solutions and efficient exploitation of fittest solutions are guaranteed by employing the three known genetic operators: crossover, mutation and copying. Therefore, using the initially generated population, multiple reproductions are to be done by using the genetic operators until no further improvements of offspring chromosomes could be attained.

Applying formulas (1) and (2), the membership functions μ_{App} and μ_{Trans} are computed and thereby, the Coverage ratio is evaluated by (3). The simulation subroutine is run several times in order to collect sufficient statistics to evaluate the Expected Value and the Variance of the Coverage for a chromosome. The fitness of a chromosome is thus evaluated by (6).

5. MONTE CARLO SIMULATION

The Simulation subroutine of the proposed computational model starts with generating the following Random Variates in each zone:

$$T_{Acc}(j) = Clock_j + \left(\frac{1}{\lambda_j}\right) Ln\left(\frac{1}{1-r_1}\right) \quad (7)$$

$$X_{Acc}(j) = S_j + r_2\, L_j \quad (8)$$

Where, r_1 and r_2 are random numbers [0-1] uniformly distributed and $Clock_j$ is the simulation clock for zone *j*. Having an accident in a position $X_{Acc}(j)$ in zone *j*, the facility with shortest proximity time is called to perform the rescue. *Facility Proximity Time* (***FPT***) of a facility ***i*** located in location *Pos*(*i*) is evaluated as follows:

$$FPT(i,j) = \begin{cases} T_{Acc}(j) + (1/V) * Abs\big(X_{Acc}(j) - Pos(i)\big);\ if\ T_{Avail}(i) \leq\ T_{Acc}(j) \\ T_{Avail}(i) + (1/V) * Abs\big(X_{Acc}(j) - Pos(i)\big);\ otherwise \end{cases} \quad (9)$$

The approach time to zone j is thereby evaluated as follows:

$$T_{App} = MFPT(j) - T_{Acc}(j) \quad (10)$$

Where, MFPT(j) is the minimum facility proximity time to zone *j*. Similarly, the transfer time to a hospital is evaluated as follows:

$$T_{Trans} = (1/V) * MHP(j) \quad (11)$$

Where, $MHP(j) = \underbrace{Min}_{k} \{abs[X_{Acc}(j) - HP(k)]\}$

The time taken by a facility i to return back from the hospital to its assigned location is evaluated as follows:

$$T_{Return} = (1/V) * \big[Abs\big(Pos(i) - HP(k^*)\big)\big] \quad (12)$$

$HP(k^*)$ is the location of k^*- the hospital nearest to the scene of accident in zone j. Updating the time at which a facility *i* is available –if not failed- for next rescue operation is evaluated as follows:

$$T_{Avail}(i) = \begin{cases} T_{Avail}(i) + T_{App} + T_{Trans} + T_{Return};\ if\ \ T_{Avail}(i) > T_{Acc}(j) \\ T_{Acc}(j) + T_{App} + T_{Trans} + T_{Return};\ if\ \ T_{Avail}(i) \leq T_{Acc}(j) \end{cases} \quad (13)$$

6. CASE STUDY

Given the following data for the Cairo-Alexandria highway in Egypt; total length = 200 km, number of zones *U* = 6, and average speed of the ambulance *V* = 60 - 80 km/hr.

Several values for the available budgeted number of ambulances M is taken in order to show the sensitivity of the Coverage to the increase in number of facilities. The start, length and accident rate of each zone are shown in Table 1. The locations of hospitals are given in Table 2. The preset times for rescue is taken as shown in Table 3.

The results of calculations are given in Table 4. The effects of budgeted number of ambulances, preset normal approach time and the availability of ambulances on the expected coverage ratio are clearly demonstrated in Table 4.

Table 1: Location of Zones and Accident Rate.

Zone	Start S_j (km)	Length L_j (km)	Accident rate λ_j (Accidents/hr)
1	0	60	0.03
2	60	5	0.06
3	65	45	0.03
4	110	10	0.06
5	120	75	0.03
6	195	5	0.06

Table 2: Location of hospitals.

Hospital	Location from start of road (km)
1	40
2	70
3	130
4	200

Table 3: Preset Times.

	Preset Times in min.	
	Normal	Maximum
Approach	15	30
Transfer	30	60

The effects of number and availability of ambulances and preset normal approach time on expected coverage ratio are shown in Table 4.

Table 4: Coverage Ratios for different inputs.

Number of Ambulances	Preset Normal Approach Time in minutes								
	10			15			20		
	Availability of Ambulance			Availability of Ambulance			Availability of Ambulance		
	0.7	0.9	0.95	0.7	0.9	0.95	0.7	0.9	0.95
2	0.52	0.67	0.73	0.53	0.71	0.77	0.52	0.82	0.89
3	0.84	0.85	0.85	0.86	0.86	0.88	0.95	0.95	0.96
4	0.94	0.94	0.95	0.95	0.98	0.98	0.96	0.97	0.98
5	0.95	0.97	0.97	0.97	0.99	0.99	0.98	0.99	1.00
6	0.97	0.98	0.98	0.99	1.00	1.00	0.99	1.00	1.00

Figure 1 shows that increasing the number of ambulances more than 4 will not bring any considerable improvement to the expected coverage ratio irrespective of preset normal approach times and facility availability. However, increasing of the number of ambulances more than 4 will enhance the robustness of the system and the coverage ratio COV will decrease as the number of ambulances increases from 4 to 6 (Figure 2).

On the other hand, for smaller number of ambulances 2 and 3, the effect of availability of ambulance is considerable and the COV approaches unity as the number of ambulances is 2 units and ambulance availability gets down to 0.7.

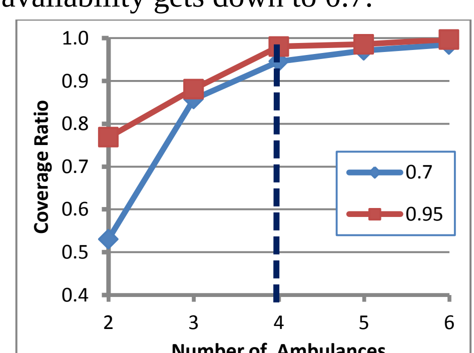

Figure 1: Coverage ratios versus number of facilities.

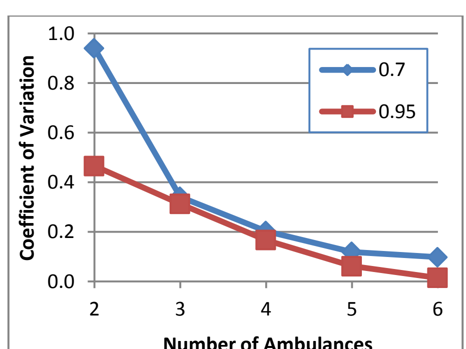

Figure 2: Coverage ratio coefficient of variation.

7. CONCLUSIONS

The well-known Maximal Coverage Facility Location Problem under uncertainty has been considered in this work with advancing the concept of continuous demand space and a new definition of the coverage extended by a system of emergency facilities (ambulances). This problem has been modified to account for demand that is defined randomly in space and time. Where, a coverage ratio has been presented and is treated as a random variable. This leads to formulating the objective of optimization as maximizing the emergency facility system robustness in covering demands arising randomly in space and time.

The effects of significant factors such as size of the system, reliability or availability of the facility, speed of the mobile emergency facility and the preset standard times are explicitly demonstrated. The most outstanding conclusion, in this respect, is that there is a limit for the number of facilities to be allocated beyond which no conceivable improvements could be achieved. Of course, this limit is depending and evaluated by the leading particulars of the demand space and the proposed eligible locations of the emergency facilities and other assets contributing to the rescue operations such as hospitals.

REFERENCES

[1] Church R. and Revelle C. (1974) “The Maximal Covering Location Problem” Papers of Regional Science Association, 32, 101-118.

[2] Daskin M. 1983 “A Maximum Expectation Covering Location Model: Formulation properties and Heuristic solutions” Transportation Science, 17, 48-70

[3] Luce Broctorne, Gilbert Laporte and Fredric Semet (2003) “Ambulance Location and Relocation Models” European Journal of Operational Research, 147, 451-463

[4] Revelle C. and Hogan 1989 “The Maximum Availability Location Problem” Transport Science, 23

[5] Toregas C. R., Swain R., Revelle C. S. and Bergman L (1971) “The Location of Emergency Service Facilities”, Operations Research, 19, 1363-1373