

## Erbium Doped Fiber Amplifier for Different Concentration Profiles

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### ABSTRACT

In this paper, a comparative study for the gain of erbium doped fiber amplifier is presented. Different doping methods are used to obtain different concentration profiles in silica fibers. Affecting parameters including amplifier length, emission cross section and doping concentration are considered in amplifier gain calculations.

### I. INTRODUCTION

Silica (SiO<sub>2</sub>) is the basic optical fiber material. It has been extensively used to make various kinds of optical fibers, amplifiers and fiber lasers with suitable doping materials since the late 1960's. In order to optimize waveguide bending radius and to minimize fiber waveguide insertion loss, local doping of optical material (like aluminum and germanium in SiO<sub>2</sub>) is necessary.

In last years, it has been focused on realizing active optical devices based on rare earth elements, which play a significant role in the progressive development of the optical communication systems due to their high gain, low noise, long transmission distances and increased channel capacity. One of the famous rare earth materials is erbium, which has gained acceptance in telecommunications. It has the characteristic  $^4I_{13/2} \rightarrow ^4I_{15/2}$  transition of Er<sup>3+</sup> emits at  $\lambda=1.55 \mu\text{m}$ , a wavelength useful for fiber communications. Er-doping in LiNbO<sub>3</sub> is studied by F. Caccavale et al. [1], where a detailed study of Er diffusion in LiNbO<sub>3</sub> is presented from a thin metal film using various doping methods, such as thermal diffusion, ion implantation and step-like.

In the present work, we studied Er-doping in different types of silica (germanosilicate and germano-aluminosilicate with two different concentrations [2]). The same described doping methods in Ref.[1] are used to study the effect of concentration on the amplifier gain. Three different values of concentration for erbium, namely;  $\rho_1 = 2 \times 10^{18} \text{ at/cm}^3$ ,  $\rho_2 = 4 \times 10^{19} \text{ at/cm}^3$ , and  $\rho_3 = 4 \times 10^{19} \text{ at/cm}^3$  [3 and 4] are examined. The effect of the amplifier length is also included through this study.

### II. THEORY

The principle of energy conservation can be expressed in terms of photon flux. If

$$\phi_p^{out,in} = p_p^{in} / h\nu_p, \quad (1)$$

is the flux of pump photons input to the amplifier, and

$$\phi_s^{out,in} = p_s^{out,in} / h\nu_s, \quad (2)$$

is the flux of signal photons output from the amplifier, where  $P_p^m$ ,  $P_s^m$ ,  $P_s^{out}$  are, respectively, input pump, input signal, and output signal powers,  $h$  the Plancks constant,  $\nu_p$ , and  $\nu_s$  the pump and signal frequency [2], one must have

$$\phi_s^{out} \leq \phi_p^m + \phi_s^m \quad (3)$$

Equation (3) can also be expressed in terms of input and output powers as

$$P_s^{out} \leq P_p^m + \frac{\lambda_p}{\lambda_s} P_s^m \quad (4)$$

with  $\lambda_p$  the pump wavelength,  $\lambda_s$  the signal wavelength.

Equation (4) can also be expressed in terms of the amplifier gain,  $G$ , as

$$G \leq 1 + \frac{\lambda_p}{\lambda_s} \frac{P_p^m}{P_s^m} = 1 + \frac{\phi_p^m}{\phi_s^m} \quad (5)$$

Equation (5) shows that the upper limit for the gain corresponds to the flux ratio  $\phi_p^m / \phi_s^m$ . Thus, the maximum possible amplifier gain corresponds to the case where each pump photon is converted into one signal photon. For a very large input signal power value such that  $P_s^m \gg (\lambda_p / \lambda_s) P_p^m$ , the maximum amplifier gain reaches the limit of unity, which corresponds to a condition of medium transparency. This last relation also shows that for achieving a maximum gain  $G_m$ , the signal input or input flux cannot exceed a certain value, i.e.  $P_s^m \leq (\lambda_p / \lambda_s) P_p^m / (G_m - 1)$  or  $\phi_s^m \leq \phi_p^m / (G_m - 1)$ .

The upper limit for the amplifier gain, given by consideration of energy conservation or Eq.(5), can be reached only if all pump photons are actually absorbed by the amplifying medium. In actual amplifiers, the finite number of rare earth ions existing in the medium limits the absorption of pump photons and hence, the amplifier gain can be represented by

$$G \leq \exp(\rho \sigma_e L) \quad (6)$$

where  $\rho$  is the rare earth concentration, which can be expressed in different methods and  $\sigma_e$  is the signal emission cross section. The peak value of the emission cross section can be expressed by [5]

$$\sigma_e^{peak} = \frac{\langle \lambda \rangle^2}{8\pi n^2 c \tau \Delta \lambda_e^{eff}} \quad (7)$$

with  $\langle \lambda \rangle$  the mean wavelength,  $n$  the fiber core refractive index which is a function of both temperature and presser,  $\Delta \lambda_e^{eff}$  the effective line width of transition cross section,  $\tau$  the spontaneous emission lifetime (which is equal to the radiative lifetime), and  $c$  is the speed of light in free space.

The fiber amplifier length,  $L$ , Eq.(6) has an optimal value  $L_{opt}$ , which gives the maximum gain, given by [2]

$$L_{opt} = \frac{1}{\alpha_p} \left( \frac{P_p^m}{P_{sat}(\lambda_p)} - \frac{1 + \eta_p}{\eta_s - \eta_p} \right) \quad (8)$$

where  $p_p^{\text{in}}$  is the input pump power,  $\eta_p$ , and  $\eta_s$  are the ratio of emission to absorption cross section at  $\lambda_p$  and  $\lambda_s$  and  $P_{\text{sat}}(\lambda_p)$  is the saturation power at  $\lambda_p$  normalized to photon energy and can be expressed by [2]

$$P_{\text{sat}}(\lambda_p) = \frac{2\pi^2 v_p^2 h}{[\sigma_e(v_p) + \sigma_a(v_p)]\tau} \quad (9)$$

where  $v_p$  is the pump and signal frequency,  $\sigma_e$  total emission cross section at  $v_p$ , and  $\sigma_a$  is the total absorption cross section at  $v_p$ .

The Er-doping absorption coefficient  $\alpha_p$  at  $\lambda_p$  is given by [2]

$$\alpha_p = \rho_0 \sigma_{ap} \Gamma_p \quad (10)$$

with  $\Gamma_p$  the overlap factor at  $\lambda_p$  ( $= \alpha_0 / w_p^2$ ) for confined doping,  $a_0$  the doped core radius,  $\rho_0$  peak dopant density and  $\sigma_{ap}$  the absorption cross section at  $\lambda_p$ . The gain expressed by Eq.(6) cannot be enhanced by increasing the fiber length or the doping concentration, due to the principle of energy conservation Eq.(3), which limits the output signal power  $p_s^{\text{out}}$ , Eq.(4).

From Eq.(6), it is clear that the concentration of the rare earth element affects the signal gain. So, one can start study the erbium concentration profiles obtained by various doping methods. A detailed study of erbium concentration profile in silica as a thin metal has been performed as a function of the initial width,  $w$ , and the diffusion depth,  $d$ , by F. Caccavale et al. [1] as follows:

#### A. Thermal diffusion Method:

In this part, the signal gain is studied as a function of the initial erbium film width ( $2w$ ) and diffusion depth ( $d$ ). The erbium concentration profile is given by the solution of the diffusion equation [1]

$$\rho(w, d) = \frac{\rho_0}{2} * \exp\left[-\left(\frac{5}{d_x}\right)^2\right] * \left\{ \text{erf}\left(\frac{5+w}{d_x}\right) - \text{erf}\left(\frac{5-w}{d_x}\right) \right\} \quad (11)$$

where  $\rho_0$  is the concentration of the erbium,  $d_x$  is the lateral diffusion width.

#### B. Ion implantation Method:

The erbium concentration is given by [1]

$$\rho(w, d) = \frac{\rho_0}{2} * \exp\left[-\left(\frac{5-d}{a_y}\right)^2\right] * \left\{ \text{erf}\left(\frac{5+w}{a_x}\right) - \text{erf}\left(\frac{5-w}{a_x}\right) \right\} \quad (12)$$

with  $a_y$  the tail width of semi-Gaussian ion depth distribution and  $a_x$  the spreading length of the lateral distribution.

#### C. Step-Like Method:

The erbium concentration is given by [1]

$$\rho(w) = \frac{\rho_0}{2} * \left\{ \text{erf}\left(\frac{5+w}{a_x}\right) - \text{erf}\left(\frac{5-w}{a_x}\right) \right\} \quad (13)$$

### III. RESULTS AND DISCUSSION

First, one must get the emission cross section for the three types of silica fibers under study (germanosilicate type I, and germano-aluminosilicate type II, and III, where alumina concentration is greatest in type III) using experimental data [2] to calculate the signal gain. Curve fitting is used with this data to get Gaussian line shapes  $i$  of central wavelength  $\lambda_i$  with full width at half maximum FWHM  $\Delta\lambda_i$ , and peak values  $a_i$  in the form [2]

$$C(\lambda) = \sum_i a_i \exp\left\{-4 \log 2 \frac{(\lambda - \lambda_i)^2}{\Delta\lambda_i^2}\right\} \quad (14)$$

where  $C(\lambda)$  is the emission cross section at a wavelength  $\lambda$ .

Using the values of the signal emission cross-section  $\sigma_e$ , one can calculate the signal gain at different values of erbium concentration  $\rho$  with different values of length  $L$  using Eq.(6) [2]. This is done for each type of the optical fiber, where  $L_1 = 22.4$  cm,  $L_2 = 15.7$  cm and  $L_3 = 10.5$  cm [1] at different concentration values and the results are displayed in Fig 1.

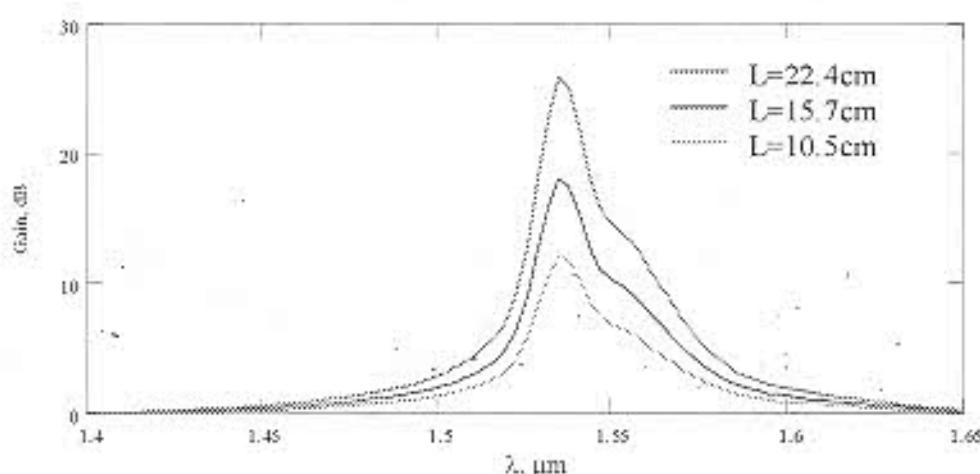


Fig.1. Signal gain of type I at  $\rho = 4 \times 10^{19}$  at/cm<sup>3</sup>.

Figure 1 shows three shapes for the signal gain of type I at different  $L$ , and concentration equals  $4 \times 10^{19}$  at/cm<sup>3</sup>. The zero value of the three shapes starts at  $1.4 \mu\text{m}$  and increases exponentially until reaching  $1.5358 \mu\text{m}$  at which the signal gain reaches its maximum value  $G(L_1) = 24.702$  dB,  $G(L_2) = 17.314$  dB, and  $G(L_3) = 11.579$  dB, then shapes becomes to decrease. For a concentration  $2 \times 10^{18}$  at/cm<sup>3</sup>,  $G(L_1) = 1.304$  dB,  $G(L_2) = 0.914$  dB, and  $G(L_3) = 0.611$  dB. For a concentration  $2.5 \times 10^{20}$  at/cm<sup>3</sup> the maximum value of signal gain for type I equal  $G(L_1) = 154.389$  dB,  $G(L_2) = 108.21$  dB and,  $G(L_3) = 72.37$  dB.

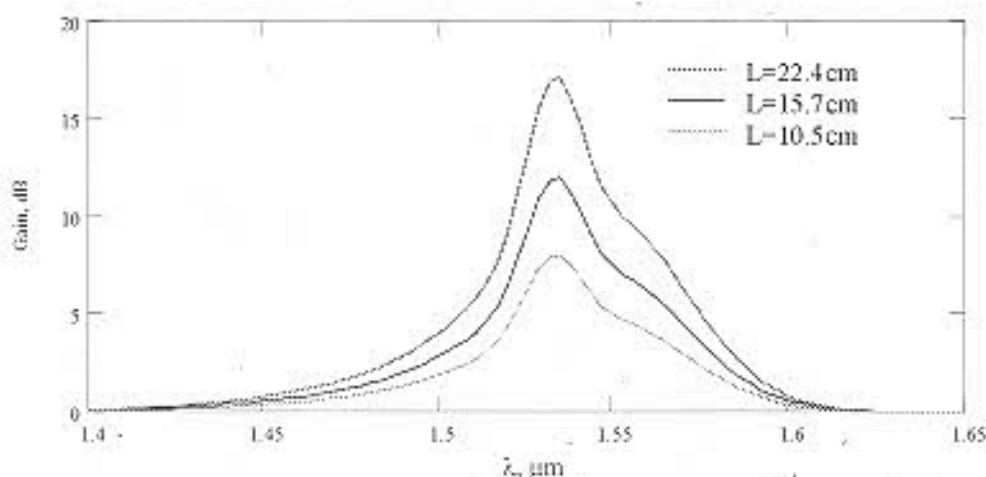


Fig.2. Signal gain of type II at  $\rho = 4 \times 10^{19}$  at/cm<sup>3</sup>.

The three shapes for the signal gain of type II at different values of  $L$ , and a concentration  $4 \times 10^{19}$  at/cm<sup>3</sup> are shown in Fig.2. The three shapes start at  $1.41 \mu\text{m}$  with zero value and increase until reaching maximum at wavelength equal  $1.534 \mu\text{m}$  where  $G(L_1) = 17.128$  dB,  $G(L_2) = 12.005$  dB, and  $G(L_3) = 8.029$  dB, then decays to reach zero again at  $1.63 \mu\text{m}$ . At a concentration  $2 \times 10^{18}$  at/cm<sup>3</sup>  $G(L_1) = 0.856$  dB,  $G(L_2) = 0.6$  dB, and  $G(L_3) = 0.401$  dB. For  $2.5 \times 10^{20}$  at/cm<sup>3</sup> erbium concentration in type II the maximum value of signal gain equals  $G(L_1) = 107.049$  dB,  $G(L_2) = 75.03$  dB, and  $G(L_3) = 50.179$  dB.

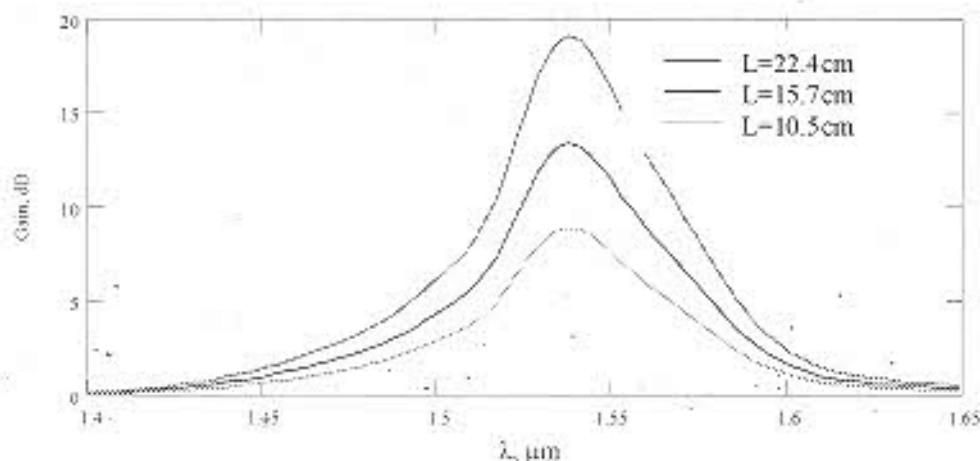


Fig.3. Signal gain of type III at  $\rho = 4 \times 10^{19}$  at/cm<sup>3</sup>.

Figure 3 shows the signal gain of type III at the different values of  $L$  and a concentration  $4 \times 10^{19}$  at/cm<sup>3</sup>. The three shapes similarly like Gaussian shape starts and end at  $1.41 \mu\text{m}$  and  $1.65 \mu\text{m}$  with zero value and its maximum at wavelength equal  $1.5385 \mu\text{m}$  where  $G(L_1) = 19.062$  dB,  $G(L_2) = 13.361$  dB and  $G(L_3) = 8.935$  dB. At a concentration  $2 \times 10^{18}$  at/cm<sup>3</sup>,  $G(L_1) = 0.953$  dB,  $G(L_2) = 0.668$  dB and  $G(L_3) = 0.447$  dB. In type III, for  $2.5 \times 10^{20}$  at/cm<sup>3</sup>, the maximum signal gain is:  $G(L_1) = 119.139$  dB,  $G(L_2) = 83.504$  dB and  $G(L_3) = 55.846$  dB.

Second, we start a comparative study of the optical signal gain as a function of the initial width ( $w$ ), and diffusion depth ( $d$ ) using different Er-doping methods, for the three types of silica fibers at three different concentrations.

One starts to plot the signal gain against the initial width ( $w$ ) and the diffusion depth ( $d$ ) for the three different values of erbium concentration and the cross section at wavelengths corresponding to maximum gain using Math Lab programs for the three difference doping methods.

#### A. Thermal Diffusion Method

Equation (11) is used with Eq.(6) at  $L = 22.4$  cm to plot the signal gain. In Fig. 4 the signal gain profile depends on both  $w$  and  $d$ . The gain  $G_s$  has a value of zero dB for small values of  $d$  for all values of  $w$  and then starts to increase till approaching its maximum at  $d = 10$   $\mu\text{m}$ . From the figure, it seen that the increase of the diffusion depth does not affect the gain in the tree types of silica fibers at different concentrations.

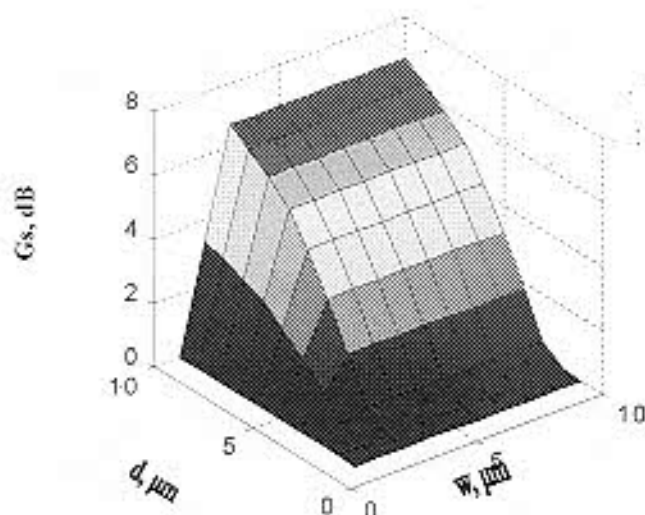


Fig. 4 Signal gain of type I at  $\rho = 2 \times 10^{18}$  at/cm<sup>3</sup> using diffusion method.

Table 1 displays the maximum values of the signal gain for each type of silica fiber using thermal diffusion method at the three different concentration values.

Type	$\rho_1 = 2 \times 10^{18}$ at/cm <sup>3</sup>	$\rho_2 = 4 \times 10^{19}$ at/cm <sup>3</sup>	$\rho_3 = 2.5 \times 10^{20}$ at/cm <sup>3</sup>
I	6.75 dB	134.9 dB	843.13 dB
II	4.429 dB	488.59 dB	553.69 dB
III	4.933 dB	98.66 dB	616.62 dB

Table 1. Maximum signal gain using thermal diffusion method

#### B. Ion Implantation Method:

We put the value of the tail width of semi-Gaussian ion depth distribution  $a_y$ , and the value of the spreading length of the lateral distribution  $a_x$  to equal 0.3  $\mu\text{m}$ , and 0.5  $\mu\text{m}$ , respectively in Eq.(12). It is then used in Eq.(6) to plot the signal gain by at  $L_{opt} = 15.7$  cm [1].



Figure 5 shows the signal gain for ion implanted silica waveguides. It increases with implantation depth ( $d$ ) for any given implantation strip width ( $w$ ). For low values of  $d$ , the rate of increase is high till it reaches saturation at  $d = 4 \mu\text{m}$  and then decays slowly. In Fig. 5, the gain starts at zero and then increases rapidly till approaching its maximum value, 6.07 dB, at  $d = 4 \mu\text{m}$  and then starts to decay for  $d > 10 \mu\text{m}$  till reaching 3.03 dB.

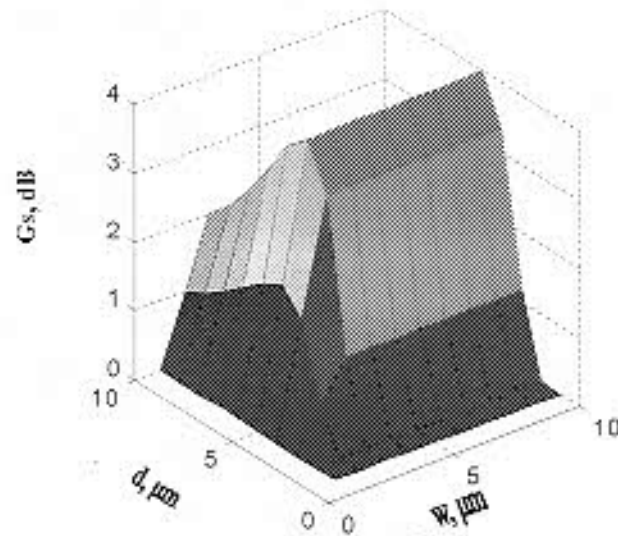


Fig. 5: Signal gain of type I at  $\rho = 2 \times 10^{18} \text{ at/cm}^3$  using ion implantation method.

Table 2 shows the maximum values of the signal gain for the three types of silica fibers using ion implantation method at the different concentration values.

Type	$\rho_1 = 2 \times 10^{18} \text{ at/cm}^3$	$\rho_2 = 4 \times 10^{19} \text{ at/cm}^3$	$\rho_3 = 2.5 \times 10^{20} \text{ at/cm}^3$
I	6.07 dB	112.4 dB	758.78 dB
II	3.98 dB	79.73 dB	498.31 dB
III	4.43 dB	88.79 dB	554.9 dB

Table 2. Maximum signal gain using ion implantation method

### C. Step-Like Method: -

In Eq.(13) we put the value of the spreading length of the lateral distribution  $a_x$  to be  $0.5 \mu\text{m}$  and use the obtained results in Eq.(6) to plot the signal gain  $G_s$ , using an optimum fiber length equal  $10.5 \text{ cm}$  [1]. This is repeated for the three types of silica at different concentrations and the results are shown in Fig.6.

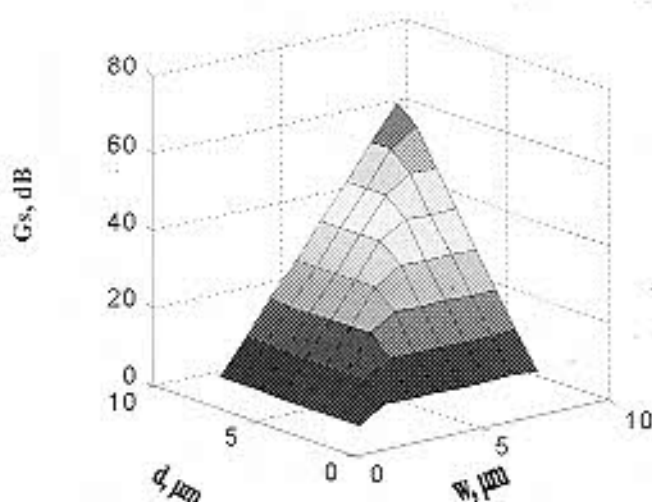


Fig. 6 Signal gain of type I at  $\rho = 2 \times 10^{18}$  at/cm<sup>3</sup> using step-like method.

In Fig. 6, the gain dependence on the width ( $w$ ) and the depth ( $d$ ) is displayed. It is seen that the gain has a proportion relation with both  $w$  and  $d$ . For small values of  $w$  and  $d$ , the value of gain is small and then it increases with both  $w$  and  $d$ . Figure 6 shows that the gain  $G_s = 4.3$  dB at  $w$  and  $d = 1 \mu\text{m}$  and approaches its maximum, 64.935 dB, at  $w$  and  $d = 10 \mu\text{m}$ .

Using step-like method the maximum values of the signals gain represents in Table 3 for the different types of optical glass at the three different concentrations.

Type	$\rho_1 = 2 \times 10^{18}$ at/cm <sup>3</sup>	$\rho_2 = 4 \times 10^{19}$ at/cm <sup>3</sup>	$\rho_3 = 2.5 \times 10^{20}$ at/cm <sup>3</sup>
I	64.94 dB	910.26 dB	3043.9 dB
II	42.64 dB	597.78 dB	1999 dB
III	47.49 dB	665.71 dB	226.1 dB

Table 3. Maximum signal gain using step-like method

#### IV. CONCLUSION

From Eq.(6), it seen that the signal gain has a direct proportion with the concentration and the optical fiber amplifier length  $L$ . This is fairly satisfied in Figs. 1-3, and in Tables 1-3. One must keep in mind that, practically, higher erbium concentration increases optical losses in glass fibers with a consequent decrease in the gain [5]. In addition, the amplifier length corresponding to maximum gain is related to the pump power [1].

Comparing different doping methods, Figs 4-6, it can be seen that for diffusion, and ion implantation methods, the erbium film width  $w$  does not have any effect on gain. It is also observed that if one considers a small depth, the case of ion implantation would be the best method used. So, in this way it is observed that for each application one can use a suitable doping method from which the signal gain could have a maximum value.



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