



TRANSMISSION CHARACTERISTICS OF GAUSSIAN PROFILE OPTICAL FIBERS

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ABSTRACT

The transmission characteristics of the Gaussian profile optical fibers are studied. The rms value of the chromatic dispersion is calculated and minimized over the wavelength range (1.25 μm -1.6 μm). The fiber parameters at which the rms value of the chromatic dispersion is minimum yield a small bending loss.

I. INTRODUCTION

Dispersion of signal-carrying medium makes the spectral components of the signal travel through the medium with different velocities. This results in distortion of the signal at the receiver end compared with the transmitter end [1]. Several studies concerning the dispersion in optical fibers have been conducted. Chang [2] evaluated the waveguide dispersion needed for the cancellation of the material dispersion in single-mode step-index optical fibers. Wemple [3] calculated the material dispersion as a function of the average electronic energy gap, the electronic oscillator strength and the lattice oscillator strength. Hatton and Nishimura [4] studied the dependence of the temperature on the zero dispersion wavelength. Bending a fiber will perturb the geometry of the profile. Grath [5] demonstrated that the unperturbed propagation constant will be corrected for a bent fiber by a small term that is proportional to the radius of curvature. The dispersion flattening in optical fibers were studied by Lundin [6,7]. He showed that step-index and W-fibers are capable of dispersion flattening by minimizing the root mean square of the chromatic dispersion in these fibers.

Radiation losses occur whenever an optical fiber undergoes a bend of finite radius of curvature [8]. The consideration of the radiation loss caused by the curvature of single-mode fiber is very important from the point of view of long-distance transmission of optical signals. Several theories on curvature loss have been proposed; most of which assume that the fields at a bend can be approximated by those of straight fibers [9]. Marcuse [10] derived a simplified formula for the leaky HE_{11} mode in the curved, doubly clad fiber. Kaufman et al. [11] derived an extension of the bending loss coefficient equation derived by Marcuse. Morgan et al. [12] studied the bend induced optical loss in a monomode fiber as a continuous function of wavelength. Vendeltorp-Pomener and Povlsen [13] presented the field distribution for both the fundamental mode and the whispering gallery modes in a bent fiber. Verrier and Goure [14] studied the influence of bends on light propagation in step-index fibers theoretically and experimentally.

Bending loss resistance of single-mode fibers with zero dispersion near 1300 nm can be well characterized through fiber parameters such as mode field radius and cutoff wavelength because the refractive index profiles of these fibers are almost of a step shape. However, indirect

evaluation methods of bending loss are required for many types of dispersion modified fibers, because it seems difficult to characterize the bending loss of these fibers through the basic fiber parameters. One of these indirect evaluation methods is based on the mode field measurement [15].

In the present work, we investigate the performance of the single-mode optical fibers with a Gaussian refractive index profile. The Gaussian profile is unique because it has, approximately, the same shape as the field intensity inside the core. Furthermore, this profile approximates those fibers for communication that undergo significant diffusion at the core-cladding interface during the manufacturing process. More importantly, it provides us with a very simple model for understanding all aspects of propagation in single-mode fibers. In addition, when the fields of the fundamental mode are approximated by a Gaussian shape, the approximate solution is simple and accurate [16]. The Gaussian profile fibers has a larger cutoff value of the normalized frequency than the step index profiles which results in a larger core radius and refractive index difference between the core and the cladding. The investigation is devoted to minimize the chromatic dispersion and the bending loss. The physical parameters of the fiber satisfying these purposes have been determined.

II. ANALYSIS

II.1. Chromatic Dispersion

II.1.1. Approximate Refractive-Index Model

The Gaussian profile is given by [16]:

$$n^2(r) = n_2^2 + (n_1^2 - n_2^2) \exp\left(-\frac{r^2}{t^2}\right), \quad (1)$$

where r is the radial distance measured from the axis of the fiber, and t is a scaling parameter. In weakly guiding fibers, the variation between n_1 and n_2 is very small, typically $\leq 1\%$ and $n(r) \approx n_2$ at $r = 2t$. Therefore, the core radius, a , assigns this value in the present work.

Considering silica glass, the relation between the refractive index, n , the vacuum wavelength, λ , and the germania mole fraction X is represented by the three-term Sellmeier dispersion relation [17]:

$$n^2 - 1 = \sum_{i=1}^3 \frac{[a_{is} + X(a_{ig} - a_{is})] \lambda^2}{\lambda^2 - [b_{is} + X(b_{ig} - b_{is})]^2}, \quad (2)$$

where $a_{is}, b_{is}, a_{ig}, b_{ig}$ are the Sellmeier coefficients for silica and germania glasses. The core refractive index can be written as:

$$n(r, \lambda) = N(r, \lambda) n_2(\lambda), \quad (3)$$

where $n_2(\lambda)$ is the cladding refractive index and $N(r, \lambda)$ is the normalized refractive index (the relative refractive-index increase) which can be approximated as a function of the radial coordinate only [6], i.e.,

$$n(r, \lambda) = N(r) n_2(\lambda). \quad (4)$$

Using Eq. (1), one can get a normalized Gaussian index profile in the form:

$$N(r) = \begin{cases} \sqrt{1 + (N_1^2 - 1) \exp\left(-\frac{r^2}{t^2}\right)} & \text{for } r \leq t \\ 1 & \text{for } r > t, \end{cases} \quad (5)$$

where N_1 is defined through the axial refractive index, n_1 , as:

$$n_1 = N_1 n_2 \quad (6)$$

II.1.2 Minimization of the Chromatic Dispersion

To minimize pulse broadening in an optical fiber, the chromatic dispersion should be low over the wavelength range used. Such fiber is called dispersion flattened fiber (DFF). The wavelength range (1.25 μm -1.6 μm) is chosen because the attenuation in pure silica glass is low [18]. The chromatic dispersion, D_T , is defined as [6]:

$$D_T = -\frac{\lambda}{c} \frac{d^2 n_e}{d\lambda^2} \quad (7)$$

where n_e is the effective refractive index and c is the speed of light in vacuum. The rms value, f , of the chromatic dispersion of the fundamental mode over a wavelength range is given by:

$$f = \sqrt{\frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} D_T^2(\lambda) d\lambda} \quad (8)$$

The minimization problem can be carried out by seeking the parameters N_1 and t that minimize f in the vacuum wavelength range ($\lambda_1 = 1.25 \mu\text{m}$, $\lambda_2 = 1.6 \mu\text{m}$) subjected to the constraint [6]:

$$\lambda_c = 1.25 \mu\text{m} \quad (9)$$

where λ_c is the vacuum cutoff wavelength for the first higher-order mode at which the V number equals 2.59. Using the definition of the normalized frequency, this constraint at definite value of X turns the rms value f into a function of only one parameter, t or N_1 , where:

$$t = \frac{0.515}{n_2 \sqrt{N_1^2 - 1}} \quad (10)$$

II.2 Bending Loss

The formula derived by Sakai and Kimura [15] for the uniform bending loss of single-mode fibers is written as follows:

$$\alpha_b = \frac{\sqrt{\pi} A_{cl}^2 \exp(-\sqrt{2} \lambda^2 R / 3 \pi^2 n_1^2 W_\infty^3)}{2 p_t (\sqrt{2}/W_\infty)^{3/2} (R + 2 \pi^2 n_1^2 W_\infty^2 a / \lambda^2)^{1/2}} \quad (11)$$

where R is the bending radius, n_1 is the axial value of refractive index of the fiber core, a is the core radius, λ is the operating wavelength and W_∞ is the mode field radius representing the bending sensitivity, which was introduced by Petermann [15]. The mode field radius, W_∞ , is defined through the propagation constant, β , of the LP_{01} mode and the wavenumber in the cladding, k_2 , as follows:

$$W_\infty = \frac{\sqrt{2}}{\sqrt{\beta^2 - k_2^2}} \quad (12)$$

where k_2 is the wave number in the cladding and β is the propagation constant of the Gaussian profile [16]. The field in the cladding region of the matched clad fiber is described by the modified Bessel function [16]:

$$\phi_2(r) = A_{cl} K_0(\sqrt{2}r/W_\omega) , \quad r > a \quad (13)$$

where A_{cl} is the coefficient of the field function $\phi_2(r)$ in the cladding. The integrated field intensity, p_t , is defined by the following equation [15]:

$$p_t = \int_0^\infty \phi^2(r) r dr . \quad (14)$$

The lower limit of integration suggests that the field function in the core region should also be used, so:

$$p_t = \int_0^a \phi_1^2(r) r dr + \int_a^\infty \phi_2^2(r) r dr , \quad (15)$$

where $\phi_1(r)$ is the field function in the core. If the cladding radius, b , is selected such that the field function can be neglected for $r > b$. Therefore, the integrated field intensity can be written as:

$$p_t = \int_0^a \phi_1^2(r) r dr + \int_a^b \phi_2^2(r) r dr . \quad (16)$$

The coefficient A_{cl} of the field function can be obtained by equating $\phi_1(r)$ and $\phi_2(r)$ at $r = a$. According to the work of Snyder [16], $\phi_1(r)$ is given by:

$$\phi_1(r) = \exp[-\frac{1}{2}(r/r_0)^2] , \quad (17)$$

where

$$r_0 = \eta(V - 1)^{1/2} , \quad (18)$$

therefore:

$$A_{cl} = \frac{\exp[-\frac{1}{2}(a/r_0)^2]}{K_0(aF)} , \quad (19)$$

where

$$F = \frac{\sqrt{2}}{W_\omega} . \quad (20)$$

Hence

$$p_t = \frac{r_0^2}{2} [1 - \exp-(a/r_0)^2] + \frac{A_{cl}^2}{F^2} s_3 . \quad (21)$$

where

$$s_3 = F^2 \int_{aF}^{bF} y K_0^2(y) dy . \quad (22)$$

III. RESULTS AND DISCUSSION

III.1 Chromatic Dispersion

The rms value, f , of the chromatic dispersion is calculated numerically as a function of the normalized refractive index, N_1 , by using a computer program for different values of the germania mole fraction, X . The variation of f with N_1 is given in Fig. 1. The minimum value for f is:

$$\begin{aligned}
 f_{\min} &= 6.9 \text{ ps/(nm.km)} , \\
 \text{and the corresponding values of } N_1 \text{ and } t \text{ are:} \\
 N_{1\min} &= 1.0093 , \\
 t_{\min} &= 2.6 \text{ } \mu\text{m.}
 \end{aligned}
 \tag{23}$$

Therefore, Gaussian profile fiber whose refractive-index increase is equal to 1.0093 represents a dispersion flattened fiber, since it has the minimum chromatic dispersion over the considered wavelength range. More importantly, the present work is consistent with that of Lundin for step-index and W fibers [6,7]. All curves have the same behavior, and their minimum values are around the $N_1 = 1.01$.

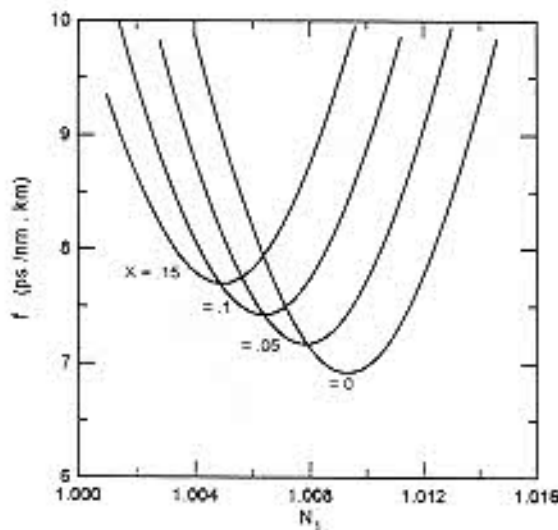


Fig. 1: The rms value f as a function of the refractive index increase at different values of the mole fraction X .

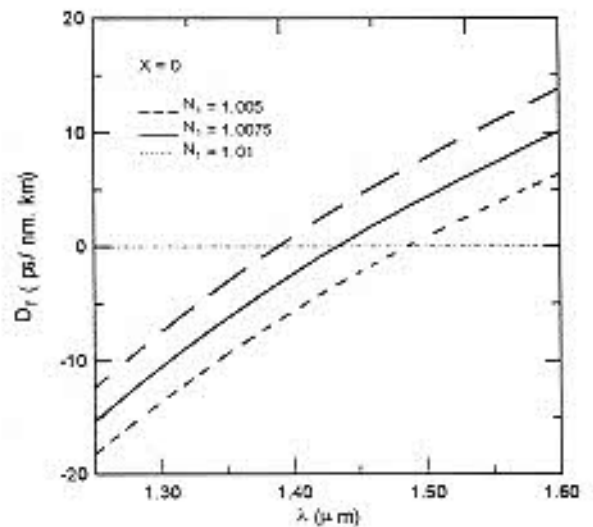


Fig. 2: The chromatic dispersion as a function of the wavelength at different values of N_1 .

It is clear that as X increases, the minimum value of f increases with a decrease in the corresponding value of N_1 . The decrease in $N_{1\min}$ means that the fiber becomes more weakly guiding because the variation between the maximum and minimum values of $n(r)$ decreases. The material dispersion, D_m , is directly proportional to N_1 . So, it is insensitive to the variations in N_1 since these variations are very small. On the other hand, the waveguide dispersion, D_w , is sensitive to the changes of N_1 . If X is fixed, then D_T will be affected by the dependence of D_w

on N_1 . Figure 2 depicts the variation of the chromatic dispersion with the wavelength at $X = 0$, at different values of N_1 . As N_1 increases, D_w takes higher negative values, so, the zero chromatic dispersion wavelength, λ_0 , will be displaced towards higher values. When λ_0 is located approximately at the middle of the wavelength range, the rms value, f , of the chromatic dispersion will be minimum. Displacing this point by increasing or decreasing the relative refractive index increase results in an increase in the value of f . This explains the relation between f and N_1 .

The chromatic dispersion as a function of the wavelength at two values of N_{1min} is plotted in Fig.3. The two curves have zero dispersion point in the middle of the wavelength range approximately. However, they diverge slightly such that the curve which corresponds to $X = 0.08$ has a higher rms value. The increase of the minimum value of f with X is due to this divergence.

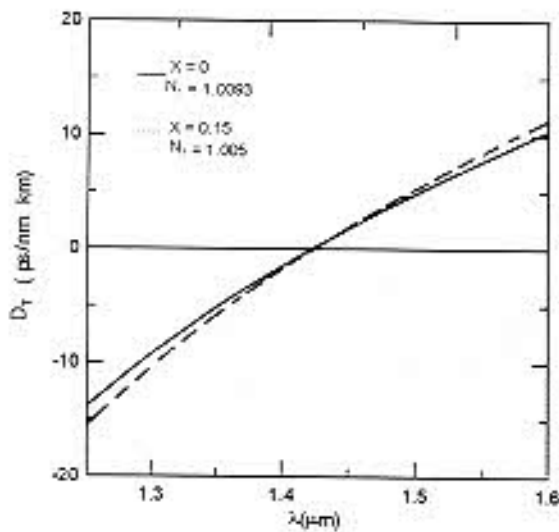


Fig. 3: The chromatic dispersion as a function of the wavelength at different values of N_1 and a corresponding to different values of f_{min} .

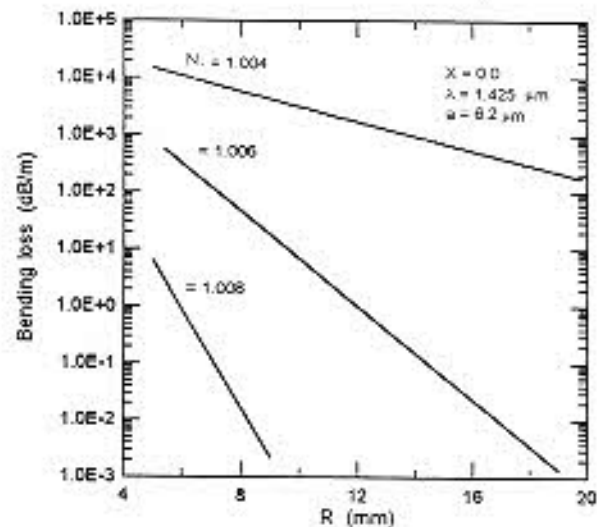


Fig. 4: The bending loss as a function of the bending radius at different values of N_1 .

The shift of the minimum value of f towards lower values of N_1 as the mole fraction X increases is due to the displacement of the zero material dispersion point towards the higher values of λ . As mentioned above, when the zero point of the chromatic dispersion is in the middle of the wavelength range, the rms value, f , will be minimum. Increasing X makes the value of λ_0 to displace towards the middle region; therefore, it will reach the middle region at lower values of N_1 .

III.2 Bending Loss

The bending loss, α_b , is calculated mainly as a function of the bending radius R . In order to study the effect of the parameter N_1 on the behavior of α_b with R , the graph $\alpha_b(R)$ is depicted in Fig. 4 for three different values of N_1 . This process is repeated for different values of a , λ , and X . Furthermore, the same relation is drawn in Fig. 5 for the values of these parameters which

entail that the rms value of the chromatic dispersion is minimum. Fortunately, the parameters N_1 and a which give the lowest minimum value of f yield the curve with the lowest values of the bending loss.

It is found that as N_1 increases, the bending loss decreases, and this is true for a and X as well. However, α_b increases when the operating wavelength, λ , increases. The dependence of the bending loss on the fiber parameters can be interpreted by considering the normalized frequency V . Depending on the value of V , a significant portion of the modal power will propagate in the cladding region. For instance, with V values less than 1.4, over half of the modal power propagates in the cladding [18]. Since the bending loss is due to the radiated energy from the part of the mode which is on the outside of the bend, then the increase of this part is associated with an increase in the bending loss. It is obvious that the normalized frequency increases with N_1 , X , and a . Consequently, the modal power is mainly confined in the core and α_b will be small. Whereas, V decreases as λ increases, so, the bending loss is large.

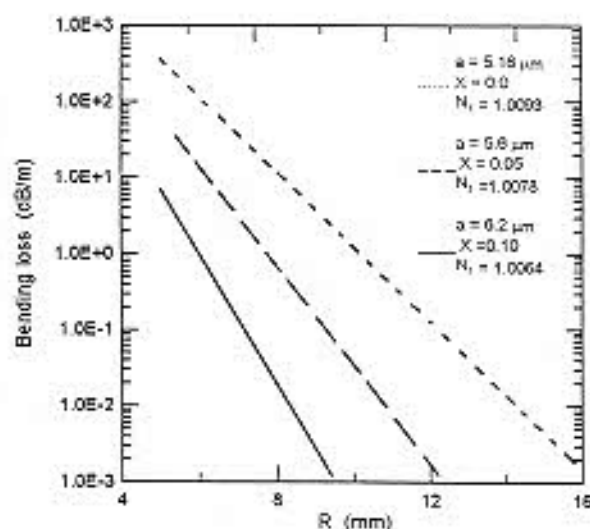


Fig. 5: The bending loss as a function of the bending radius at values of the parameters a , X , and N_1 corresponding to different values of f_{min} .

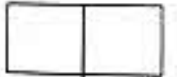
IV. CONCLUSION

We have investigated the suitability of the Gaussian profile fibers as a transmission medium. To minimize the rms value of the chromatic dispersion, λ_0 should be at the middle of the considered wavelength range (1.25 μm -1.6 μm). The value of, the zero dispersion wavelength, λ_0 , is sensitive to the variations in both t and N_1 . The obtained dispersion flattened fiber shows a small bending loss.

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خصائص الانتشار للموجات في ألياف بصرية أحادية النسق ذات معامل انكسار جاوسي القَطَاع

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الملخص العربي

في هذا البحث تم فيه دراسة خصائص الانتشار للموجات في ألياف بصرية أحادية النسق ذات معامل انكسار جاوسي القَطَاع، وتشمل هذه الخصائص كلا من التشتت اللوني، وفقد الانحناءات الكبيرة. وقد كرس هذا البحث لتقليل كل منهما قدر الإمكان. للحصول على أفضل أداء للألياف البصرية أحادية النسق، لا بد أن ينطبق طول موجة التشغيل مع الطول الموجي الذي ينعدم عنده التشتت اللوني. وعليه، يجب اختيار البارامترات الفيزيائية للألياف بحيث تعطي أقل تشتت لوني ممكن. وقد تم تحديد نصف قطر القلب والزيادة النسبية في معامل انكساره اللذين يعطيان أقل جذر متوسط مربع للتشتت اللوني في منطقة محددة من الأطوال الموجية (1.25 - 1.60 μm) تتميز خلاله السليكا النقية بتوهين الطاقة الضوئية المقيدة بالألياف بصورة ضئيلة. والطريقة المستخدمة هي دراسة تأثير البارامترات الفيزيائية لزجاج الليف ولغلافه على جذر متوسط المربع للتشتت اللوني خلال مدى الأطوال الموجية المذكور. يعتبر فقدان الانحناءات الكبيرة معياراً مهماً عند تصميم ليف بصري كي يستخدم في مدى واسع في مجال الاتصالات. لذلك فإن من الضروري تصنيع ليف بصري بحيث تقلل هذا الفقدان للمستوى الأدنى. وقد تم دراسة فقدان الطاقة نتيجة انحناءات كبيرة مختلفة تحت تأثير تغير بارامترات الليف. ووجد أن تلك البارامترات التي تعطي أقل تشتت لوني تكون مناسبة جداً عند اعتبار فقدان الانحناءات الكبيرة.