ERBIUM-DOPED OPTICAL FIBER AMPLIFIER

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ABSTRACT

An optical fiber doped with erbium is studied and investigated as an amplifier (EDFA) for optical pulses. The model of the wave equation of the carrier envelope and the two energy level system is used to study the evolution of solitons in this fiber amplifier. We adapted the Fourier-split method to include the polarization-induced term representing the effect of doping atoms on the propagating optical field. This method is used to solve the system of Maxwell-Bloch equations saving more computation time compared with any finite difference method. Third order dispersion coefficient, self steepening and effect of delayed nonlinear response during pulse propagation are considered. It is studied also the effect of modulation instability of the medium of anomalous dispersion at $\lambda = 1.55\ \mu m$ on amplification of optical pulses in the fiber. The results show that amplification is more affected by the doping level, pulse duration, relaxation time of induced atomic dipole polarization of the doping atoms, dispersion and nonlinearity of the fiber material rather than the fiber core index profile. The obtained EDFA can be used as an optical repeater within the optical communication system, with a recommended length not greater than the dispersion length.

Keywords: Soliton, Nonlinear Effects, Maxwell-Bloch System, Fourier Split Method, Optical Amplifier.

1. INTRODUCTION

Several kinds of optical amplifiers were studied and developed during the 1980s. Semiconductor laser amplifiers were used initially, but the interest shifted toward fiber-based amplifiers because of the practical issues related to coupling losses, polarization sensitivity, and interchannel crosstalk. Fiber-Raman amplifiers require high power (0.5-1W) that are not easily available from semiconductor lasers. Fiber-Brillouin amplifiers can operate at low pump powers, but have bandwidths too small to be useful as in line amplifiers in lightwave systems. A new kind of fiber amplifier based on silica fibers doped with rare-earth elements, was developed in the late 1980s and turned to be most suitable for lightwave system applications. The advent of the erbium-doped fiber amplifier (EDFA) has revolutionized the field of optical communications [1].

When these rare-earth elements are doped in silica or other glass fibers, they become triply ionized by the removal of two outer 6s electrons and an inner 4f electron. Optical properties of such dopants are determined from the partially filled inner 4f shells and are relatively unaffected (except for gain spectrum broadening) by the host because of shielding provided by the outer complete 5s & 5p shells. Many different rare-earth elements, such as erbium, holmium, neodymium, samarium, thulium, and ytterbium, can be used to make fiber amplifiers operate at different wavelengths covering a wide range from visible to infrared. EDFAs have attracted most attention because they operate near 1.55 μm, the wavelength corresponding to minimum loss in silica fibers, and are therefore ideal for 1.55 μm fiber optic communication systems.

The technique of doping silica fibers with rare-earth elements constitutes a relatively new technology that can be used to make fiber laser and amplifiers. Although such fibers were studied as early as 1964, their use became practical only in 1988, after the techniques of fabrication and characterization of low-loss, rare-earth-doped fibers

were perfected. Amplifier characteristics such as the operating wavelength and the gain bandwidth are determined by dopants rather by the fiber, which plays the role of a host medium.

Optical amplifiers amplify incident light through stimulated emission, the same mechanism used by lasers. Indeed, an optical amplifier is just a laser without feedback. Its main ingredient is the optical gain realized when the amplifier is pumped to achieve population inversion. EDFAs make use of a three-level pumping scheme.

A. Icsevgi and W. Lamb [2] discussed the propagation of light pulses in a laser amplifier. Frederic A.Hopf and Marlan O. Scully [3] discussed the theory of an inhomogeneously broadened laser amplifier, where they investigated the propagation of ultrashort electromagnetic pulses in solid state laser amplifier.

The effect of nonlinearity and dispersion on propagation of a light pulse including propagation in optical fibers is discussed in many contexts [1, 4]. In a glass fiber material of negative dispersion coefficient, the light pulse tends to propagate as a soliton.

The propagation of a light pulse in a fiber amplifier, where the pulse is affected by amplification (due to the lasing medium) and nonlinearity (due to Kerr effect in glass) and dispersion of the glass material, was studied mathematically and numerically by many researchers. I.V.Mel'nikov ,R.F.Nabiev and Nazarkin [5] studied the coherent amplification of ultrashort solitons in doped fibers, they used low level doping to achieve adiabatic amplification of soliton. Barry Gross and Jamal T. Manassah [6] numerically and generally discussed the evolution of soliton in doped fibers.

In the present work, the evolution of solitons in EDFAs is studied. The described model of Agrawal [1]of wave equation of envelope and two energy level system is used to study in this fiber amplifier. The study is started with the study of the propagation of optical pulses in a fiber amplifier without including material effects of the fiber, i.e., dispersion, nonlinearity, ..etc. The lasing medium is modeled as a collection of atoms of two energy levels. The contribution of these atoms to the wave equation of pulse propagating in this medium is adding a nonlinear induced polarization term. This term is responsible for the amplification of optical pulses in such a medium. Then, the effect of nonlinearity and dispersion on pulse propagation in this passive optical fiber is investigated. It is discussed also the modulation instability in optical fibers and its effect in generating two side bands around the optical carrier. Finally, the effect of induced polarization of doping atoms was included in the wave equation of the envelope mentioned by Agrawal [1] to get the Maxwell-Bloch system of equations. Using the adapted Fourier-split method, this system of equations was solved numerically, to get pulse evolution shape through the EDFA.

2. MATHEMATICAL MODEL

2.1 Analysis

The W-fiber under consideration is characterized by a biquadratic refractive index profile for its core in the form:

$$n^2(r) = n_1^2\{1-2m(r/a)^2+2m\alpha_1(r/a)^4\}, \quad 0 \le r \le a \quad (1\text{-a})$$
$$= n_1^2\{1-2m+2m\alpha_1\}, \quad 0 \le r \quad (1\text{-b})$$

where n_1 is the axial value, α_1 and m are two tailoring parameters.

For this fiber type, the propagation constant, β, is derived for single-mode operation using the WKB approximation as [7]:

$$\beta^2 = n_1^2k^2\{1 + \frac{8m(1-\sqrt{1+3\alpha_1/V}}{3\alpha_1}, \quad (2)$$

where k is the wave number (= 2π / λ), λ is the wavelength, V is the normalized frequency.

Analysis of optical pulse propagation through an EDFA requires the use of full Maxwell-Bloch model [1, 2]:

$$\frac{\partial A}{\partial z} + \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2} + \frac{\alpha}{2}A = i\gamma|A|^2A + (\frac{\mu_o\omega^2}{2\beta})P, \quad (3)$$

$$\frac{\partial P}{\partial T} = (1/T_2)P + i(\bar{\omega}-\omega)P + (\frac{P^2}{\hbar})AN, \quad (4)$$

$$\frac{\partial N}{\partial T} = (N_o - N)/T_1 - \frac{1}{2}\theta(AP^* + A^*P)/\hbar\bar{\omega}, \quad (5)$$

with

$$\theta = \frac{\int\int_{area} \Re(x,y)|^4 dxdy}{\int\int_{area} \Re(x,y)|^2 dxdy} \quad (6)$$

where z is the propagation distance, T (= t - z/V_g) is the time in the retarded frame and V_g is the group velocity, A = A(z,t) is the slowly varying envelope of the optical carrier, β_2 is the second derivative of the propagation constant β w.r.t. the carrier frequency ω, α is the attenuation constant, μ_o is the free space permitivity, P is the partial polarization produced by the doping atoms, T_1 and T_2 are the population and polarization relaxation times, p is the dipole moment of the doping atom, N (= N_2 -N_1) is the population inversion density with an initial value N_o, $\bar{\omega}$ is the atomic transition frequency, $\Re(x,y)$ is the electric field distribution function in the fiber core and finally, γ is the coefficient of nonlinearity given by [1]:

$$\gamma = \frac{n_2\omega}{n_1 c}\frac{\int\int_{area} \Re(r,\theta)|^4 n(r) r dr d\theta}{\int\int_{area} \Re(r,\theta)|^2 r dr d\theta}, \quad (7)$$

where n(r) is the refractive index profile in the fiber core and n_2 is the Kerr coefficient.

The initial conditions of these equations are as follows; we assume a certain shape of pulse A(0,T), i.e. at z=0 and an initial value N_o (at T = 0) for the population inversion density and zero polarization at T = o and z = o.

The system of Eqns. (3-5) holds for a pulse width ≥ 100 fs, where the effects of higher order nonlinearity and dispersion are neglected. However, for pulse widths in the range from 10 to 100 fs, higher order dispersion and nonlinearity effects should be included and Eq. (1) is modified in the form:

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A + \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2} - \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3}$$

$$= i\gamma[|A|^2 A + \frac{2i}{\omega_o}\frac{\partial}{\partial T}(|A|^2 A) - T_R A\frac{\partial |A|^2}{\partial} T] + (\frac{\mu_o \omega^2}{2\beta})P, \quad (8)$$

where the term containing β_3 represents the third order dispersion. An important higher-order nonlinear effect is self-steepening governed by the second term on the right hand side of this equation. Self steepening of an optical pulse results from the intensity dependence of the group velocity.

2.2 *Numerical Solution*

The propagation equation of the carrier envelope (A), Eq. (8), is a nonlinear partial differential equation that does not generally lend itself to analytic solutions except for some specific cases in which the inverse scattering method can be employed [4]. A numerical approach is therefore necessary for understanding the nonlinear effects in fibers. The method that has been used extensively to solve the pulse propagation problem in nonlinear dispersive media is the split-step Fourier method [8]. The relative speed of this method compared with most finite-difference methods can be attributed in part to the use of the Finite Fourier Transform (FFT) algorithm. Here, we will describe the split -step Fourier method and its modification as they apply to the pulse propagation problem in the EDFA. To understand the philosophy behind the split-step Fourier method, it is useful to write Eq. (8) in the form:

$$\frac{\partial A}{\partial z} = (D + Q_{NL})A + P_{NL}, \quad (9)$$

where D is a differential operator that accounts for dispersion and absorption in a linear medium and O_{NL} is a nonlinear operator that governs the effect of fiber nonlinearities on pulse propagation. Comparing Eq. (9) with Eq. (8), both operators D and O_{NL} can be easily obtained. Here, P_{NL}, corresponding to the second term on the right hand side in Eq. (8), accounts for the effect of nonlinear polarization due to excitation of doping atoms.

In general, dispersion, nonlinearity and gain act together along the fiber length. The split-step Fourier method obtains an approximate solution by

assuming that when the optical field propagates a small distance h, dispersion and nonlinear effects can be pretended to act independently. More specifically, propagation from z to z+h is carried out in three steps. First, the nonlinearity acts alone, and D = 0. Second, dispersion acts alone and $O_{NL} = 0$. Finally, amplification (gain) acts alone and both nonlinearity and dispersion are set to zero. So, the envelope after a distance h, can be represented by:

$$A(z+h,T) \simeq \exp(hD)\exp(hO_{NL}).(A(z,T)+P_{NL}h) \quad (10)$$

The execution of the exponential operator exp(hD) is carried out in the Fourier domain using the prescription [1]:

$$\exp(hD)B(z,T)=F^{-1}\{\exp(hD)(i\omega)F\{B(z,t)\}\}. \quad (11)$$

where F denotes the Fourier-transform operation, D(iω) is obtained from the operator D by replacing the differential operator $\partial/\partial T$ by iω, and ω is the frequency in the Fourier domain. The accuracy of the split-step Fourier method can be improved by adopting a different procedure to propagate the optical pulse over one segment from z to z+h. In this case, Eq. (10) will have the form:

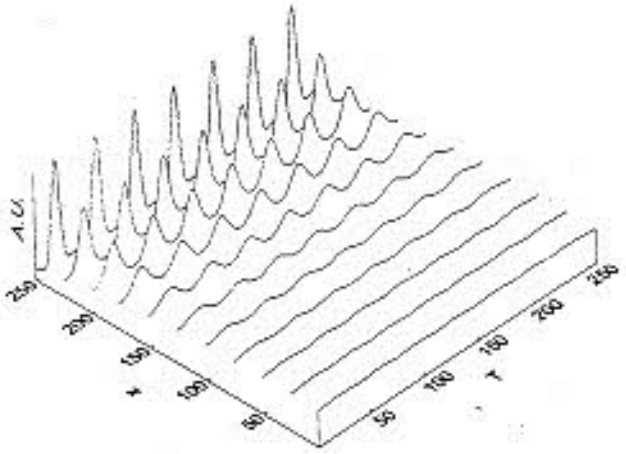
$$A(z+h,T) \simeq \exp\left[\frac{h}{2}D\right]\exp\left[\int_{z}^{z+h} N(z')dz'\right]\left[\left(\exp\left[\frac{h}{2}D\right]A(z,T)\right)+hP_{NL}\right]. \quad (12)$$

The procedure of evaluation of pulse propagation begins with calculating the initial values of both nonlinearity operator, $\exp(O_0h)$, and the nonlinear polarization induced by doped atoms, hP_{NL0}, from the initial value of A(z,T). In the first half of the segment, i.e., from z to z+h/2, we apply the dispersion operator on the initial profile of the optical pulse. At the h/2 point, we save the shape of pulse (A_o), add hP_{NL0} and multiply by $\exp(O_0h)$. At the remaining part of the segment, we apply the dispersion operator and get A(z+h,T). To improve accuracy, two more iterations are performed, this is because the nonlinearity operator is evaluated using the initial value of A which varies along the propagation distance h, so the previous calculated value of nonlinearity operator does not reflect exactly the contribution of nonlinearity effect to the propagation of pulse. The obtained value A(z+h,T) is used again to calculate iterated values hP_{NL1} and O_1h. We add $(hP_{NL0} + hP_{NL1})/2$ to the previously stored value A_0 after first half of segment and the resulting value is multiplied also by $\exp\{(O_0+O_1)h/2\}$. In the last half we apply dispersion only on signal to get an improved value of A(z,T). This iteration is repeated once more to get a more improved value of A(z,T) at the end of the segment. Our contribution to Fourier-split method is adding a nonlinear polarization term due to doping atoms to the equation and the way of using this term in iteration procedure used to improve calculation. The iteration could be done more times to improve calculations but it is time consuming because no appreciable improvement is obtained.

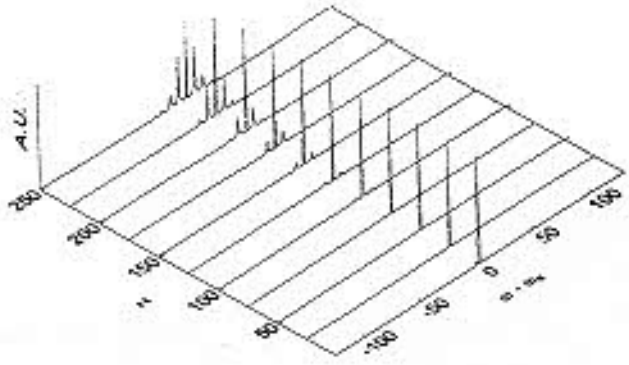
3. RESULTS AND DISCUSSION

Based on the described model, a computer program is performed to carry out calculations and to find the evolution of optical pulses propagating through the Er-doped fiber amplifier (EDFA). Initially, a light soliton pulse is used with a sech function shape [4]. Steps in z direction and time are taken equal to (1/50) of the dispersion length, L_D and (1/16) of the initial soliton width, T_o, respectively, unless other scale is mentioned. The time axis is divided into 256 parts. This is because we used fast Fourier transform (FFT) to account for dispersion. The wavelength, λ is taken to be equal to 1.55 μm, corresponding to region of anomalous dispersion and minimum loss [9]. The dipole moment of erbium, $p = 8.339\times10^{-32}$ C.m [5]. The W-fiber under consideration, Eq. (1), has a core radius, a = 90 μm, $n_1 = 1.5$, $\alpha_1 = 0.9$ and m = 0.001.

Figure (1a) shows the amplification of small sine wave ripples added to a carrier of constant amplitude E_o equal to electric field amplitude needed to support a fundamental soliton in an undoped fiber. Here, we used a step $\tau = T_o/8$. The figure shows a large gain contribution to the ripples in which the peaks can attain a value of $2E_o$ at a distance of $5L_D$. In the frequency domain, Figure (1b), the peak around ω_o is accompanied by two other small peaks (like side bands) at $\pm\ \omega_c$, which means an energy transfer from low frequency components to high frequency ones.



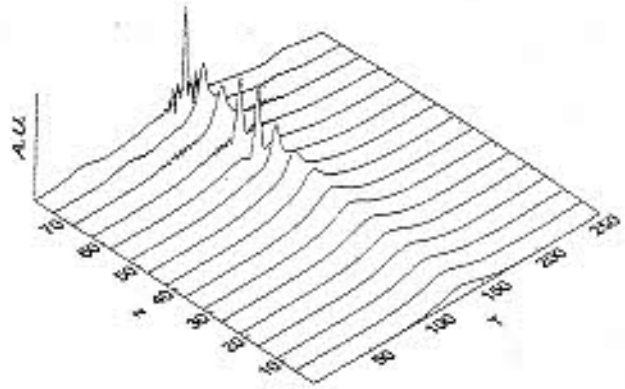
(a)



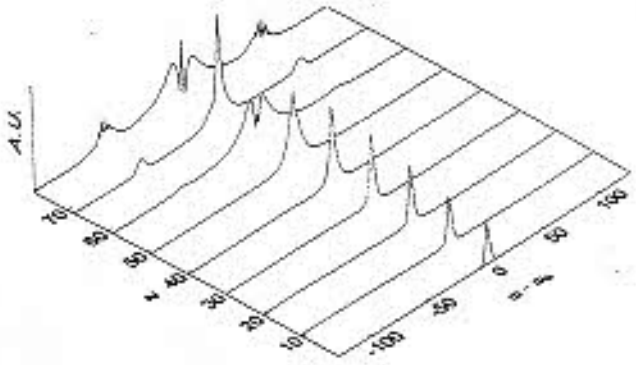
(b)

Figure 1. Evolution of a small perturbation in an undoped fiber (a. time domain, b. frequency domain).

Figure (2a) and (b) -obtained when higher order nonlinearity, dispersion, and Raman effects are neglected- shows the amplification of the fundamental soliton in a doped fiber. A doping level of 2.5×10^{23} atoms/m^3 and a soliton width of 3×10^{-13}s are used. Dipole relaxation time, T_2, of the doping material is 10^{-13} s. This data gives a dispersion length = 0.43 m and a gain = 1, and hence the signal is amplified about 2.718 times in one dispersion length.



(a)



(b)

Figure 2. Evolution of the fundamental soliton in the fiber amplifier, T_o=0.3 ps, T_2=0.1 ps, the doping level is 2.5×10^{23} atoms/m^3.(a. time domain, b. frequency domain).

The evolution of the pulse in time domain in Figure (2a) shows a dramatic increase in the soliton peak accompanied by a decrease in its width after one dispersion length. This behavior or soliton evolution is in consistence with the work done by Agrawal [1] with an approximated wave equation and pulse width >> relaxation dipole time T_2 . It is noted that this process is suitable for pulse compression. The pulse suffers after that from a decrease in its peak with a generation of a small subpulse until it reaches another maximum. This behavior of subpulse formation also coincides with that found in Ref. [1]. In frequency domain, Figure (2b), the frequency spectrum of initial soliton widens and increases until the end of the first dispersion length. Some deformation occurs in which

components near zero frequency decrease and formation of two side bands far away from soliton spectrum appears. These two side bands generate ripples near tails of soliton at z = 60 (= 1.2 L_D) in time domain. It is clear that side bands are far away from any peak and they can not be formed by modulation instability. We suggest a mechanism such that components at $\pm \omega_c$ and their harmonics plus components of zero frequency interact together through the nonlinearity of the medium to generate a band of frequencies forming these two side bands. It appears also that such mechanism acts slowly when the pulse spectrum does not reach this frequency range. However, when the pulse spectrum broadens and reaches these side bands, the mechanism is accelerated so that the side bands build up through small evolution distance.

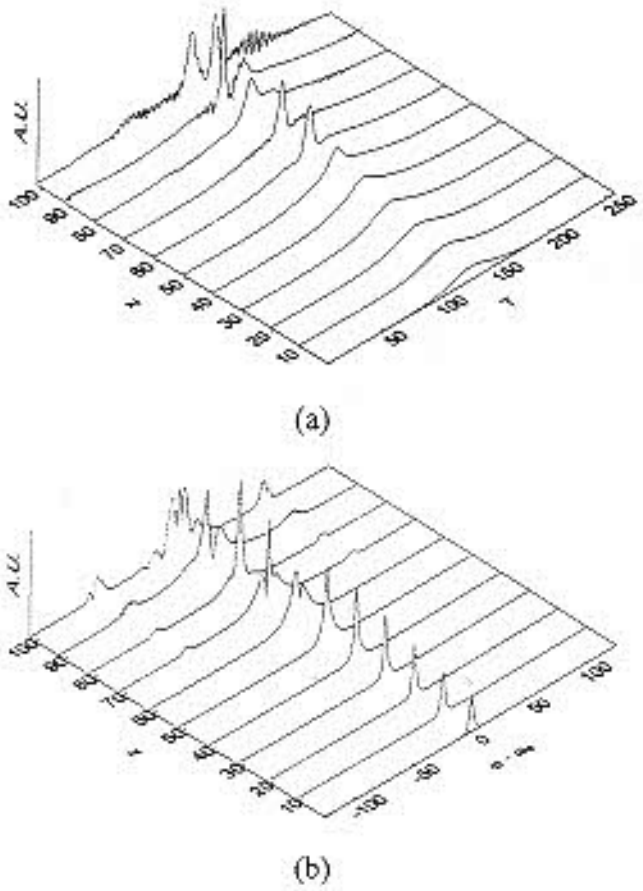


Figure 3. Evolution of the fundamental soliton in the fiber amplifier, T_o=T_2=0.1 ps, the doping level is 2.5x10^{24} atoms/m^3. (a. time domain. b. frequency domain).

The calculations are repeated at different data and similar behavior is obtained and a sample of results is given in Figure (3) (a and b), which shows propagation for a soliton of initial width T_o =T_2. The doping level was 2.5x10^{24} atoms/m^3 to achieve the same gain; g = 1. The pulse is amplified and compressed, so, its width becomes << T_2 . Hence, the coherence effect of the medium -in which the induced dipole polarization still memorizes the pulse after its passage [3]- shows itself by a second pulse formation at z = 2L_D; an echo phenomenon.

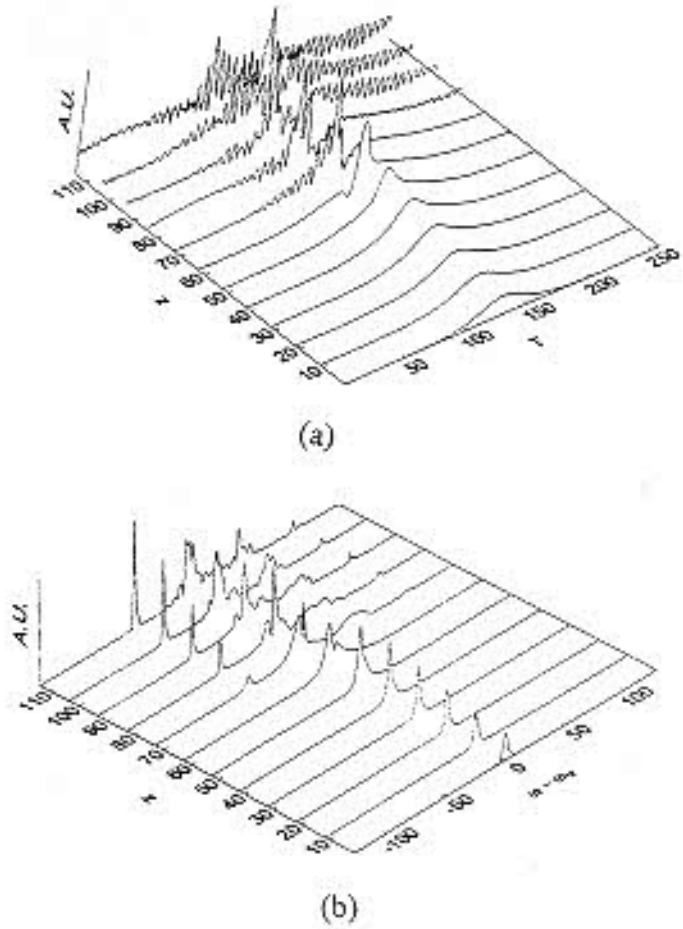


Figure 4. Evolution of the fundamental soliton in the fiber amplifier, T_o=T_2=0.1 ps, the doping level is 2.5x10^{24} atoms/m^3. The complete model is used, where all effects are considered a. time domain, b. frequency domain.

Finally, all higher order dispersion, nonlinearity and Raman effects are considered in the wave equation

of the envelope, Eq. (8). A sample of results are shown in Figure (4) (a and b), where $T_o = T_2$ and a doping level of 2.5×10^{24} atoms/m^3 is used. The Raman effect appears in time domain, Figure (4a), in splitting the original pulse into two pulses; the first one contributes to the original signal and the other is the Raman pulse propagating in the opposite direction. This result agrees with that obtained by J. Manassa et al. [6]. In the frequency domain, Fig. 4.b, the lower frequency components are amplified more; this is a clear evidence of Raman scattering effect; in which the incident photon is scattered to a lower frequency one, while in the same time the molecule makes a transition between vibrational states. It appears that the Raman effect enhances ripples formation by amplifying lower side band frequencies.

4. CONCLUSION

In this study, an erbium-doped optical fiber is used as an amplifier (EDFA). We showed that by using a suitable doping level, signal could be amplified to about 2.8 times in one dispersion length. It appears that after one dispersion length, the signal is distorted by several effects; specially generation of higher order harmonics due to nonlinearity of the medium, generation of subpulses due to coherence effect for ultra short pulses and Raman scattering in which Raman pulse is generated after a distance of about one dispersion length. So, it is advisable to use it for only one dispersion length as a fiber amplifier.

Because of relatively high doping, the gain achieved here is a nonadiabatic one, i.e. the soliton does not maintain its order after amplification. Therefore, a notice must be kept when the EDFA is used as a repeater that after amplification, the fundamental soliton propagates as a higher order soliton. In consistence with pervious work, it is noted that the gain factor is a function of the doping level, pulse duration, relaxation time of induced atomic dipole polarization of the doping atoms, which are related to the physical properties of the medium. The obtained system can be used as an optical repeater within the optical communication system, with a recommended length for the EDFA not greater than the dispersion length.

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الملخص العربي

مكبر الليف البصري المُطَعَّم بعنصر الإربيوم

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تعتمد فكرة مكبر الليف البصري على استخدام ألياف بصرية مُطَعَّمة بشوائب من العناصر الأرضية النادرة (rare-earth elements) مثل الإربيوم، ويقوم مصدر ضوئي ثنائي ليزري (laser diode) بعملية ضخ ذرات الشوائب لمستوى الطاقة العلوي وجعلها في حالة توزيع تزاحمي معكوس (population inversion). فعند مرور الإشارة الضوئية بالمكبر فإنها تَستحِثُّ ذرات الشوائب المُثارة على إطلاق طاقتها، ومن ثَمَّ تعود هذه الذرات لمستوى الطاقة الأرضي. والطاقة المُشَعَّة تضاف إلى طاقة الإشارة الضوئية، وبذلك يتم تكبيرها.

وعندما تكون نسبة الشوائب في الألياف صغيرة، فإن الموجة المتفاردة (soliton) تُكَبَّر تكبيراً أدياباتيكياً (adiabatic). وقد تمت دراسة هذا النوع من التكبير من قبل دراسةً تحليلية، وأمكن استنتاج معادلات تقريبية تصف انتشار الموجات المتفاردة في هذه الحالة. أما عندما يكون مستوى الشوائب مرتفعاً، فإن الطرق التحليلية لا تصلح، ويتطلَّب الأمر استخدام طرق عددية لدراسة انتشار الموجات المتفاردة في هذا النوع من المكبرات. وهذا ما نحتاجه في بحثنا، حيث تم تعديل إحدى الطرق المستخدمة في دراسة انتشار الموجات في الألياف البصرية العادية لتناسب دراسة انتشار الموجات المتفاردة في الألياف البصرية المُكبِّرة عن طريق حل مجموعة معادلات (Maxwell-Bloch). والطريقة المُعدَّلة هي طريقة (Fourier split method) . وقد دُرِسَت عدة حالات أظهرت اعتماد التكبير على العرض الزمني للموجة (duration)، وعلى زمن الاسترخاء (relaxation time) لذرات الشوائب، وعلى تشتت ولا خطية مادة الألياف أكثر من اعتمادها على شكل معامل الانكسار لقلب هذه الألياف.