SOLITON PROPAGATION IN OPTICAL FIBERS WITH W- TAILORED REFRACTIVE INDEX

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Abstract

Soliton transmission of optical pulses in nonlinear inhomogeneous media with W- tailored refractive index is modeled and parametrically analyzed. Two kinds of inhomogeneities are simultaneously considered and investigated :
the biquadratic variation of refractive index and the boundary conditions of the cladded fiber. The wave equation is solved in the presence of these inhomogeneities assuming the radial dependence under a simple series form. The tailored parameters of the refractive index spatial profile have a remarkable influence on the minimum power , high bit rate , and the high group velocity required to achieve a stable soliton transmission.

I. Introduction

The term soliton has recently been coined to describe a pulselike nonlinear wave. The balance between the nonlinearity effect from one side and the dispersion effect from the other side creates a solitary wave. The dispersion of a medium , such as an optical fiber , (in the absence of nonlinearity) makes the various frequency components propagate at different velocities; while the nonlinearity (in the absence of dispersion) causes the pulse energy be continually injected (via harmonic generation) into higher frequency modes. That is to say , the dispersion effect results in the broadening of the pulse , while the nonlinearity tends to sharpen it (1). The optical soliton transmission leads to high bit rate and automatic pulse reshaping (2,3,4).

In this paper a method for soliton transmission in inhomogeneous media with W-tailored refractive index is modeled (2) and parametrically analyzed. Two kinds of inhomogeneities are simultaneously considered : (a) Biquadratic variation of the refractive index (W-tailored radial profile) , and (b) Boundary conditions of the cladded fiber.

II. Basic Model and Analysis:

Starting with the three-dimensional wave equation including nonlinearity , dispersion , and two kinds of inhomogeneities , and on the same spirit of Ref. (2) , the amplitude of the field A (r,z,t) is obtained by solving :

$$\Big[\nabla_{\perp}^2 + \frac{\partial^2}{\partial z^2} + 2jq \frac{\partial}{\partial z} - q^2 + (1-2\alpha\rho^2 + 2m\alpha\rho^4) \left\{ (k_0^2 + 2j\, k_0 k_0' \frac{\partial}{\partial t}) - (k_0'^2 + k_0 k_0'') \frac{\partial^2}{\partial t^2} \right\} \Big] A\,(r,z,t)$$

$$= - \frac{2n_2}{n_0} k_0^2 \, |A|^2 \, A\,(r,z,t), \qquad (1)$$

with q the propagation constant and both nonlinearity and dispersion effect are assumed through the refractive index,

$$n\,(r,w,E\,) = n\,(r,w\,) + n_2\,|E|^2 \,. \qquad (2)$$

where n_2 is the Kerr coefficient ($\simeq 1.2\times10^{-22}\ m^2/V^2$ for quartz).
The inhomogeneity of the medium is assumed under the biquadratic (W- tailored) form ,

$$n^2(r,w) = n_o^2(w)\,(1-2\alpha\rho^2 + 2m\alpha\rho^4) \qquad (3)$$

where both α and m are positive tailoring parameters , $\rho = r/R$, and R is the fiber radius.

In equation (1)

$$K(w) = wn(w)/c \ ,$$
$$K_o = w_o\, n(w_o)/c \ ,$$
$$K_o' = \partial K/\partial w \big|_{w_o} \ , \text{ and}$$
$$K_o'' = \partial^2 K/\partial w^2 \big|_{w_o} .$$

The " r-dependence " is treated in a linear fashon. Considering small nonlinear effects and using the separation of variables ,

$A(r,z,t) = U(r)\varphi(z,t)$, we obtain:

$$\varphi(z,t)\nabla^2 U(r) + U(r)\Big[\frac{\partial^2}{\partial z^2} + 2jq\frac{\partial}{\partial z} - q^2 + (1-2\alpha\rho^2+2m\alpha\rho^4)\Big\{\Big(K_o^2 + 2j\,K_o K_o'\frac{\partial}{\partial t}\Big) - (K_o'^2 + K_o K_o'')\frac{\partial^2}{\partial t^2}\Big\}\Big]\varphi(z,t)$$

$$= -\frac{2n_2}{n_o}\,K_o^2\,|U\varphi|^2\,U(r)\varphi(z,t) \qquad (4)$$

The " r-dependent " part , $U(r)$ is taken as the solution of the steady state wave equation for a linear inhomogeneous medium

$$\Big\{\nabla_\perp^2 - P_o^2 - K_o(1-2\alpha\rho^2 + 2m\alpha\rho^4)\Big\}\,U_o(\rho) = 0 \qquad (5)$$

where P_0 is the propagation constant calculated through the perturbation theory to the third order for a single mode (5) as :

$$P_o^2 = K_o^2 - \frac{2k_o\sqrt{2\alpha}}{R} + \frac{2m}{R^2} + \frac{9m^2}{2R^3\, K_o\sqrt{2\alpha}} + \frac{79m^3}{8\, R^2\, K_o^2\,\alpha} \quad (6)$$

$U_o(\rho)$ is obtained under a simple series form.

$$U_o(\rho) = e^{-\rho^2/2} \sum_{i=0}^{i=\infty} a_{2i}\,\rho^{2i} \quad (7)$$

In these calculations, the expansion is truncated at i=7 with a negligible error. The use of equation (7) in equation (5) yields the coefficients a_{2i}'s as functions of K_o R, α, and m, with

$$Z1 = (P_o^2 - K_o^2)\, R^2 \quad (8\text{-}a)$$

$$Z2 = 2\alpha R^2\, K_o^2 - 1 \quad (8\text{-}b)$$

$$Z3 = -2m\alpha R^2\, K_o^2 \quad (8\text{-}c)$$

These coefficients are obtained under the forms:

$$a_2 = (1/4)\,[\,2 + Z1] \quad (9\text{-}a)$$

$$a_4 = (1/16)[(6+Z1)\, a_2 + Z2] \quad (9\text{-}b)$$

$$a_6 = (1/36)[(10+Z1)\, a_4 + Z2\, a_2 + Z3] \quad (9\text{-}c)$$

$$a_8 = (1/64)[(14+Z1)\, a_6 + Z2\, a_4 + Z3\, a_2] \quad (9\text{-}d)$$

$$a_{10} = (1/100)[(18+Z1)\, a_8 + Z2\, a_6 + Z3\, a_4] \quad (9\text{-}e)$$

$$a_{12} = (1/144)[(22+Z1)\, a_{10} + Z2\, a_8 + Z3\, a_6] \quad (9\text{-}f)$$

$$a_{14} = (1/196)[(26+Z1)\, a_{12} + Z2\, a_{10} + Z3\, a_8] \quad (9\text{-}g)$$

In equation (4) , the function U(r) is replaced by the function $U_o(\rho)$ to get :

$$U_0(\rho)\,[(P_0^2 - q^2) + \frac{\partial}{\partial z^2} + 2jq\,\frac{\partial}{\partial z} + (1 - 2\alpha\rho^2 + 2m\alpha\rho^4)$$

$$\cdot\left\{2j\,K_0K_0'\,\frac{\partial}{\partial t} - (K_0'^2 + K_0K_0'')\,\frac{\partial^2}{\partial t^2}\right\}]\,\varphi(z,t)$$

$$= -\,\frac{2n_0}{n_0}\,K_0^2\,|U_0|^2\,|\varphi|^2\,U_0(\rho)\,\varphi(z,t) \qquad (10)$$

Multiplying both sides by $2\pi\rho\,U_0(\rho)\,d\rho$ and integrating from o to 1 , we get:

$$[\,(P_0^2 - q^2) + \frac{\partial^2}{\partial z^2} + 2jq\,\frac{\partial}{\partial z} + (1 - \alpha + \frac{2}{3}\alpha m)$$

$$\left\{2j\,K_0K_0'\,\frac{\partial}{\partial t} - (K_0'^2 + K_0K_0'')\,\frac{\partial}{\partial t^2}\right\}]\,\varphi(z,t)$$

$$= -\,\frac{2n_2}{n_0}\,K_0\,\delta\,|\varphi|^2\,\varphi(z,t) \quad , \qquad (11)$$

where $= \int_0^1 U_0^{\;3}(\rho)\;d\rho^2$.

For soliton solution to exist , $\varphi(z,t)$ must be real . Thus

$$2q\,\frac{\partial\varphi}{\partial z} + 2k_0K_0'\,(1 - \alpha + \frac{2}{3}\,m\,\alpha)\,\frac{\partial\Phi}{\partial z} = 0 \qquad (12)$$

For this to be possible , $\varphi(z,t)$ must be a function of ζ where

$$\zeta = (V_g t - z)\,/\,V_g\,\tau \, , \qquad (13)$$

where τ is the width of the pulse and V_g is the group velocity given by :

$$V_g = q\,/\,[\,K_0K_0'\,(1 - \alpha + \frac{2}{3}\,m\alpha)\,] \qquad (14)$$

In this expression , P_0 could be substituted for q with a negligible error to get:

$$V_g = P_0\,/\,[\,K_0K_0'\,(1 - \alpha + \frac{2}{3}\,m\alpha)\,] \qquad (15)$$

The use of equations (12) , (13) and (15) in equation (11) yields:

$$a \frac{d^2\varphi}{d\,\eta^2} + b\varphi + c\varphi^3 = 0 \qquad (16)$$

where

$$a = (1/V_g^2) - (1 - \alpha + \frac{2}{3}\alpha m)(K_o'^2 + K_o K_o'') \qquad (17\text{-a})$$

$$b = q^2 - P_o^2 \qquad (17\text{-b})$$

$$c = 2n_2 K_o^2 \tau^2 \delta / n_o \qquad (17\text{-c})$$

The light pulse soliton given by

$$\varphi(\eta) = \varphi_o \operatorname{sech}\eta \qquad (18)$$

is assumed as a solution of equation (16).

The use of equation (18) in equation (16) yields:

$$(a+b) + (c\varphi_o^2 - 2a)\operatorname{sech}^2\varphi = 0 \qquad (19)$$

which necessiates :

$$a + b = 0 \qquad (20\text{-a})$$

$$c\varphi_o^2 - 2a = 0 \qquad (20\text{-b})$$

or:

$$q^2 = P_o^2 + (1/V_g^2) - (1 - \alpha + \frac{2}{3}\alpha m)(K_o'^2 + K_o K_o'') \qquad (21)$$

$$\varphi_o^2 = \frac{(1/V_g^2) - (1 - \alpha + \frac{2}{3}\alpha m)(K_o'^2 + K_o K_o'')}{n_2 K_o^2 \tau^2 \delta / n_o} \qquad (22)$$

For soliton to exist , φ_o^2 must be a positive quantity which in turns necessiates the positivity of a , b , and c in equation (16). In terms of φ_o^2 , the peak power P_o of a soliton is given by:

$$P_o = \tfrac{1}{2} V_g \varphi_o^2 \epsilon_o S n_o^2 \quad . \qquad (23)$$

where ϵ_o is the permittivity of free space, and S is the cross - sectional area of the fiber core . This is the peak power in watts, of a laser source to obtain a light soliton pulse. This power is a function of the fiber tailoring parameters and the dispersion conditions.

III. Results and Discussion

Both the peak power P_o (or the quantity ϕ_o^2) and the group velocity V_g are functions of the radius R, the tailoring parameters (α and m), and the dispersion conditions.
The quantity K'', which depends basically on the wavelength λ, determines the dispersion conditions. According to Marcuse (6), K'' was approximated by Gloge (7) under the form:

$$K'' = -\sigma\lambda_n \; (1-\lambda_n^{-1}) \; , \qquad (24)$$

where σ is a positive constant and $\lambda_n = \lambda/ 1.27$, with λ in μm.
The kerr coefficient, n_2, is usually encountered positive. Thus, for anomalous dispersion, $K_o''<0$ ($\lambda_n > 1$), the quantity ϕ_o^2, and hence the power P_o, is positive.
For normal dispersion, $K_o'' > 0$ ($\lambda_n < 1$), the light soliton is also possible if the values of the tailoring parameters (α and m) are adjusted with the dispersion conditions K_o' and K_o'' namely if

$$K_o'^2 \alpha \left(\frac{2}{3} m - 1 \right) > \left(1 - \alpha + \frac{2}{3} \alpha m \right) K_o K_o''$$

This necessiates at least that

$$2\alpha \left(\frac{2}{3} m - 1 \right) > 1$$

Form equation (23), it is clear that the quantity $P_o \tau^2/B^2$ is a function of the set $\{K_o, K_o', K_o'', \alpha, m\}$, say f (w_o, α, m), and consequently $P_o/B^2 R^2$ is also a function of the same set of parameters , where B is the bit rate . Thus for a given fiber employed as a communication channel at a certain frequency , the bit rate , B , is proportional to $(P_o/R^2)^{\frac{1}{2}}$, or

$$B = 0.5 \, [\, P_o \, / R^2 \, f \, (W_o \, , \alpha , \, m \,) \,]^{0.5} \quad .. \qquad (25)$$

Thus , B will be of maximum value if P_o is of maximum value and both R and f are of minimum values.

The variations of both the peak power P_o and the group velocity V_g with the parameter α at different values of m and the assumed set of parameters are displayed respectively in Figs. 1-4 . From these figures and the other obtained data , it is found that both P_o and V_g possess respectively positive and negative correlations with the parameter α whatever the value of the parameter m .

The results clarified in Fig 5 , where the radius R appears as a parameter , assert that , for any assumed set of parameters , there is a threshold value α_{th} to achieve a stable soliton transmission. α_{th} increases as the fiber radius increases , Fig 6.

IV. Conclusions;

Whatever the type of dispersion , one can design a fiber with W-tailored refractive index for stable light soliton transmission which suites well for a quality process (high bit rate , minimum power , and high group velocity) , the parameter α must be tailored with a minimum value , i.e. , α_{th} ; while the parameter m must be tailored with a maximum value .

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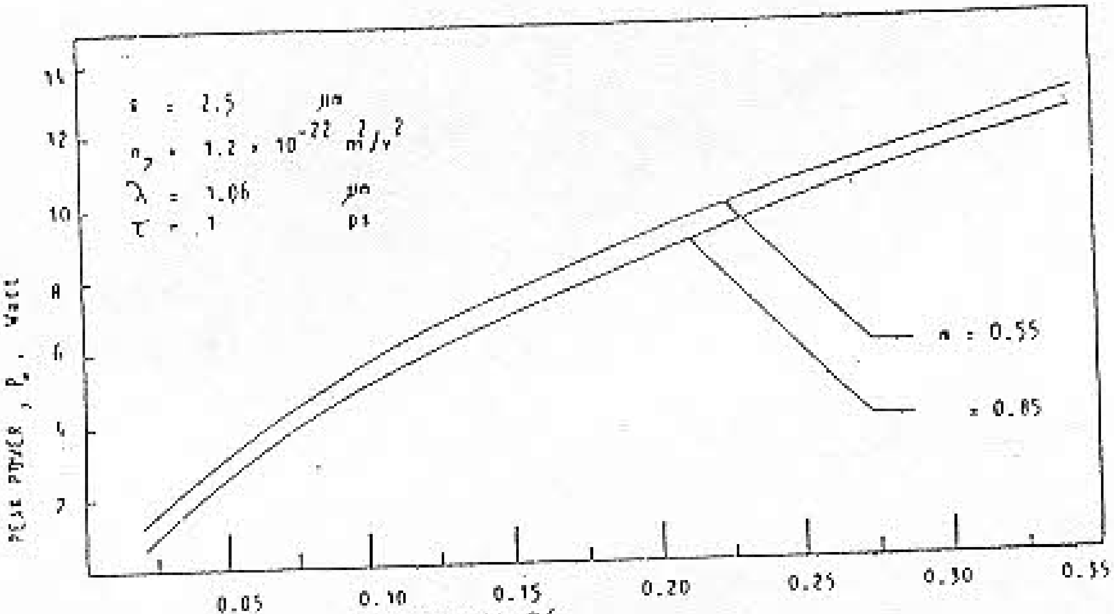


Fig.1. Variation of P_o with α for different values of m and the assumed set of other parameters.

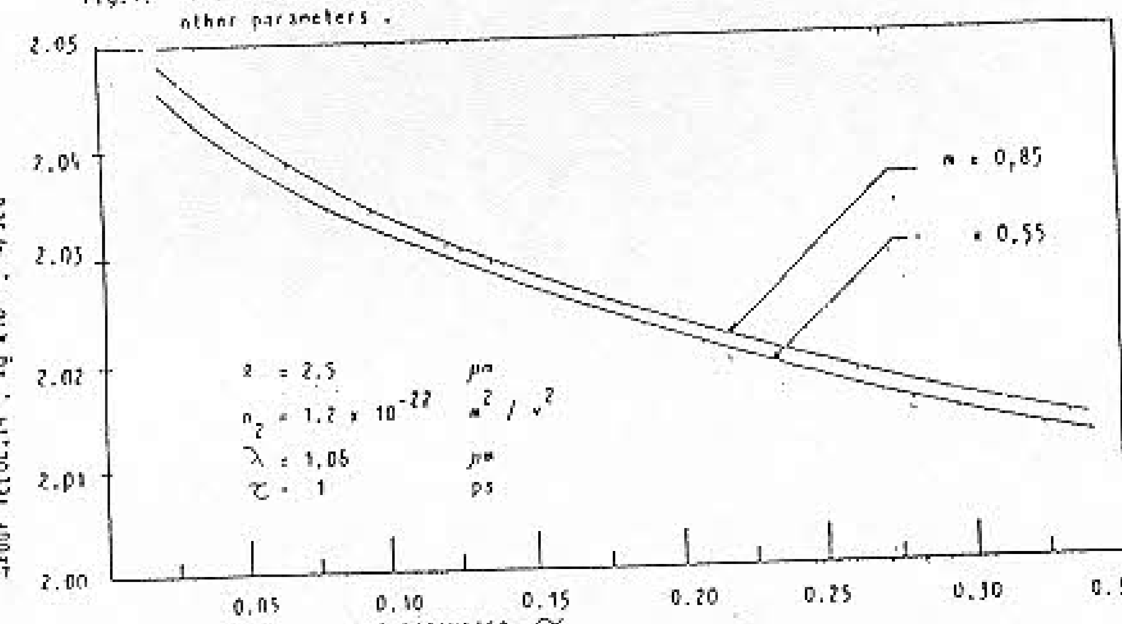
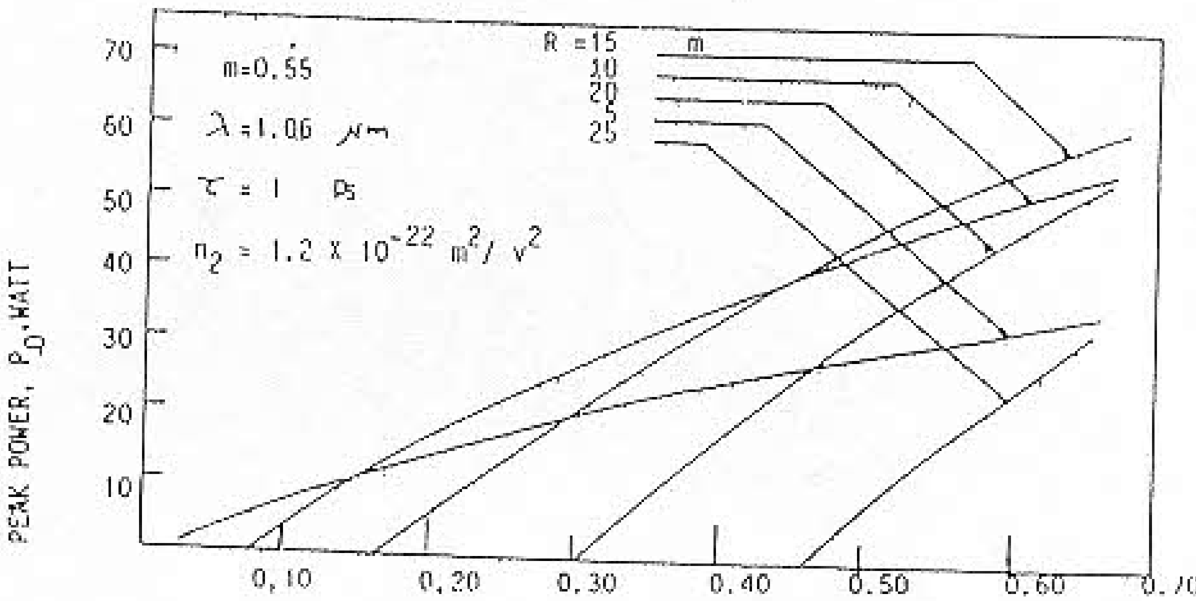
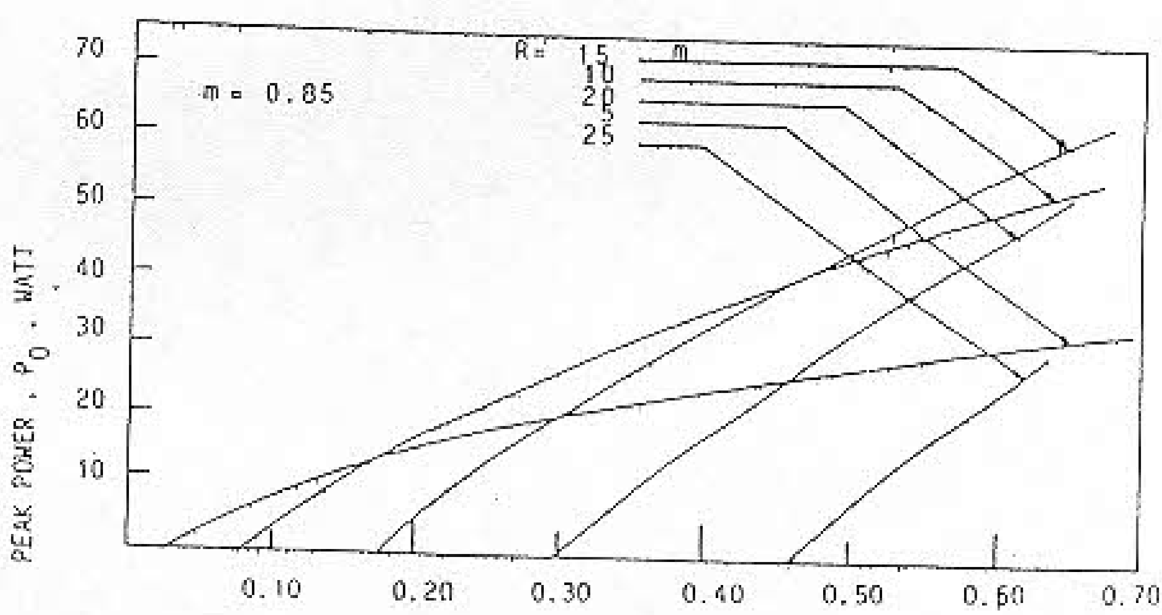


Fig.2. Variation of Vg with α for different values of m and the assumed set of other parameters.



THE PARAMETER α

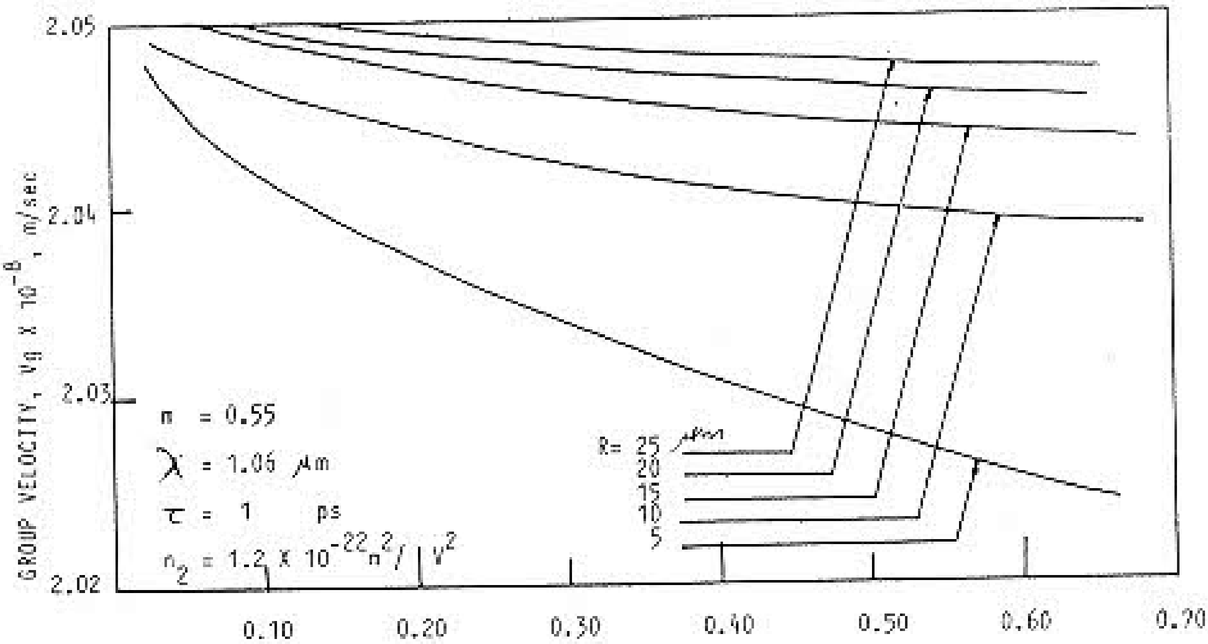
(A)



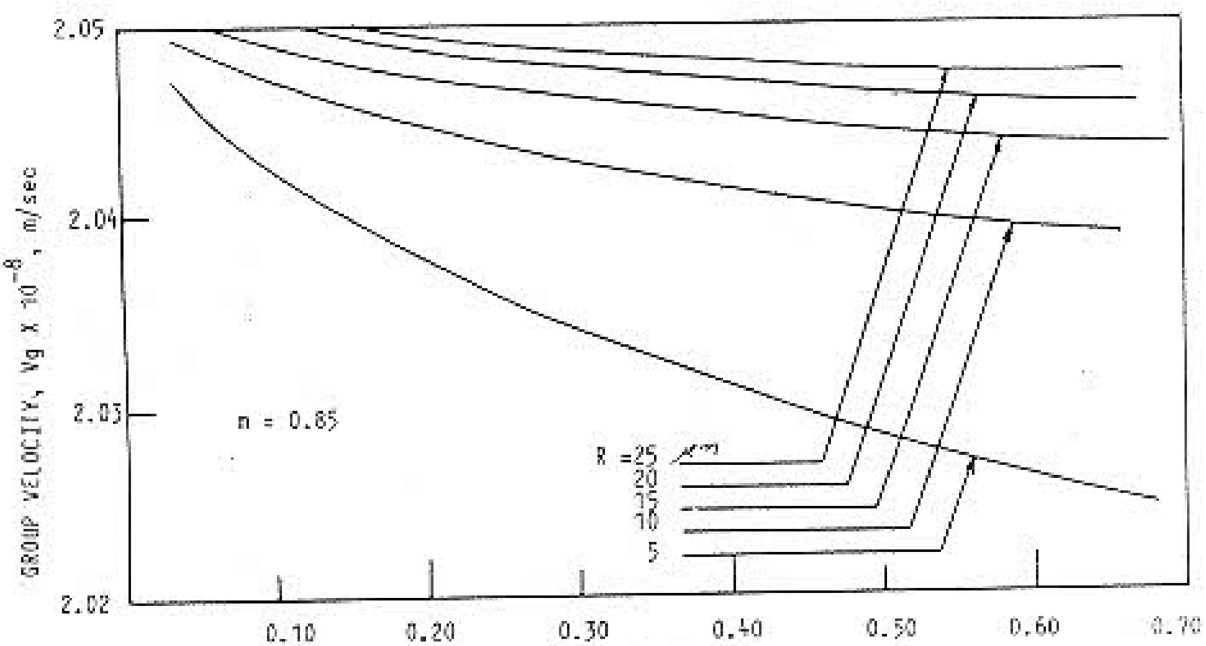
THE PERAMETER α

(B)

Fig.3 : Variation of P_o with for different values of m,R, and the assumed set of other parameters .



(A)



(B)

Fig. 4 : Variation of Vg with for different values of m , R , and the assumed set of other parameter .

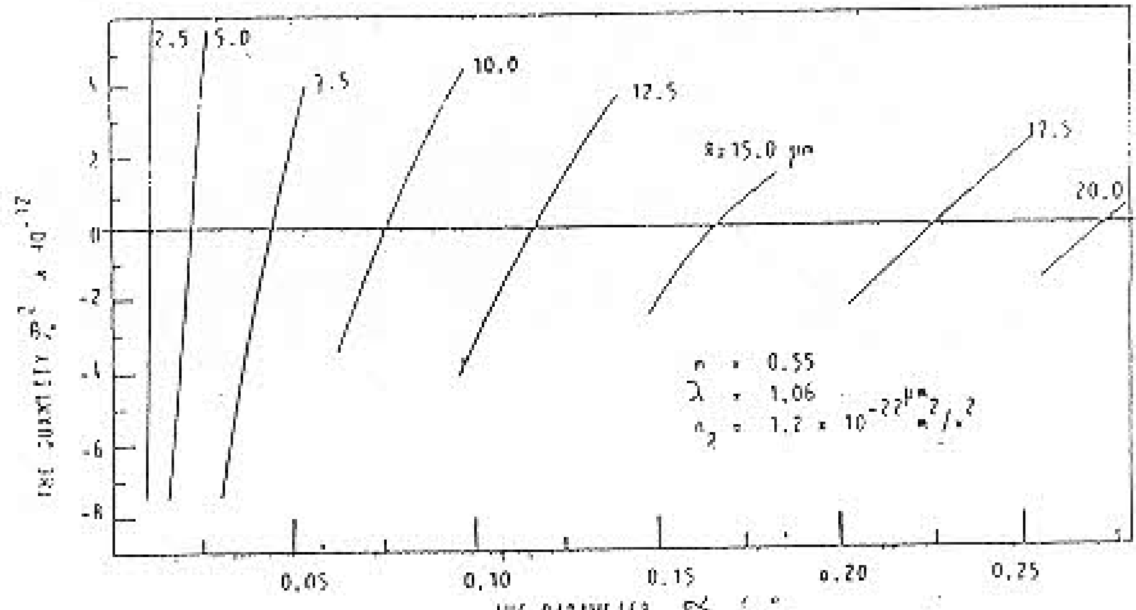


Fig.5. Variation of φ^2 with α for different values of R and the assumed set of other parameters .

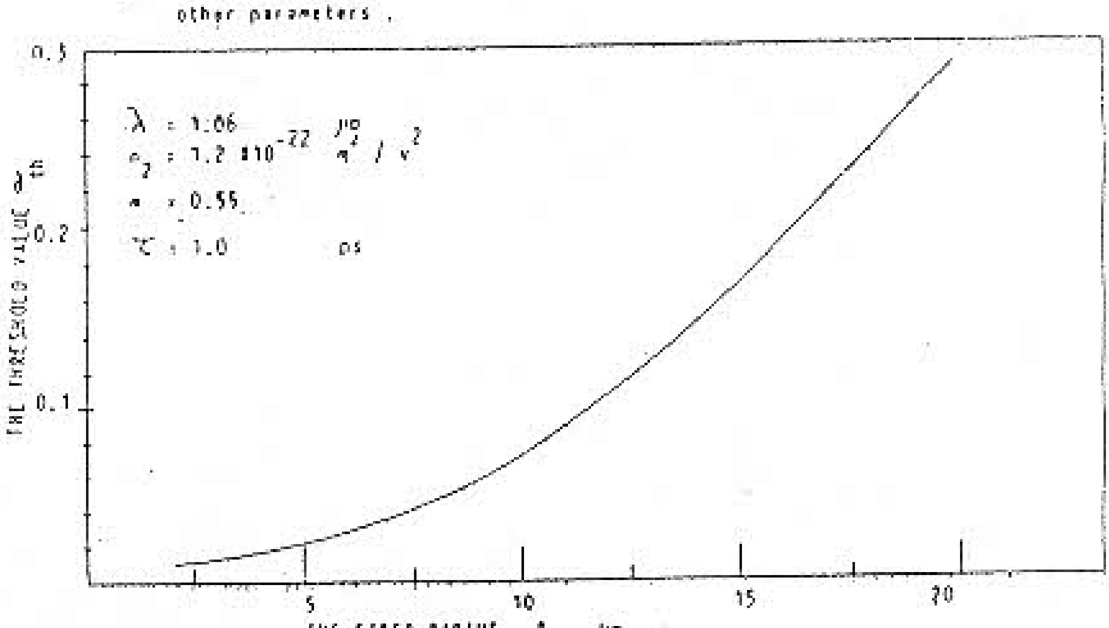


Fig.6. Variation of α_{th} with R at the assumed set of parameters .

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