

SOLITON TRANSMISSION IN INHOMOGENEOUS
MEDIA WITH W-TAILORED REFRACTIVE INDEX

Farag Z. El-Halafawy
Fac. of Electronic Eng.
Menouf, 23951, Egypt

El-Sayed A. El-Badawy, Mohammed A. El-Gammal,
and Mostafa H. Aly
Fac. of Eng.
Alexandria University
Alexandria, Egypt

In this paper, a method for soliton transmission in inhomogeneous media with W-tailored refractive index is modeled and parametrically analyzed. Two kinds of inhomogeneities are simultaneously considered: (a) Biquadratic variation of the refractive index (W-tailored radial profile), and (b) Boundary conditions of the cladded fiber.

The achievable bit rate of such important transmission was shown [1] to comfortably match with and could be made an order of magnitude better than that of the best linear transmission.

The refractive index, n , is W-tailored under the biquadratic nonlinear form,

$$n(\rho, \lambda, E) = n(\lambda) [1 - \alpha \rho^2 + m \alpha \rho^4] + n_2 |E|^2 \quad (1)$$

where $\rho = r/R$ and R, λ, E , and n_2 are respectively fiber radius, carrier wavelength, electric field, and nonlinear coefficient of the refractive index. The parameters α and m control both the values of minimum and maximum refractive index and the radial position of the first ρ_{\min} where:

$$n_{\max} = n(\lambda) (1 - \alpha + \alpha m), \quad n_{\min} = n(\lambda) (1 - \alpha/4m), \quad \text{and} \quad \rho_{\min}^2 = 1/2 m$$

The wave equation for the electric field, E , is given by [2]

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 \bar{D}_L}{\partial t^2} = \frac{2n_2 n_0}{c^2} \frac{\partial^2}{\partial t^2} (|E|^2 \bar{E}) \quad (2)$$

where the electric field in the fiber $E(r, z, t)$ is given by

$$E(r, z, t) = \text{Re} [\Phi(z, t) U(\rho) \exp [j(qz - \omega t)]] \quad (3)$$

where $U(\rho)$ is the solution of the steady state wave equation for the linear inhomogeneous medium,

$$[\nabla_{\perp}^2 - P^2 + k_0^2 (1 - 2\alpha\rho^2 + 2\alpha m\rho^4)] U(\rho) = 0 \quad (4)$$

where P is the eigenvalue of the propagation constant and

$$k = (w + w_0) n(w + w_0), w = 2\pi c/\lambda, \text{ and } k_0 = \lim_{w \rightarrow 0} k$$

the solution of Eq.(4) is assumed under the series form,

$$U(\rho) = \exp(-\rho^2/2) \sum_{i=0}^{\infty} a_{2i} \rho^{2i} \quad (5)$$

with $a_0 = 1$. It is found that a_{2i} 's ($i > 1$) are functions of a_2 , α , and m where $a_2 = 0.5 + (P^2 - k_0^2) R^2/4$

P_0^2 was calculated through the perturbation theory to the third order [3] under the form.

$$P_0^2 = k_0^2 - 2k_0 \sqrt{2\alpha} / R + 2m / R^2 + 4.5m^2 / R^3 k_0 \sqrt{2\alpha + 79} m^3 / 8\alpha R^2 k_0^2 \quad (7)$$

The function $\Phi(z,t) = \Phi(\eta)$ possesses a light solitary solution of the form

$$\Phi(\eta) = \Phi_0 \operatorname{sech}(\eta) \quad (8)$$

$$\text{where } \Phi_0^2 = \frac{Vg^{-2} - Q(k_0' + k_0 k_0'')}{(n_2/n_0) k_0^2 \tau^2 \delta}$$

with a propagation velocity Vg ,

$$Vg = P_0 / Q k_0 k_0' \quad (10)$$

where $\eta = (Vg\tau - z) / Vg\tau$, τ is the pulse duration, k_0' , k_0'' are the derivatives of k w.r.t. w , $Q = 1 - \alpha + \frac{1}{2}\alpha m$, and δ is a function of R , α , and m ,

$$\delta = \int_0^{\infty} U(\rho) d\rho^2 \quad (11)$$

It is not the same as reported by [2].

The peak power P_0 is given by [1]

$$P_0 = \frac{1}{2} Vg \epsilon_0 n_0^2 S \Phi_0^2$$

where ϵ_0 is the permittivity of free space and S is the cross sectional area of the fiber core.

A sample of the calculations based on this model will be discussed. The variations of the peak power P_0 and the group velocity Vg with the parameter α at different values of m and the assumed set of parameters are displayed respectively in Figs.1 and 2. From these figures and the other obtained data, it is found that both P_0 and Vg possess exponential correlations with the parameter α . The results clarified in Fig.3, where the radius R appears as a parameter, assert that, at any assumed set

of parameters, there is a threshold value α_{th} to achieve a stable soliton transmission where α_{th} increases as the fiber radius increases, Fig. 4.

In conclusion, to design a W-tailored refractive index fiber for stable soliton transmission of high bit rate, minimum power and high group velocity, the parameter α must be of minimum value while the parameter m must be of maximum value.

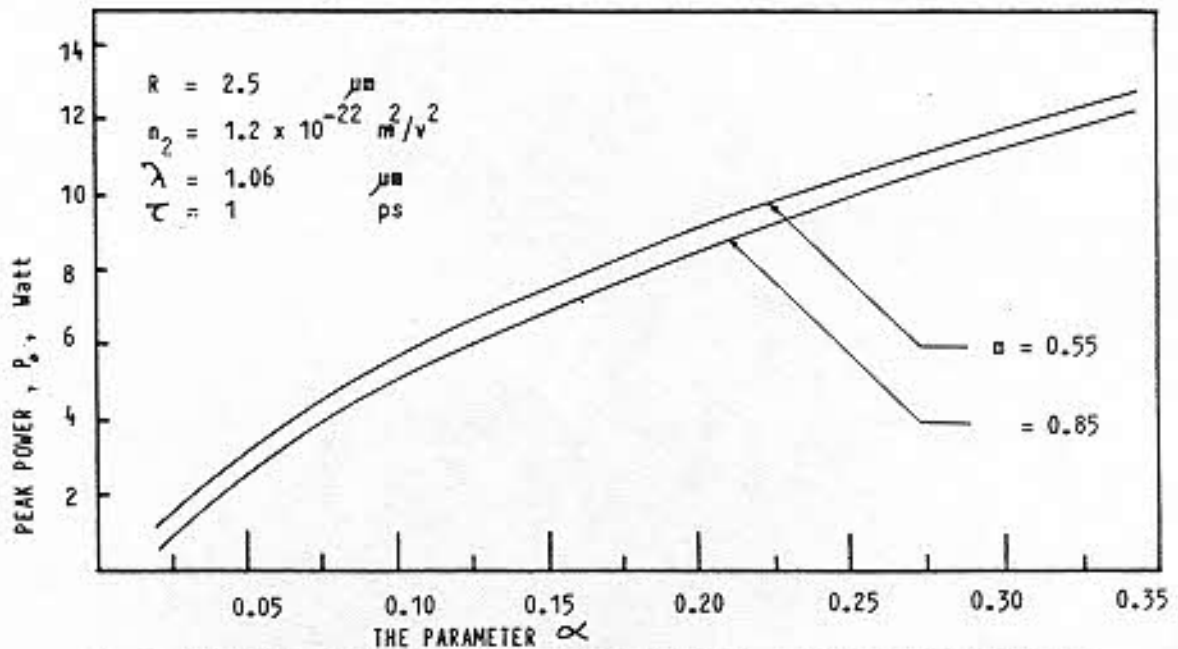


Fig.1. Variation of P_0 with α for different values of m and the assumed set of other parameters.

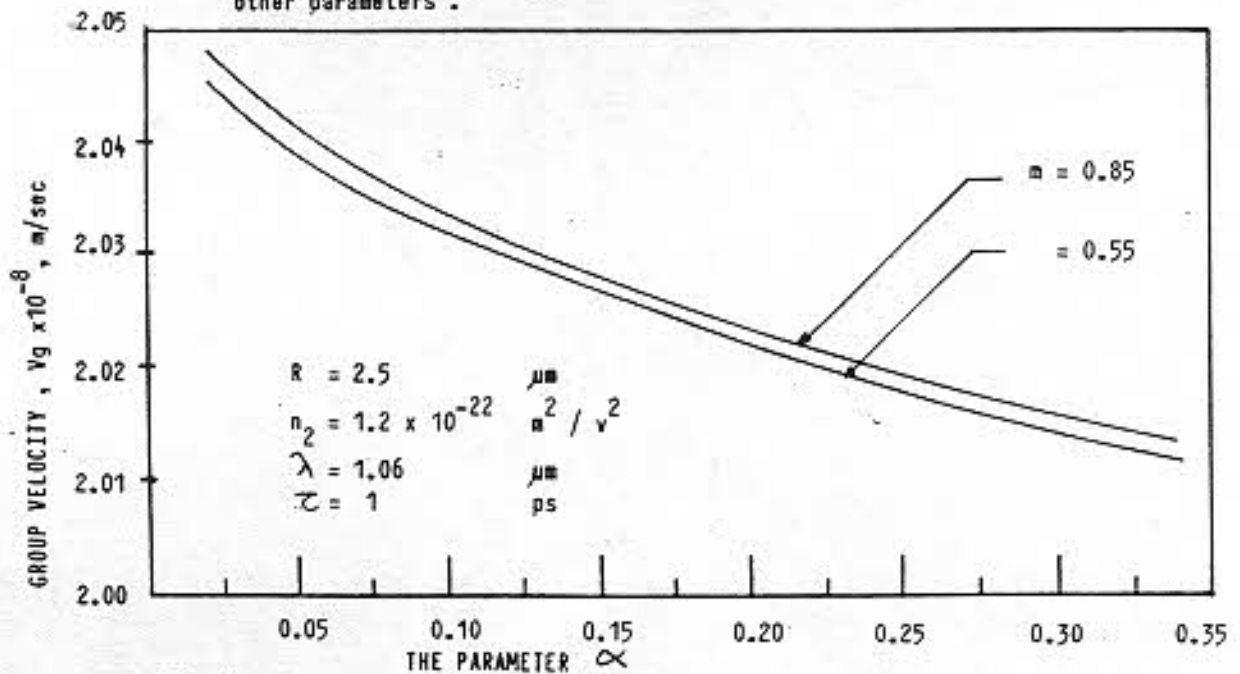


Fig.2. Variation of V_g with α for different values of m and the assumed set of other parameters.

- [1] A. Hasegawa and Y. Kodama, Proc. IEEE, Vol. 69, No 9, PP. 1145 - 1150, Sep. 1981.
 [2] M. Jain and N. Tzoar, J. Appl. Phys., Vol. 49, No. 9, PP. 4649 - 4654, Sep. 1978.
 [3] G. Jacobsen and J.J.R. Hansen, Appl. Opt., Vol. 18, No. 16, PP. 2837 - 2842, 15 Aug. 1979.

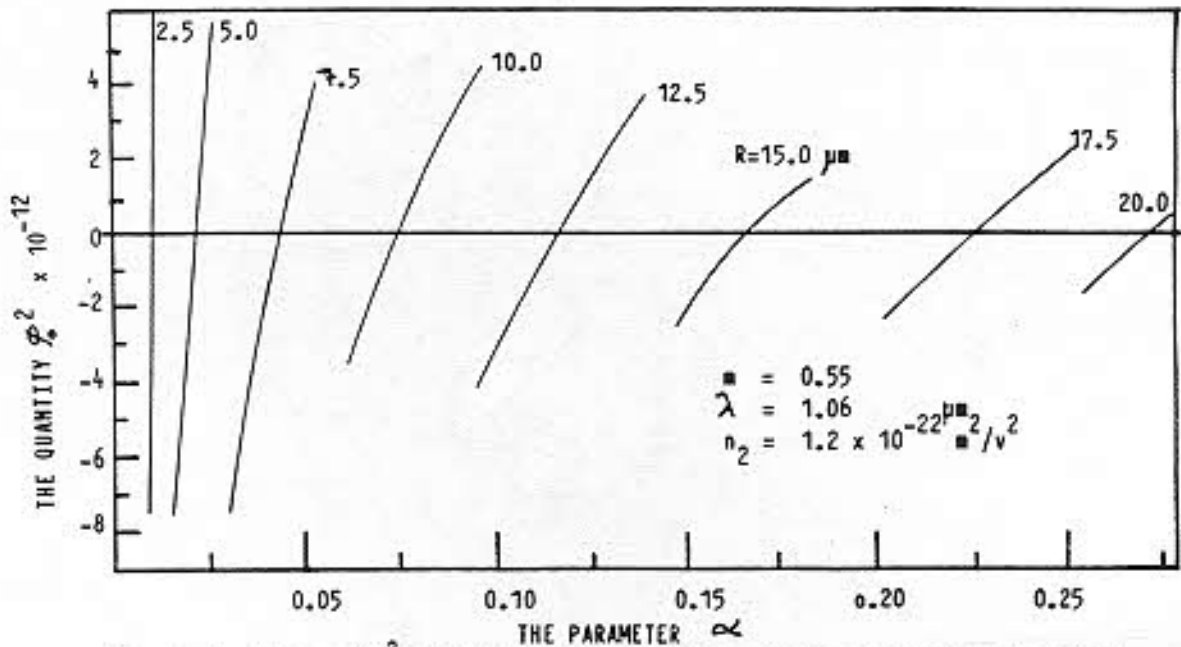


Fig.3. Variation of ϕ_0^2 with α for different values of R and the assumed set of other parameters.

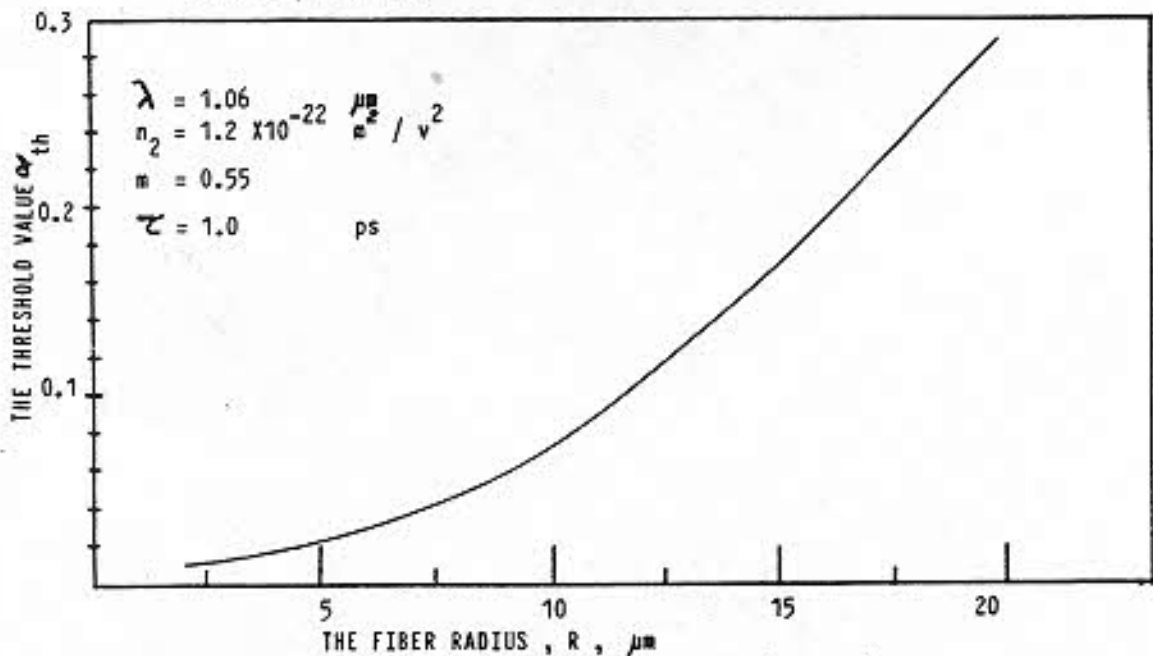


Fig.4. Variation of α_{th} with R at the assumed set of parameters.