



## ZERO DISPERSION IN RADIALLY INHOMOGENEOUS SINGLE-MODE $\text{SiO}_2\text{-GeO}_2$ OPTICAL FIBERS

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### ABSTRACT

The cancellation between material and waveguide dispersions for weakly guiding single-mode fibers has been analyzed to predict the optimum wavelength for zero chromatic (total) dispersion. The fibers under consideration are characterized by a radially inhomogeneous circular core with a polynomial refractive index profile. Zero dispersion wavelength of this type is studied for  $\text{SiO}_2\text{-GeO}_2$  glasses and is depicted around  $0.7 \mu\text{m}$  and  $0.9 \mu\text{m}$  for silica and germania glasses, respectively.

### I. INTRODUCTION

Single-mode fibers has been established as an excellent medium for long distance telecommunications, providing long repeater spacings and large bandwidths. The length of a repeaterless optical fiber link and the bit rate that can be transmitted over it are limited by two quantities. These are the power attenuation due to losses in the fiber and the pulse dispersion which occurs because of the finite spectral spread of the light source. For efficient communication, it is desirable that the minimum loss wavelength coincides with the wavelength of zero chromatic dispersion or that dispersion is sufficiently low over a large range of wavelengths [1, 2]. Therefore, the study of chromatic dispersion in single-mode fibers is of great importance for ensuring the optimum performance of data transfer rate of long-haul communication systems.

Chromatic dispersion can be derived through the wave equation by numerical methods [2]-[5]. In a recent work [6], we have studied the chromatic dispersion in single-mode fibers with core refractive index of a biquadratic variation with the radial distance.

In this paper, we present a simple analytical study to obtain the chromatic dispersion in single-mode fibers of a radially inhomogeneous circular core with a polynomial profile to the sixth order. The method used to obtain the waveguide dispersion is based on the Kaushal ansatz for the eigen function [7], while the other used to obtain the material dispersion is based on the three-term Sellmeier dispersion relation [8]. The  $\text{SiO}_2\text{-GeO}_2$  glass type is considered because it is currently of great interest. The interest in high germania side is highly increasing because previous calculations have shown that the  $\text{GeO}_2$  glass exhibits a lower optical loss window than that of high  $\text{SiO}_2$  containing glasses as a result of reduced Rayleigh scattering at longer wavelengths [8].

## II. MATHEMATICAL MODEL

### II.1. Propagation Constant

It is well known that  $\phi(r)$ , the part of the electric field depending on the radial distance  $r$ , in an optical fiber, under the weak guidance conditions, satisfies the scalar wave equation [9] :

$$\frac{d^2\phi(r)}{dr^2} + \frac{1}{r} \frac{d\phi(r)}{dr} + \left( n^2(r) k^2 - \beta^2 - \frac{m^2}{r^2} \right) \phi(r) = 0 \quad , \quad (1)$$

where  $k (= 2\pi/\lambda)$  is the free space wave number,  $\lambda$  is the operating wavelength,  $\beta$  is the propagation constant,  $m$  is the azimuthal mode number ( $= 0, 1, 2, 3, \dots$ ) and  $n(r)$  is the refractive index as a function  $r$ .

The fiber under consideration is characterized by a core refractive index of a polynomial profile of the form [7] :

$$n^2(r) = n_0^2 \{ 1 - f(r) \} \quad , \quad r \leq r_0 \quad (2-a)$$

with

$$f(r) = \frac{1}{n_0^2 k^2} \left( a_1 (r/r_0)^2 - b_1 (r/r_0)^4 + c_1 (r/r_0)^6 \right) \quad , \quad (2-b)$$

where  $n_0$  is the axial value of the core refractive index,  $r_0$  is the core radius,  $a_1$ ,  $b_1$  and  $c_1$  are positive constants. This profile is consistent with the experimental work of Rawson et al. [10] and looks like a double clad W-fiber as indicated in the graphs given below, Fig. 1 through Fig. 4.

Solving Eq. (1), Kaushal has obtained the propagation constant,  $\beta$ , in the form [7] :

$$\beta^2 = n_0^2 k^2 + (1 \pm m) b / \sqrt{c} \quad , \quad (3)$$

where

$$b = b_1 r_0^2 \quad \text{and} \quad c = c_1 r_0^2 \quad .$$

Kaushal also showed that the constant  $b$  is given by:

$$b = 8 c^{3/2} (m + 2) + 4 a c \quad , \quad (4)$$

where

$$a = a_1 r_0^2 \quad .$$

Here, the constants  $a$ ,  $b$  and  $c$  are dimensionless.

### II.2 Chromatic Dispersion

Based on the additive effect of the material dispersion,  $D_m$ , and the waveguide dispersion,  $D_w$ , the chromatic dispersion,  $D_c$ , is obtained [9]. Detailed analysis for both dispersion types are given in the following subsections.

### II.2.1. Material Dispersion

The material dispersion is included in the wavelength dependence of the axial value of the refractive index,  $n_0$ , of the fiber material having the form [9] :

$$D_m = \frac{\lambda}{2 \pi c} \frac{d^2 n_0}{d\lambda^2} \quad (5)$$

where  $c$  is the free space speed of light.

The value of  $n_0$  is represented by the three-term Sellmeier equation [8] :

$$n_0^2 = 1 + \sum_{i=1}^3 \frac{A_i \lambda_i}{\lambda^2 - \ell_i^2} \quad (6)$$

where  $A_i$  are coefficients related to the material oscillator strength and  $\ell_i$  are the corresponding oscillator wavelengths. The values of  $A_i$  and  $\ell_i$ , in terms of the germania percentage,  $x$ , in the wavelength range  $0.365 \leq \lambda$  ( $\mu\text{m}$ )  $\leq 4.28$ , are given by [8] :

$$A_1 = 0.6961663 + (0.11070342) x \quad (7-a)$$

$$A_2 = 0.4079426 + (0.31021588) x \quad (7-b)$$

$$A_3 = 0.8974794 - (0.04331109) x \quad (7-c)$$

$$\ell_1 = 0.0684043 + (0.00056) x \quad (7-d)$$

$$\ell_2 = 0.1162414 + (0.03772565) x \quad (7-e)$$

$$\ell_3 = 9.8961610 + (1.94577) x \quad (7-f)$$

### II.2.2. Waveguide Dispersion

Assuming only the fundamental mode propagation, the waveguide dispersion can be obtained from the relation [2, 4, 5, 11] :

$$D_w = \frac{-1}{2 \pi c} \left( 2 \lambda \frac{d\beta}{d\lambda} + \lambda^2 \frac{d^2\beta}{d\lambda^2} \right) \quad (8)$$

## III. RESULTS AND DISCUSSION

The values of the constants  $a$  and  $c$  together with the core radius,  $r_0$ , and the wavelength,  $\lambda$ , are chosen to give a positive value for  $n^2(r)$ . For single-mode operation, the value of  $r_0$  is chosen between  $3 \mu\text{m}$  and  $5 \mu\text{m}$  and the constants  $a$  and  $c$  were obtained to have the range of values:

$$1500 \leq a \leq 2000$$

and

$$2000 \leq c \leq 4000$$

It was found that increasing the value of  $a$ ,  $c$  or both over this range (keeping the fiber radius in the range of single-mode operation) will decrease the value of the core refractive index below the well known values for glass, Fig. 1. The same result was obtained at higher values of  $\lambda$ , Fig. 2.

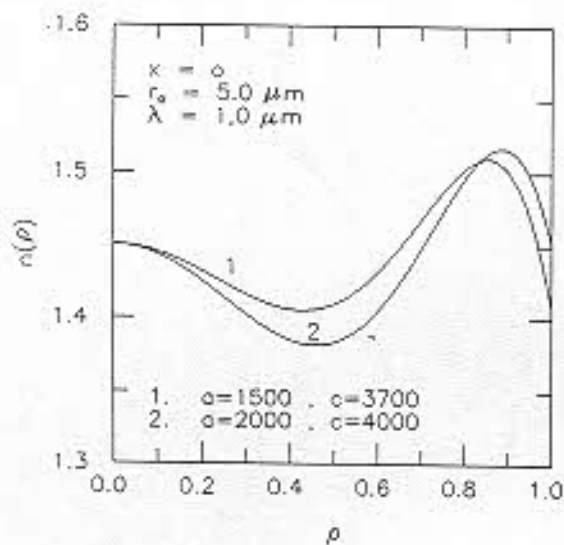


Fig. 1. Core refractive index, where  $\rho = r / r_0$ .  
 {Effect of the constants a and c}

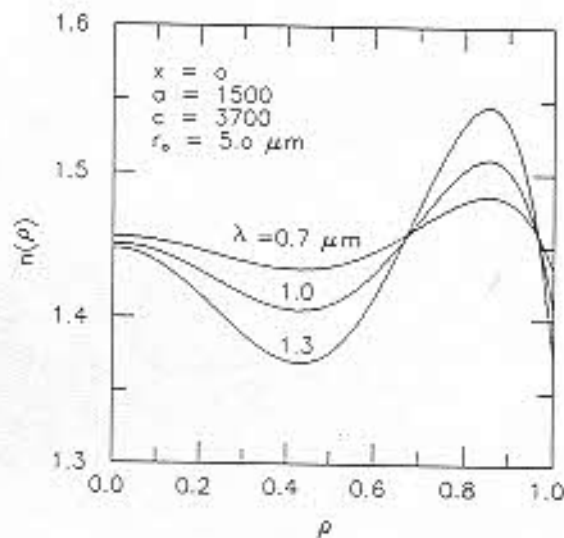


Fig. 2. Core refractive index, where  $\rho = r / r_0$ .  
 {Effect of the wavelength  $\lambda$ }

Figure 3 shows that a core radius of 5  $\mu\text{m}$  gives a better profile for the core refractive index than thinner ones, giving less values for the refractive index near the core-cladding interface. This is the reason for the more emphasis given to this radius in our study.

The  $\text{SiO}_2\text{-GeO}_2$  glass was studied through different values of the germania percentage,  $x$ , Fig. 4. It is observed that the increase in the value of  $x$  yields a shifted profile as expected because the refractive index of the germania glass is greater than that of the silica one.

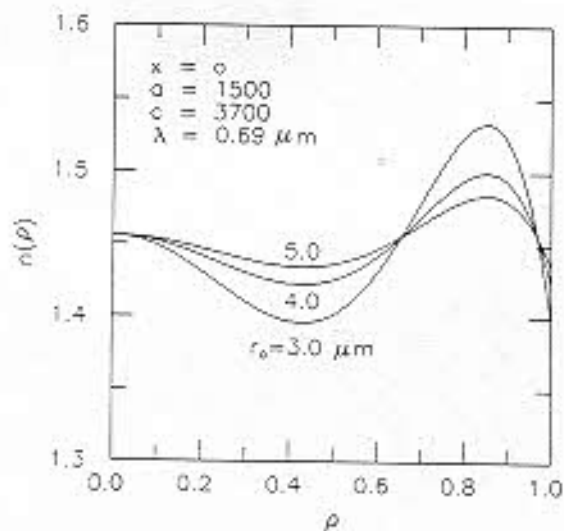


Fig. 3. Core refractive index, where  $\rho = r / r_0$ .  
 {Effect of the core radius,  $r_0$ }

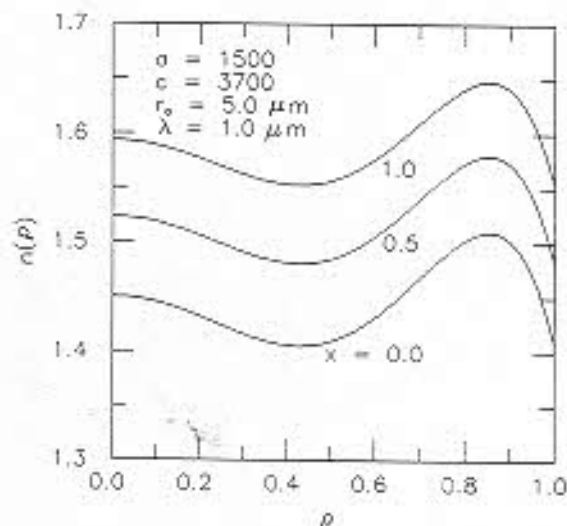


Fig. 4. Core refractive index, where  $\rho = r / r_0$ .  
 {Effect of the germania percentage,  $x$ }

The material dispersion was directly obtained through Eqs. (5-7) and the waveguide dispersion was obtained through Eqs. (3) and (8). The negative sign in Eq. (3) is chosen because the positive sign results in a propagation constant,  $\beta$ , greater than  $n_0k$ , which is forbidden.

Adding both dispersion types, the chromatic dispersion was then obtained. Figures 5 and 6 represent the obtained results for both  $\text{SiO}_2$  ( $x=0$ ) and  $\text{GeO}_2$  ( $x=1$ ) glasses, respectively. It is observed that, for  $\text{SiO}_2$  glass, the material dispersion tends to zero at a wavelength,  $\lambda_0 = 1.27 \mu\text{m}$ , in consistence with the value found in Ref. [9], while the corresponding value in the  $\text{GeO}_2$  glass is observed at  $1.74 \mu\text{m}$ . From both figures, it is noted that waveguide dispersion, when considered, results in a decrease in the value of  $\lambda_0$ .

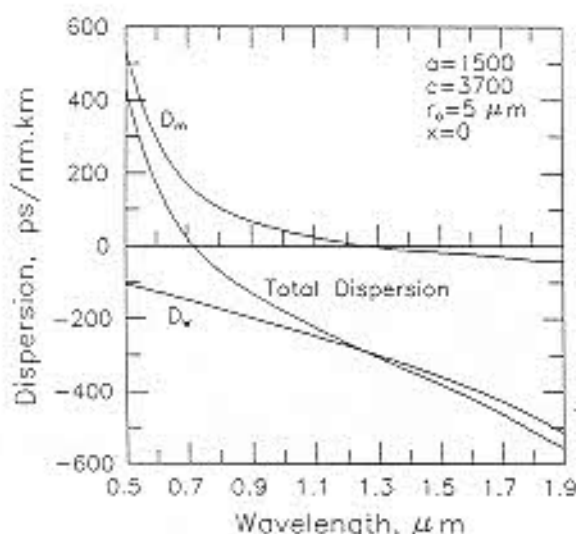


Fig. 5. Chromatic (Total) Dispersion for Silica Fibers.  
 $D_m$  is the material dispersion.  
 $D_w$  is the waveguide dispersion.

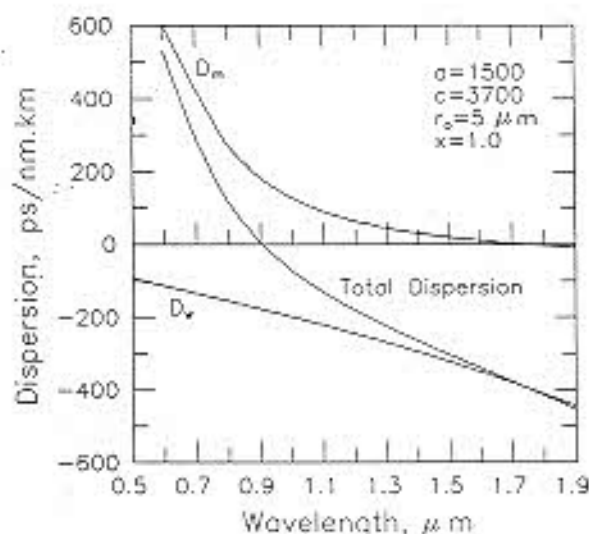


Fig. 6. Chromatic (Total) Dispersion for Germania Fibers.  
 $D_m$  is the material dispersion.  
 $D_w$  is the waveguide dispersion.

The zero chromatic dispersion wavelength,  $\lambda_0$ , is displayed against the germania percentage,  $x$ , in Fig. 7. The upper curve is drawn when only the material dispersion,  $D_m$ , is considered while the other curves are drawn when the waveguide dispersion is added to the material dispersion. The figure shows that the value of  $\lambda_0$  increases linearly with  $x$  at constant radius. At a core radius of  $5 \mu\text{m}$ , as an example, this variation is:

$$\lambda_0 (\mu\text{m}) = 0.1836 x + 0.7221 \quad (9)$$

It is clear, from Fig. 7, that increasing the core radius,  $r_0$ , will shift the  $\lambda_0$  line up, i.e., the effect of the waveguide dispersion is less. This continues until the line of the material dispersion is reached for higher values of the core radius corresponding to multimode

operation, at which the waveguide dispersion has a negligible effect in agreement with that obtained in Ref. [9]. Figure 7 shows also that the increase in the value of the constants  $a$  and  $c$  yields a very small effect on the value of  $\lambda_0$ .

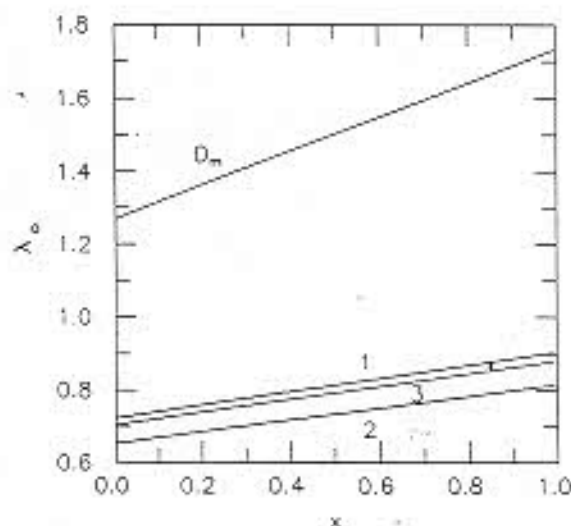


Fig. 7. Zero Dispersion wavelength,  $\lambda_0$ , against the germania percentage,  $x$ .

$D_m$  denotes the variation of  $\lambda_0$  when only the material dispersion is considered.

Curves 1, 2 and 3 give the variation of  $\lambda_0$  for total dispersion.

In curve 1,  $r_0 = 5 \mu\text{m}$ ,  $a = 1500$ ,  $c = 3700$

In curve 2,  $r_0 = 4 \mu\text{m}$ ,  $a = 1500$ ,  $c = 3700$

In curve 3,  $r_0 = 5 \mu\text{m}$ ,  $a = 2000$ ,  $c = 4000$

## V. CONCLUSION

This work represents the calculation of the chromatic dispersion,  $D_c$ , for inhomogeneous fibers of  $\text{SiO}_2\text{-GeO}_2$  glasses. It is found that the value of the zero dispersion wavelength,  $\lambda_0$ , increases with the germania ratio,  $x$ . According to the obtained results and for the considered fiber type, one can conclude that better operation is achieved around  $0.7 \mu\text{m}$  for silica and around  $0.9 \mu\text{m}$  for germania glasses.

## REFERENCES

- [1] A.V. Belov, A.S. Kurov, V.A. Semenov and A.V. Chicolini, "The Measurement of Chromatic Dispersion in Single-Mode Fibers by Interferometric Loop", J. Lightwave Technol., LT-7, 863-867, 1989.

- [2] A. Sharma and S. Banerjee, "Chromatic Dispersion in Single-Mode Fibers with Arbitrary Index Profiles: A Simple Method for Exact Numerical Evaluation", *J. Lightwave Technol.*, LT-3, 1919-1923, 1989.
- [3] C.T. Chang, "Minimum Dispersion in a Single-Mode Step-Index Optical Fiber", *Appl. Opt.*, 18, 2516-2522, 1979.
- [4] A. Safaai-Hazi and L.J. Lu, "Evaluation of Chromatic Dispersion in W-Type Fibers", *Opt. Lett.*, 14, 760-762, 1989.
- [5] R. Alonso and J. Pelayo, "Analysis of Chromatic Dispersion Properties of Triple-Clad Fibers by Coupling-Mode Theory", *Opt. Lett.*, 15, 1135-1137, 1990.
- [6] Moustafa H. Aly, "Chromatic Dispersion in Graded-Index Single-Mode Optical Fibers", *Proc. IEEE Pacific Rim Conf. on Communications, Computers and Signal Processing*, Toronto, Canada, 823-826, May 1991.
- [7] R.S. Kaushal, "Solving the Problem of a Radially Inhomogeneous Circular Waveguide with a Polynomial Profile", *J. Opt. Soc. Am. A*, 8, 1245-1247, 1991.
- [8] James W. Fleming, "Dispersion in  $\text{GeO}_2\text{-SiO}_2$  Glasses", *Applied Optics*, 23, 4486-4493, 1984.
- [9] J. Senior, *Optical Fiber Communications: Principles and Practice*, Printice Hall, New York, second edition, 1992.
- [10] Eric G. Rawson and Richard G. Murray, "Interferometric Measurement of SELFOC Dielectric Constant Coefficients to Sixth Order", *IEEE J. Quantum Electron.*, QE-9, 1114-1119, 1973.
- [11] C.D. Poole, J.M. Wiesenfeld, A.R. McCormic and K.T. Nelson, "Broadband Dispersion Compensation by Using the Higher-Order Spatial Mode in a Two-Mode Fiber", *Opt. Lett.*, 17, 985-987, 1992.