Probability of Error Performance of Free Space Optical Systems in Severe Atomspheric Turbulence Channels

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Abstract—**Atmospheric tu rbulence cau ses d egradation in th e performance of the f ree space op tical (FSO) transmission. T his turbulence is referred to as scintillation. To mitigate this effect, a multiple input multiple output (MIMO) system is employed. This paper investigates the use of multiple lasers and multiple aperture receivers in s evere atmos pheric tu rbulence when b inary p ulse position modulation (BPP M)) is employed. F irst, single input multiple output(SIMO) syst em using BPP M tec hnique is investigated with equal gain combining (EGC), s election combining (SC), and maximal ratio combining (MRC) diversity schemes. A clo sed form for t he p robability of erro r h as b een derived for both SC and MRC techniques, as well as Monte Carlo simulations. Th en, a MI MO s ystem fo r b oth z ero forcin g (ZF) equalizer and minimum mean square error (MMSE) equalizer is used. Finally, a comparison between different diversity techniques and linear equalizers is carried out.**

Index Terms —**Pulse position modulat ion, M IMO systems, maximal ratio combining, s election combining, equal g ain combining, zero for cing, minimum mean square error equalizer, and probability of error.**

I. INTRODUCTION

Free space optical (FSO) communication has been spread in the last years. This is because of the very high data rates it can provide, which is on the order of gigabits per second [1]. FSO laser beams cannot be detected with spectrum analyzers or radio frequency (RF) meters. Since the laser beams generated by FSO systems are narrow and invisible, this makes them harder to find and even harder to intercept and crack. Also, it is immune to noise, interference, and jamming from other sources. FSO requires no radio frequency spectrum licensing that is translated into unlimited bandwidth, ease, speed and low deployment cost. FSO transmits invisible, eye-safe light beams from one "telescope" to another using low power. Each telescope consists of an optical transceiver with a transmitter and a receiver to provide full-duplex capability. Each optical wireless unit uses an optical source plus a lens or telescope that transmits light through the air to another lens receiving the information. So, it is very easy to reposition the system and change its place [2]. FSO systems can function over distances of several kilometers. Though, it requires a clear line of sight between the source and the destination and enough transmitter power.

As the laser beam is so narrow, it requies accurate pointing. It also needs a tracking mechanism to overcome the buildings sway. Also, the atmosphere consists of very small particles and molecules whose sizes are comparable to the carrier wavelength, which in turn results in various effects that the beam is subjected to. Typically, these effects are not known in the radio frequency (RF) systems. One of such effects is the scintillation process, which causes random fluctuations in the received irradiance of the optical beam. This effect is typically equivalent to fading in RF systems.

Scintillation takes place as a result of heating of the earth's surfaces, which results in the rise of thermal air masses. These masses are then combined together forming regions with different densities and sizes, which cause a difference in the refractive index that varies with time [1]. Moreover, these regions cause fluctuations in the irradiance of the received laser beam. Many studies have been carried out to analyze the scintillation effect and to describe its model. The distribution of random fluctuations depends on the optical turbulence strength. In case of weak and moderate turbulence, it is lognormally distributed [1]. As the atmospheric turbulence increases, the lognormal model begins to deviate. Rayleigh distribution is considered the best scenario for MIMO in case of strong turbulence [3,4].

This paper mainly focuses on the mitigation of the scintillation effect (which will be refered to as fading) through the use of multiple lasers at the transmitter and multiple apertures at the receiver. This paper is an extension of the work presented in [1] and [2].

References [1] and [2] studied the FSO MIMO channel assuming *Q-ary* a PPM scheme. They also assumed non-ideal photodetection. In this work, first, BPPM is employed assuming non-ideal photodetection with EGC, SC, and MRC diversity schemes employed at the receiver. Then, both ZF, and MMSE equalizers will be employed.Finally, a comparison will be carried out between our work and that of [1] and [2], which only used EGC diversity scheme at the receiver.

This paper is organized as follows. Section II describes the system model. In Section III, the average probability of error performance is presented for SIMO system in case of *B*PPM with EGC, SC, and MRC. Furthermore, Section IV presents the average probability of error performance for MIMO system in case of *B*PPM using ZF equalizer. Then , section V explains the use of MMSE. This is followed by conclusion in Section VI.

II. SYSTEM MODEL AND ASSUMPTIONS

In this paper, we assume an FSO MIMO system comprising M lasers at the transmitter and N aperture receivers, as shown in Fig. 1.
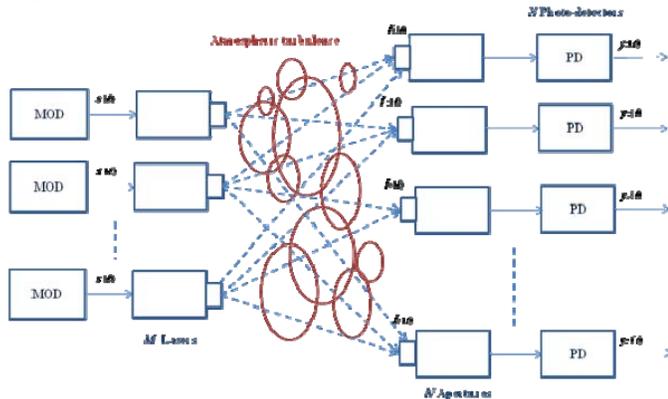

Fig.1 FSO block diagram [1].

It is worth noting that the total transmitted optical power is constant regardless of the number of lasres. The irradiances of the laser beams are added constructively at each receiving aperture (as the irradiance is optical power) [1]

$$I_n(t) = As(t) \sum_{m=1}^{M} h_{m,n}(t)$$ (1)

where A denotes the received irradiance in the absence of scintillation, $s(t)$ denotes the transmitted irradiance and $h_{m,n}(t) \geq 0$ is the irradiance fading coefficient due to scintillation between the m'th laser and n'th aperture. The separation distance between lasers is assumed to be large enough to assure that the fading paths $h_{m,n}(t)$ for $m =1,…, M$ and $n = 1,…., N$ are independent and identically distributed (i.i.d.). A non-ideal photodetection (PD) is employed, such that shot noise and thermal noise processes are well approximated by a Gaussian distribution [1]. As non-ideal PD is assumed, then the PD will have a current that is directly proportional to the received irradiance [4,5]

$$y_n(t) = \rho I_n(t) + w_n(t)$$ (2)

where ρ denotes the responsivity of the photodetector and $w_n(t)$ denotes the zero mean signal independent additive white Gaussian noise (AWGN) process with two-sided power spectral density.

III. AVERAGE PROBABILITY OF ERROR PERFORMANCE FOR BPPM

We assume one laser at the transmitter and N apertures at the receiver. In the following, BPPM is employed and the symbol time is assumed T_s. For Q symbols, T_s is divided into Q equal time slots of width T_p, (i.e. $T_p =T_s/Q$). The l'th symbol $X[l] = q$, where $q \in \{1,……, Q\}$ is being sent as a rectangular pulse. Then, the transmitted signal for viewing L frames and the presence of generally M transmitting lasers will be [1]

$$s(t) = \frac{1}{M} \sum_{l=0}^{L-1} rect\,(t - lT_s - (X[l]-1)T_p)$$ (3)

In general, if M lasers are used at the transmitter then, the transmitted power is constant and it does not depend on the number of transmitting lasers. Also, the M lasers are assumed to be separated with a large distance, which is sufficient to consider the fading paths to be totally independent from each other. In this section, we assume an SIMO system. Also, the fading coefficient $h_{n,m}(t)$ is Rayleigh distributed, corresponding to assumed severe atmospheric turbulence.

Non-ideal PD is being assumed, then by recalling Eq. (2), and letting $E_s = (\rho A)^2 T_p$, which denotes the received symbol energy with the use of matched filter at each detector, and with an integrate-and-dump (ID) filter, then, the output of the ID for the n'th aperture for the l'th symbol period for the transmitted symbol $X[l] = j$ will be [1]

$$z_{n,q}[l] = \begin{cases} \sqrt{E_s}\left(\frac{1}{M}\sum_{m=1}^{M} h_{m,n}[l]\right) + w_{n,q}[l], & q = j \\ w_{n,q}[l] & q \neq j \end{cases}$$ (4)

A. Equal Gain Combining

When EGC is employed, the output of the ID is averaged over the number of receiving apertures [5]. To analyze the average probability of error performance in case of BPPM in the presence of a severe turbulence, Monte Carlo simulations are carried out. Only theoretical probability of error equations for one and two receiving apertures are used [7,8,9]. The probability of error in case of one and two receiving apertures are, respectively, given by [7]

$$P_b = \frac{1}{2}\left[1 - \frac{1}{\sqrt{1+2/\gamma}}\right]$$ (5)

$$P_b = \frac{1}{2}\left[1 - \frac{\sqrt{\gamma/2(\gamma/2+2)}}{\gamma/2+1}\right]$$ (6)

Figure 2 displays the average probability of error performance of the BPPM against bit energy to noise ratio for different number of apertures at the receiver.
In Fig. 2, in case of $N = 1, 2$, the performance obtained theoretically and the corresponding performance obtained using Monte Carlo simulation are nearly identical.

B. Selection Combining

To analyze the probability of error performance in case of BPPM in the presence of severe turbulence using SC diversity technique, a closed-form expression for the probability of error is derived and then a Monte Carlo simulation is conducted.
Consider an N branch SC receiver. Since the probability of error is defined by

$$P_b = \int_0^\infty P_b(\gamma)p(\gamma)d\gamma$$ (7)

where $P_b = Q(\gamma)$ is the conditional bit error probability for the binary PPM modulation scheme, $Q(.)$ is the Gaussian Q-function, and γ is the SNR per channel, given by

$$\gamma_m = \{\gamma_1, \gamma_2,........,\gamma_N\}$$ (8)
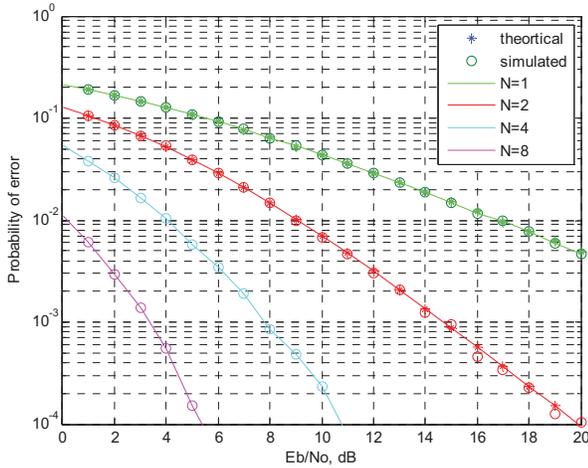
Fig.2 Probability of error at *M=1*,with increasing *N* in a Rayleigh fading channel using EGC diversity.

For (i.i.d.) random variables, the probability density function (pdf) of the maximum is

$$p_{max}(\gamma) = Np_x(\gamma)\left(\int_0^\gamma p_\gamma(x)dx\right)^{N-1} \quad (9)$$

where p_γ is the pdf of the SNR for all channels. For Rayleigh fading, γ is an exponential random variable, given by

$$p_\gamma(\gamma) = \frac{2}{\overline{\gamma}}e^{-\gamma/\overline{\gamma}}, \quad \gamma > 0 \quad (10)$$

where $\overline{\gamma}$ is the mean SNR per channel.

From Eq. (8), one can write

$$\int_0^{\gamma/2} p_\gamma(x)dx = \int_0^{\gamma/2} \frac{2}{\overline{\gamma}}e^{-2x/\overline{\gamma}}dx$$
$$= \int_0^{\gamma/\overline{\gamma}} e^{-y}dy = e^{-y}\Big|_\gamma^0 \quad (11)$$

Thus

$$p_{\gamma max}(\gamma) = \frac{2Ne^{-\gamma/\overline{\gamma}}}{\overline{\gamma}}\left(1-e^{-\gamma/\overline{\gamma}}\right)^{N-1} \quad (12)$$

Using the Binomial Theorem and the error function

$$(1-x)^n = 1-nx+\frac{n(n-1)x^2}{2!}-\frac{n(n-1)(n-2)x^3}{3!}+\cdots+\frac{n(n-1)...}{n!}x^n \quad (13)$$

$$\int_0^\infty erfc\left(\sqrt{x}\right)e^{-\alpha x}dx = \frac{1}{\alpha}\left[1-\frac{1}{\sqrt{1-\alpha}}\right] \quad (14)$$

and substituting Eqs. (9), (13) and (14) in Eq. (7), one gets

$$P_b^{SC} = \frac{1}{2}\left[1-\sum_{n=1}^N \binom{N}{n}(-1)^{n-1}\sqrt{\frac{\overline{\gamma}/2}{\overline{\gamma}/2+n}}\right] \quad (15)$$

Figure 3 shows the probability of error performance of the BPPM versus bit energy to noise ratio at different number of receiver apertures. It also gives a comparison between the

theoretical and the simulated performance. It is obvious that, the theoretical and the simulated curves are nearly the same. As the number of receiving aperture increases from N=2 to N=4, the performance of average probability of error is improved by nearly 6 dB. Also, the performance of the probability of error is decreased by nearly 3 dB by increasing the number of receiving apertures to 8.
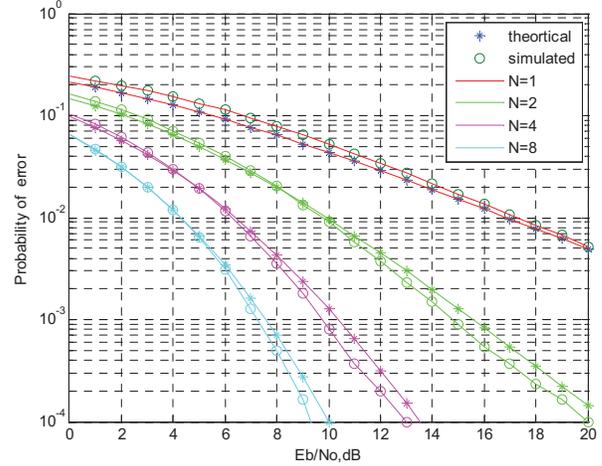

Fig.3 Probability of error at *M=1*,with increasing *N* in Rayleigh fading channel using SC diversity.

C. Maximal Ratio Combining

When MRC is employed, the output of the combiner is a weighted sum of all branches. Branches with high signal-to-noise-ratios are given weights higher than other branches [7]. To analyze the probability of error performance in case of BPPM in a Rayleigh fading channel using MRC diversity technique, a closed-form is derived for the probability of error then Monte Carlo simulations is carried out. A comparison between the theortical results and the simulated ones is done in Fig. 4. Let us assume N branch (finger) MRC receiver. For equally likely transmitted symbols, the output of the combiner is given by [10,11]

$$\gamma_t = \sum_{n=1}^N \gamma_n \quad (16)$$

where, γ_n is the SNR per symbol in each branch and γ_t is the total SNR per symbol.
For coherent binary signals, the probability of error, P_b, is given by [10]

$$P_b = (E|\{\gamma_n\}_{n=1}^N) = Q(\sqrt{2g\gamma_t}) \quad (17)$$

where $g = 1/2$ in case of BPPM and $Q(.)$ is the Gaussian *Q*-function.
By finding the pdf of γ_t, one can get

$$P_b(E) = \int_0^\infty Q(\sqrt{2g\gamma_t})p_{\gamma_t}(\gamma_t)d\gamma_t \quad (18)$$

By considering MRC a combination of N (i.i.d.) Rayleigh fading paths, the SNR per bit per path γ_n has an exponential pdf with an average SNR per bit $\bar{\gamma}$. Therefore:

$$p_{\gamma_n}(\gamma_n) = \frac{2}{\bar{\gamma}} e^{-\gamma_n/\bar{\gamma}} \tag{19}$$

Given that, the SNR per bit of the combined SNR γ_t has a chi-square pdf [10], then

$$p_{\gamma_t}(\gamma_t) = \frac{1}{(N-1)!\, \bar{\gamma}^{-N}} \gamma_t^{N-1}\, e^{-\gamma_n/\bar{\gamma}} \tag{20}$$

By applying successive integration by parts, one gets a closed-form for the average probability of error which is:

$$P_b(E) = \left(\frac{1-\mu}{2}\right)^N \sum_{n=0}^{N-1} \binom{N-1+n}{n}\left(\frac{1+\mu}{2}\right)^n \tag{21}$$

where $\mu = \sqrt{\dfrac{\bar{\gamma}/2}{1+\bar{\gamma}/2}}$

Monte Carlo simulations are conducted. Figure 4 shows a comparison between the simulated and the theoretical curves. It is clear that, both simulated and theoretical curves are nearly identical. As expected, the performance of the probability of error is improved by increasing the number of receiving apertures. It is improved by nearly 9 dB by increasing N=2 to N=4, and by 6 dB in by increasing N=4 to N=8.
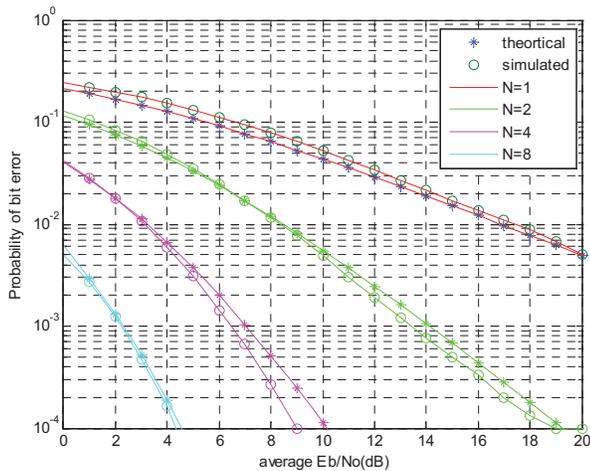

Fig.4 Probabilty of error at M=1,with increasing N in Rayleigh fading channel using MRC diversity technique.

IV. ZERO FORCING(ZF)

The ZF equalizer cancels all intersymbol interference (ISI), but can lead to a considerable noise enhancement. The received signal is given by[7]

$$Y=Hx+n \tag{22}$$

where H is the channel matrix, x is the transmitted signal, and the n is Gaussian noise at the receiving apertures.
To solve for x, it is neededed to find a matrix W which satisfies $WH=I$. The ZF linear detector for meeting this constraint is given by[7]

$$W = (H^H H)^{-1} H^H \tag{23}$$

Note that, the off diagonal terms in the matrix ($H^H H$) are not zero. Because the off diagonal terms are not zero, the zero forcing (ZF) equalizer tries to null out the interfering terms when performing the equalization. While doing so, there can be amplification of noise. Monte Carlo simulations are carried out in Fig.5 to display the average probability of error of MIMO system in strong atmospheric turbulence when employing ZF equalizer
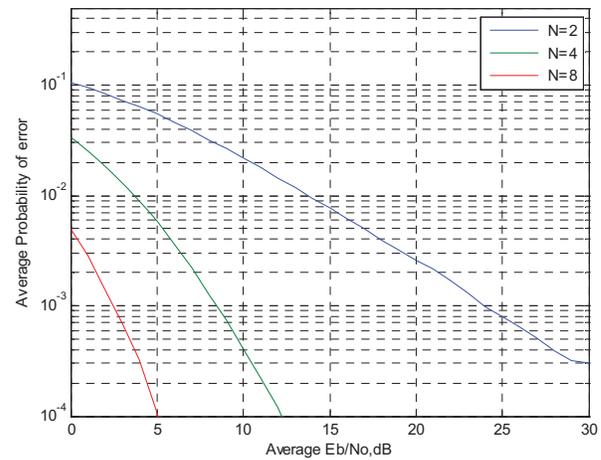

Fig. 5 Probability of error at M=2, with increasing N in Rayleigh fading channel using linear ZF equalizer

Figure 5 shows that the performance of the probability of error is improved by increasing the number of receiving apertures. By increasing the N=2 to N=4, the probability is decreased by nearly 20 dB. Also, in case of increasing the N=4 to N=8, the performance of the probability of error improves by 7 dB.

V. MINIMUM MEAN SQUARE ERROR (MMSE)

The MMSE technique minimizes the expected mean squared error between the transmitted symbol and the symbol detected at the equalizer output. Similarly, if the received signal is

$$Y=Hx+n \tag{24}$$

Then, the MMSE approach tries to find a coefficient W which minimizes the criterion [7,12].

$$E = \left\{[Wy-x][Wy-x]^H\right\} \tag{25}$$

such that W is given by [7]

$$W = \left[H^H H + NoI\right]^{-1} H^H \tag{26}$$

In fact, when the noise term is zero, the MMSE equalizer reduces to ZF equalizer.

Monte Carlo simulations are conducted in Fig.6 to show the probability of error performance of MIMO system in strong atmospheric turbulence using MMSE linear equalizer.
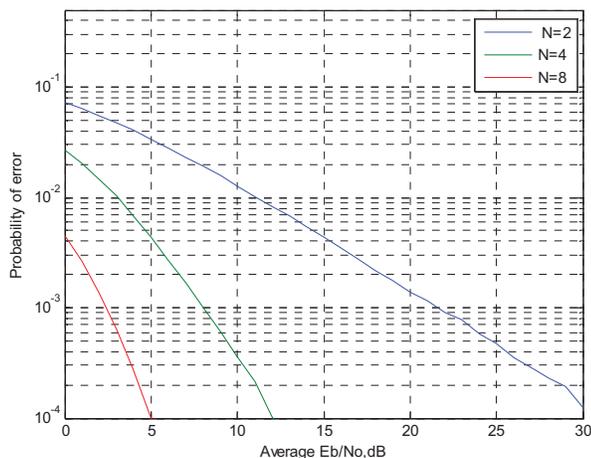

Fig. 6 Probability of error at M=2, with increasing N in Rayleigh fading channel using linear MMSE equalizer

Figure 6 proves that, the performance of the probability of error is improved by increasing the number of receiving apertures. By increasing the N=2 to N=4, the probability is decreased by nearly 18 dB. Also, in case of increasing the N=4 to N=8, the performance improves by 7 dB.

VI. CONCLUSION

In this paper, the probability of error performance analysis has been carried out in case of BPPM technique with the use of EGC, SC, and MRC. The MRC diversity technique is recommended to be used at the receiver, because it has a better performance than other diversity techniques as indicated from the obtained results (theoretically and by simulation). Then, the probability of error performance analysis took place using linear equalizers at the receiver. When comparing the performance in case of MRC to that of linear equalizers, it is clear that, MRC performance is much better. MRC diversity technique is recommended to achieve a better performance.

REFERENCES

[1] Nick Letzepis, Ian Holland and William Cowley, "The Gaussian free space optical MIMO channel with Q-ary pulse position modulation," *IEEE Ttrans. on Wireless Commun.*, vol. 7, no. 5, pp.1744-1753,May 2000.

[2] S. G. Wilson, M. Brandt-Pearce, Q. Cao and J. H. Leveque, "Free-space optical MIMO transmission with Q-ary PPM," *IEEE Trans. Commun.*, vol. 53, no. 8, pp. 1402–1412, Aug. 2005.

[3] Akhil R. Shah, Rick C. J. Hsu, Alireza Tarighat, Ali H. Sayed and Bahram Jalali, "Coherent optical MIMO (COMIMO)," *J. Lightwave Technol.*, vol. 23, no. 8, p. 2410, Aug. 2005.

[4] Neda Cvijetic, Stephen G. Wilson and Maïté Brandt-Pearce, "Receiver optimization in turbulent free-space optical MIMO channels with APDs and Q-ary PPM," *IEEE Photonics Technol Letters*, vol. 19, no. 2, pp. 103–105, Jan. 2007.

[5] J. W. Strohbehn, *Laser Beam Propagation in the Atmosphere*, Springer-Verlag, 1978.

[6] R. M. Gagliardi and S. Karp, and J. G. Proakis, *Optical Communications*, John Wiley & Sons, Inc., 1995.

[7] A. Goldsmith, *Wireless Communications*, Cambridge University press, USA, 2005.

[8] Bernard Sklar, *Digital Communications Fundamentals and Applications*, 2nd ed., Prentice Hall PTR, USA, 2001.

[9] Q. T. Zhang, "Probability of error for equal-gain combiners over Rayleigh channels," *IEEE Trans. on Communications*, vol. 45, no. 3, pp. 270-273, Mar 1997.

[10] Marvin K. Simon and Mohamed -Slim Alouini, *Digital Communication Over Fading Channels,* John Wiley & Sons, New York, 2000.

[11] J. G. Proakis, *Digital Communications*, 3rd ed, McGraw-Hill, New York, 1995.

[12] Colin Reinhardt, Yasuo Kuga, Sermsak Jaruwatanadilok and Akira Ishimaru, "Improving bit-error-rate performance of the free-space optical communications system with channel estimation based on radiative transfer theory," *IEEE J. Selected Areas in Communications*, vol. 27, no. 9, Dec. 2009