

Far Off Resonance AFBG for Dispersion Compensation in Transmission

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Abstract: The use of apodized fiber Bragg fiber gratings (AFBGs) with far off resonance for dispersion compensation, when operated in transmission, is investigated using the asymptotic form of the bandwidth as a measure for the performance of the dispersion compensator. An analytic expression for the quadratic and cubic dispersion of the fiber Bragg grating (FBG) is obtained at frequencies far from the Bragg condition, where the usual coupled mode theory (CMT) fails. This is used to investigate the effect of apodization on quadratic and cubic dispersion of the off resonance grating for eight different apodization profiles. It is also used to calculate the figure of merit (FOM), which emphasizes the same results obtained by the asymptotic form of the bandwidth method.

Key words: Apodized fiber Bragg grating, off resonance grating, quadratic and cubic dispersion, asymptotic form of the bandwidth, figure of merit.

INTRODUCTION

A fiber Bragg grating is well known to exhibit a strong reflection of an incident light pulse, provided that the pulse frequency matches the grating's Bragg frequency. Yet, even if the reflection is small due to a mismatch between these frequencies, the grating still affects the light propagation. Light can travel (in a fiber with a Bragg grating) at a group velocity much smaller than that in a uniform fiber (without a grating), even though the reflectivity is negligible^[1]. However, this effect is appreciable only over a small bandwidth, which, for typical fiber gratings, is less than a nanometer. Another effect at wavelengths away from the Bragg wavelength, and a potentially more relevant one, is that gratings lead to strong dispersion of the transmitted light with negligible reflection.

Gratings became one of the most important components for the design of optical communication systems. Since they are very attractive components for being passive, linear and compact, they are the key components for wavelength division multiplexing (WDM) applications and dispersion compensation in both reflection and transmission. The use of unchirped ramped gratings in transmission has been investigated by K. Hinton^[2]. For FBGs to operate in high performance optoelectronic applications, proper apodization is needed. Apodization helps in removing the resonant cavity effects of uniform Bragg gratings. Moreover, the transition between reflection and transmission is very abrupt^[3].

The use of apodized fiber Bragg gratings (AFBGs) in transmission is preferred sometimes over using them in reflection for several reasons: i) when operating in reflection, it requires the use of a circulator which produces losses into the system and increases the complexity and cost of the dispersion compensator, ii) the signal optical field must interact strongly with the grating, which means that any imperfection in fabrication will lead to a degradation of system performance because it will affect its property as a compensator. Oppositely, when operating in transmission the device can be spliced into the transmission link directly. Also, the interaction between the signal optical field and the grating is much weaker^[6].

A previous work on the far off resonance gratings has been investigated by J. E. Sipe *et al.*^[1]. They gave an approximate analytic description of the off resonance light propagation through a grating, leading to an expression for the wave number of light as a function of frequency. Once the wave number of light in the grating is known, it is, in principle applicable to extract the quadratic and higher order dispersion by differentiation with respect to frequency. The obtained expression was evaluated using apodization sin profile. They showed that the quadratic dispersion decreases far away from the Bragg resonance. Their expression was verified by calculating the quadratic dispersion using the same apodization profile but with an exact calculation resulting from the standard CMT, where both results were indistinguishable. The CMT results were quite good close to the Bragg resonance, but

failed far from the Bragg frequency at low frequencies compared to the Bragg resonance frequency.

The applications of this method include dispersion compensation and fiber lasers in which the quadratic dispersion is crucial in determining the properties of the emitted pulses. This is used to investigate the effect of apodization on quadratic and cubic dispersion of the off resonance grating and to calculate a variant ratio called “figure of merit (FOM)” which emphasizes the same results of the asymptotic form of the bandwidth method.

Theory: We consider a one dimensional model for the propagation of light in a fiber, where, at a given frequency ω , the electric field $E(z)$ can be taken to satisfy the differential equation[1]

$$\left[\frac{d^2}{dz^2} + \frac{\omega^2}{c^2} n^2(z) \right] E(z) = 0 \tag{1}$$

All quantities, such as the effective refractive index of the mode $n(z)$, can also depend on frequency. We now extract a constant reference refractive index \bar{n} from $n(z)$ by writing (1) as

$$\left[\frac{d^2}{dz^2} + \bar{k}^2 \right] E(z) = -\frac{\omega^2}{c^2} \left[n^2(z) - \bar{n}^2 \right] E(z) \tag{2}$$

where $\bar{k} \equiv \omega \bar{n} / c$ [1].

A formal solution of (2), using Green’s function, can be written as [1].

$$E(z) = E_+(z)e^{i\bar{k}z} + E_-(z)e^{-i\bar{k}z} \tag{4}$$

where

$$E_+(z) = \varepsilon_+ + \frac{i\omega^2}{2c^2\bar{k}} \int_{-\infty}^z e^{-i\bar{k}z'} \left[n^2(z') - \bar{n}^2 \right] E(z') dz' \tag{5-a}$$

$$\left[n^2(z') - \bar{n}^2 \right] E(z') dz'$$

and

$$E_-(z) = \varepsilon_- + \frac{i\omega^2}{2c^2\bar{k}} \int_z^{\infty} e^{i\bar{k}z'} \left[n^2(z') - \bar{n}^2 \right] E(z') dz' \tag{5-b}$$

$$\left[n^2(z') - \bar{n}^2 \right] E(z') dz'$$

where the ε_{\pm} are arbitrary constants. Differentiating

(5-a ,b) once, with respect to z , gives the differential equations

$$\frac{dE_{\pm}}{dz} = \pm \frac{i\bar{k}}{2} \left[\frac{n^2(z)}{\bar{n}^2} - 1 \right] \left[E_{\pm}(z) + E_{\mp}(z) e^{\mp 2i\bar{k}z} \right] \tag{6}$$

where (4) is used for $E(z)$.

Thus, (6) together with (4), formally solves (2).

In the following subsections, (6) is solved analytically, using an approximate scheme that makes use of the fact that $n^2(z)/\bar{n}^2$ is close to unity, that the refractive index distribution is approximately periodic, and that the field spectrum is well away from any Bragg resonance^[1]. The solution is predominantly forward propagating because the grating reflection is small. The main result is that this dominant field contribution approximately propagates as a plane wave, but with a wave number that differs from that in bare fiber, due to the presence of the grating.

Using the multiple-scales analysis, we assume that $n^2(z)/\bar{n}^2 - 1 = \eta$ is a small quantity of order $\ll 1$. We do a multiple scales analysis, in this parameter introducing $z_n = \eta^n z$ in the usual way. Further, we assume that η has a periodicity at the z_0 level with a period $d = \pi/k$, and that the amplitude of the oscillation varies, at most, at the z_l level. That is

$$\frac{n^2(z)}{\bar{n}^2} = 1 + 2\eta \sum_{all\ m} F_m(z_l) e^{2imKz_0} \tag{7}$$

where the all m designation indicates all positive and negative integers, and zero. If the F_m were constants, then $n^2(z)$ would be a purely periodic function and the grating would, thus, be uniform. By making the F_m depend on the slow parameter z_l , $n^2(z)$ is almost periodic, with slowly varying, but otherwise arbitrary, phase and amplitude modulation; this can be used, for example, to describe chirped or apodized gratings.

The prefactor 2 in (7) is introduced for later convenience and all of the $F_m(z_l)$ are assumed to be of order unity or less. We take η to be real, so that $F_{-m}(z_l) = F_m^*(z_l)$, and one can, thus, write

$$F_{\pm m}(z_l) = A_m(z_l) e^{\pm i\Phi_m(z_l)} \tag{8}$$

for nonnegative m , where $A_m(z_l)$ and $\Phi_m(z_l)$ are real, and $\Phi_0(z_l) = 0$.

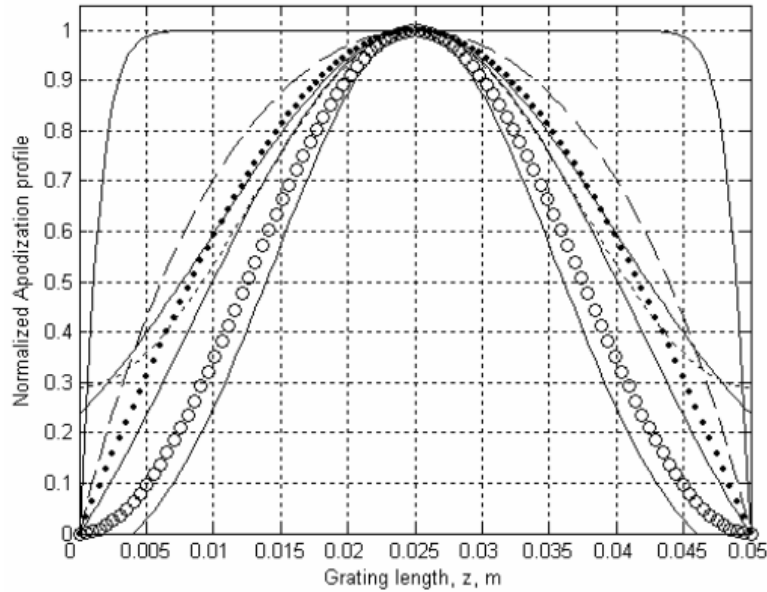


Fig. 1: Different apodization profiles versus Bragg grating length.

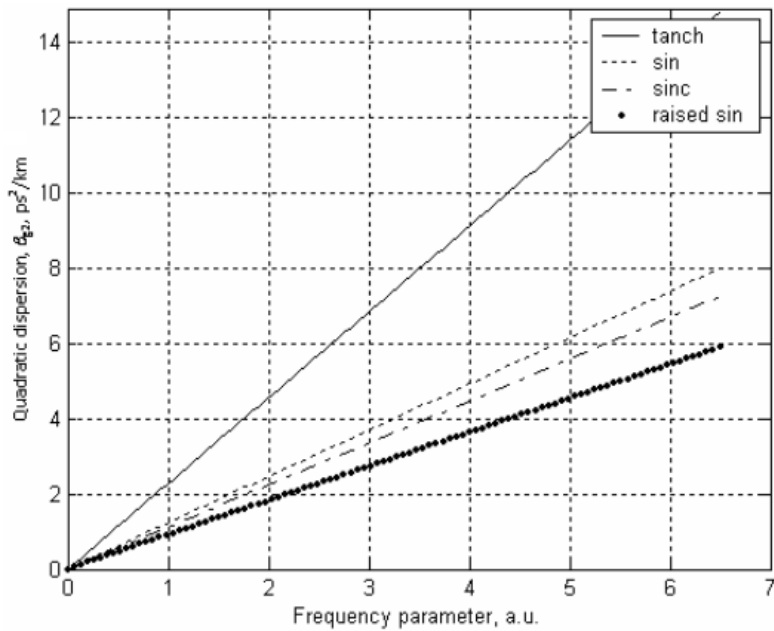


Fig. 2: Quadratic dispersion for off resonance grating (Apodization profiles group I).

The use of these expressions in (7) gives

$$\frac{n^2(z)}{n^2} = 1 + 2\eta A_0(z_1) + 4\eta \sum_{m=1}^{\infty} A_m(z_1) \cos[2mKz_0 + \Phi_m(z_1)] \quad (9)$$

We now assume that there is no resonance; that is, $[mK + \bar{k}]$ is of the order of \bar{k} for all m . This means that, as discussed, the light spectrum is well away from all grating's Bragg resonance.

Using (7) for $n^2(z)/n^2$ in (6), one obtains the pair of equations

$$\frac{dE_{\pm}}{dz} = \pm i\bar{k}\eta. \tag{10}$$

$$\sum_{all\ m} F_m(z_1)e^{2imKz_0} \left[E_{\pm} + E_{\mp}e^{\mp 2i\bar{k}z} \right]$$

Solving these equations asymptotically using the assumed forms, one can get

$$E_+(z) = E_{+0}(z_1, \dots) + \eta E_{+1}(z_0, z_1, \dots) + \dots$$

and

$$E_-(z) = \eta E_{-1}(z_0, z_1) + \dots \tag{11}$$

Accordingly, the field is predominantly forward propagating, and this field component has a wave number that is close to \bar{k} . The other smaller contributions to the field are all allowed to vary rapidly. Putting these forms in (10), one can begin to collect equations to successively higher orders in η .

First order equations and their solutions to the order η^1 , (10) gives

$$\frac{dE_{+0}}{dz_1} + \frac{dE_{+1}}{dz_0} = i\bar{k} \sum_{all\ m} F_m(z_1)e^{2imKz_0} E_{+0}(z_1, \dots) \tag{12-a}$$

and

$$\frac{dE_{-1}}{dz_0} = -i\bar{k} \sum_{all\ m} F_m(z_1)e^{2i(mK+\bar{k})z_0} E_{+0}(z_1, \dots) \tag{12-b}$$

Equation (12-a) can be solved by the assumption^[1]

$$\frac{dE_{+0}}{dz_1} = i\bar{k}F_0(z_1)E_{+0}(z_1, \dots) \tag{13}$$

and

$$\frac{dE_{+1}}{dz_0} = i\bar{k} \sum_{m \neq 0} F_m(z_1)e^{2imKz_0} E_{+0}(z_1, \dots) \tag{14}$$

Equation (14) can be immediately integrated to give

$$E_{+1}(z_0, z_1, \dots) = \sum_{m \neq 0} \frac{\bar{k}}{2mK} F_m(z_1)E_{+0}(z_1, \dots)e^{2imKz_0} \tag{15}$$

This is solved for E_{+1} in terms of E_{+0} , which still must be determined from (13).

Similarly, the second term of (12) can be integrated to determine E_{-1}

$$E_{-1}(z_0, z_1, \dots) = - \sum_{all\ m} \frac{\bar{k}}{2(mK+\bar{k})} \times F_m(z_1)E_{+0}(z_1, \dots)e^{2i(mK+\bar{k})z_0} \tag{16}$$

Quadratic and Cubic Dispersion in AFBG: The obtained expressions for E_{+1} and E_{-1} are used with the same procedure for the second order, η^2 . The local wave number is obtained as

$$k(z) = \bar{k} \left[1 + a_0(z) - \frac{1}{2}a_0^2(z) + \sum_{m=1}^{\infty} a_m^2(z) \frac{\bar{k}^2}{(m^2K^2 - \bar{k}^2)} + \dots \right] \tag{17}$$

However, the wave number of the resulting plane wave solution is not \bar{k} , associated with the reference refractive index \bar{n} , but is changed due to the off resonant grating. This shift is given by (17). The second and third term in (17) are associated with the direct current value of the grating refractive index and are not surprising. They also do not depend on frequency. The new result in (17) is the contribution due to the gratings various nonzero Fourier coefficients, a contribution that depends on frequency through \bar{k} . However, if the grating affects the wave number in a way that depends on frequency this results in dispersion.

From (17), the dispersion introduced by a grating includes the quadratic and the cubic dispersion (dispersion slope), which corresponds to taking various derivatives with respect to ω . Sipe et al. dropped all higher order resonance that have ignored effect^[1]. Also, ignoring the dc (a_0 and a_1) contribution, then the wave number shift can be obtained for the grating as

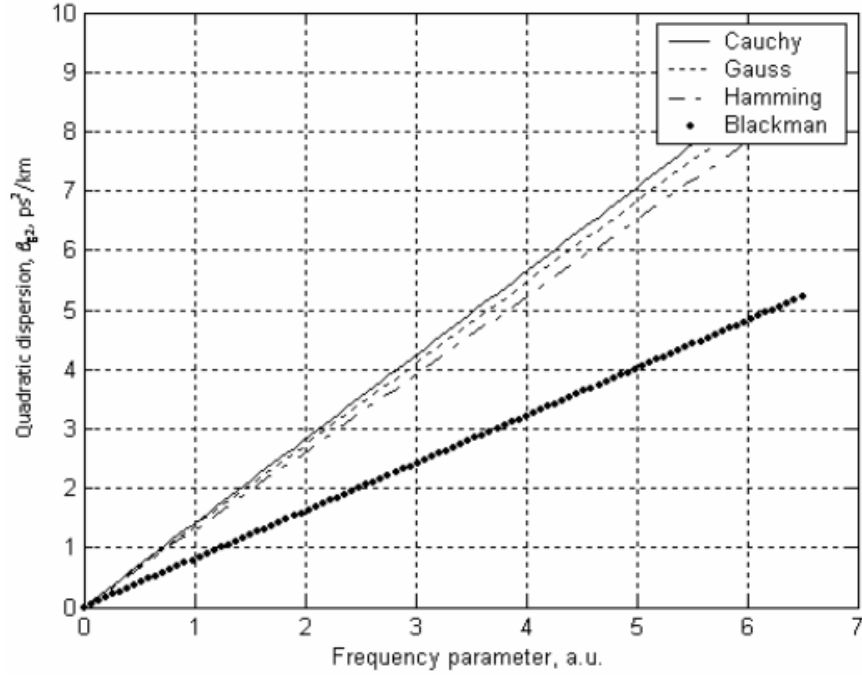


Fig. 3: Quadratic dispersion for off resonance grating (Apodization profiles group II).

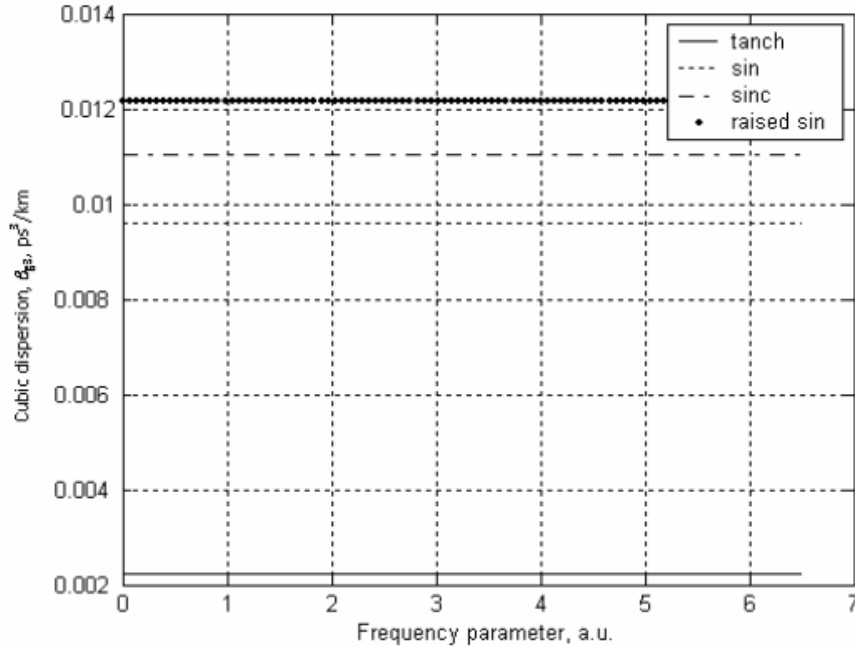


Fig. 4: Cubic dispersion for off resonance grating (Apodization profiles group I).

$$\frac{k(\omega) - \bar{k}}{a_1^2} \equiv g(\omega) \equiv \frac{\bar{k}^2}{(K^2 - \bar{k}^2)} \quad (18)$$

where $\bar{k} = \omega \bar{n} / c$ and $K = 2\pi / \Lambda$ with \bar{n} the

frequency independent component of the refractive index.

The derivative of (18) with respect to ω corresponds to the change in quadratic dispersion due to presence of an off resonance grating with unit strength. To model an AFBG and study how to minimize the quadratic dispersion, the effective medium

method is used^[3]. By this method, the refractive index variation of the AFBG, of length L , is given as the following section.

Symptotic Form of the Dispersion Compensator Bandwidth in Transmission: Here, the mathematical model of the AFBG in transmission is described. In order to model the operation of an AFBG in transmission, the "effective medium method" is used^[2]. The refractive index, at any distance, z , of the AFBG along its length, L , is variation is considered to be

$$n(z) = n_0 \{1 + \sigma(z) + 2h(z) \cdot [\cos(2K_0z + \phi(z))]\} \tag{19}$$

where n_0 is the core refractive index which depends on frequency. All quantities, such as the effective index of refraction of the mode $n(z)$, may also depend on frequency. In the following, the frequency dependent quantities will be excluded to ensure that the frequent effect is only due to the grating (not the other quantities). The quantity $\sigma(z)$ is the background refractive index variation, $K_0 = 2\pi / \Lambda_0$ is the reference Bragg wave vector (Λ_0 is the reference Bragg period), and $\phi(z)$ is the slowly varying grating phase. In case of linearly chirped gratings, $\phi(z) = K_0 Cz^2$, where C (in m^{-1}) is the chirp parameter. The total variation of the local Bragg wavelength across the entire grating length L is given by $\Delta\lambda_B = 2\lambda_0 CL$, where $\lambda_0 = (2n_0\Lambda_0)$ is the reference Bragg wavelength. The quantity $h(z)$ describes the amplitude variation of the induced refractive modulation and is, in general, expressed as $h(z) = h_0 f(z)$, where h_0 is the peak refractive index modulation and $f(z)$ is the apodization profile,

Apodized FBG: The effect of changing the apodization profiles with the quadratic dispersion parameter for off resonance grating is investigated using the following profiles^[7]:

Sine Profile:

$$h(z) = \sin\left(\frac{\pi z}{L}\right), \quad 0 \leq z \leq L \tag{20}$$

Raised Sine Profile:

$$h(z) = \sin^2\left(\frac{\pi z}{L}\right), \quad 0 \leq z \leq L \tag{21}$$

Sinc Profile:

$$h(z) = \frac{\sin(x)}{x}, \quad x = \frac{2\pi\left(z - \frac{L}{2}\right)}{L}, \quad 0 \leq z \leq L \tag{22}$$

Positive-tanh Profile:

$$h(z) = \tanh\left[\frac{2az}{L}\right], \quad 0 \leq z \leq \frac{L}{2} \quad (a = 4)$$

$$\tanh\left[\frac{2a(L-z)}{L}\right], \quad \frac{L}{2} \leq z \leq L \tag{23}$$

Blackman Profile:

$$h(z) = \frac{1 + (1+B)\cos\left(\frac{2\pi z}{L}\right) + B\cos\left(\frac{4\pi z}{L}\right)}{2 + 2B}$$

, $0 \leq z \leq L$ ($B = 0.19$) (24)

Gauss Profile:

$$h(z) = \exp\left[-G\left(\frac{z}{L}\right)^2\right], \quad 0 \leq z \leq L \quad (G = 15) \tag{25}$$

Hamming Profile:

$$h(z) = \frac{1 + H\cos\left(\frac{2\pi z}{L}\right)}{1 + H}$$

, $0 \leq z \leq L$ ($H = 0.55$) (26)

Cauchy Profile:

$$h(z) = \frac{1 - \left(\frac{2z}{L}\right)^2}{1 - \left(\frac{2Cz}{L}\right)^2}, \quad 0 \leq z \leq L \quad (C = 0.5) \tag{27}$$

The apodization functions initially proposed correspond to well known window functions employed in filter design to suppress side lobes in the reject band, there parameter values which provide an efficient way to control the characteristics of the functions, are chosen such that all the profiles have a similar characteristics, which is composed with a flat region at the grating center, and a constant slope decaying characteristics toward the grating's edges.

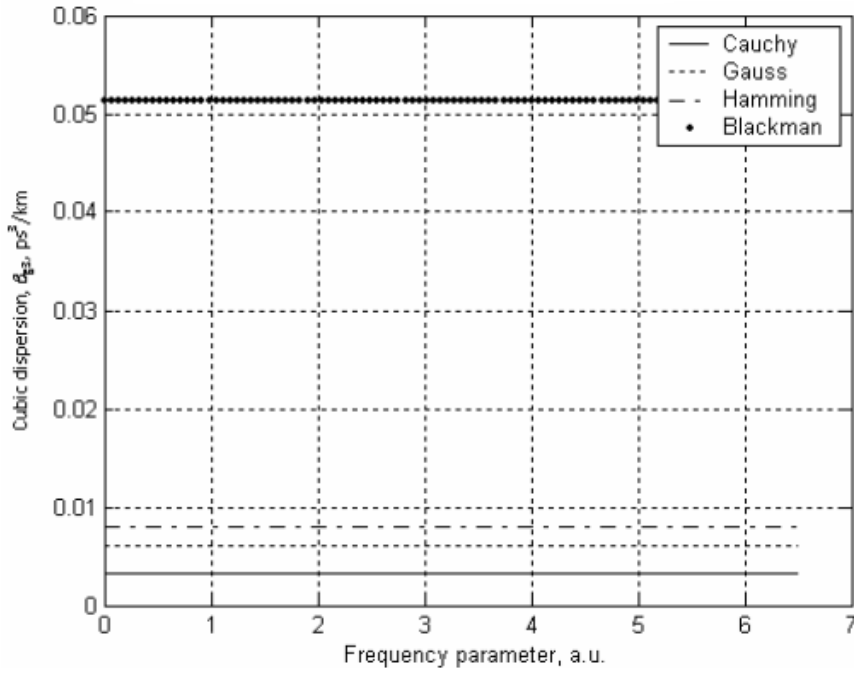


Fig. 5: Cubic dispersion for off resonance grating (Apodization profiles group II).

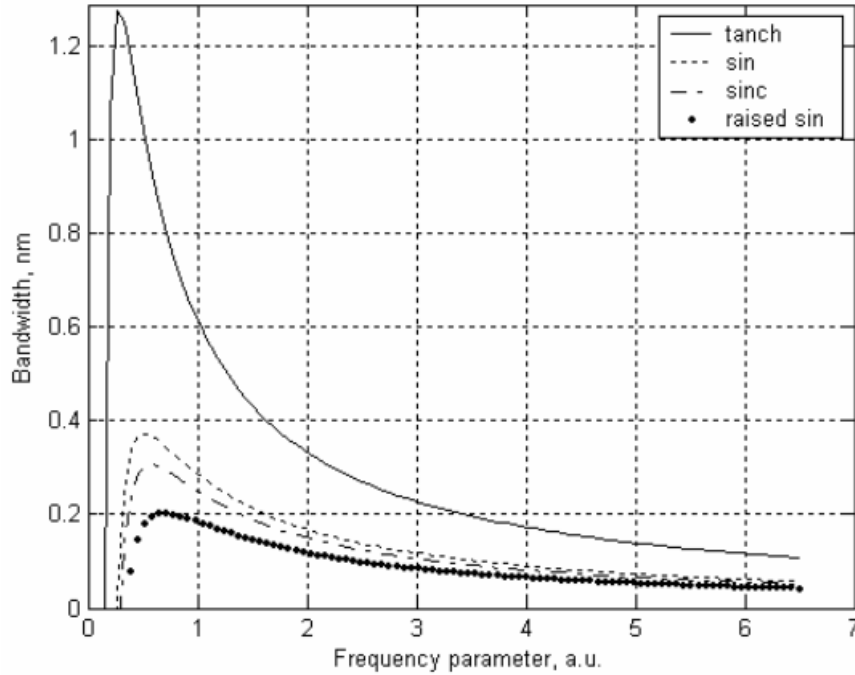


Fig. 6: Dispersion compensator bandwidth for off resonance grating (Apodization profiles group I).

Using (19) in Maxwell's equations and applying perturbation techniques, the grating can be represented as an "effective medium" with an effective refractive index n_{eff} , an effective dielectric permittivity ϵ , an effective magnetic permeability m , and an effective local impedance Z .

A local detuning of a grating, $d(z,D)$ is defined as^[2]

$$\delta(z, \Delta) = \Delta + \frac{\pi}{\Lambda} \sigma(z) - \frac{d}{dz} \phi(z), \quad (28)$$

where

$$\Delta(k) = kn_o - \frac{\pi}{\Lambda} \tag{29}$$

with $k(=2\pi/\lambda)$ the free space wave number of the incident light and

$$\kappa(z) = h(z) \frac{\pi}{\Lambda} \tag{30}$$

Both medium permittivity and permeability as functions of z are obtained as

$$\varepsilon(z, \Delta) = \delta(z, \Delta) + \kappa(z) \tag{31}$$

and

$$\mu(z, \Delta) = \delta(z, \Delta) - \kappa(z) \tag{32}$$

Therefore, the effective refractive index, $n(z, \Delta)$, is

$$n_{eff}(z, \Delta) = \sqrt{\varepsilon\mu} = \sqrt{\delta^2(z, \Delta) - \kappa^2(z)} \tag{33}$$

and the effective local impedance, $Z(z, \Delta)$, is

$$Z^2(z, \Delta) = \frac{\mu}{\varepsilon} = \frac{\delta(z, \Delta) - \kappa(z)}{\delta(z, \Delta) + \kappa(z)}. \tag{34}$$

The dispersion, $d(k)$, and the dispersion slope, $d'(k)$, of the grating are given, respectively, by^[1]

$$d(k) = -\frac{2\pi n_o^2}{\lambda^2 c} \frac{d^2}{d\Delta^2} \arg\{t(\Delta)\} \text{ ps/nm} \tag{35}$$

and

$$d'(k) = \left(\frac{2\pi n_o}{\lambda^2}\right)^2 \frac{n_o}{c} \frac{d^3}{d\Delta^3} \arg\{t(\Delta)\} \text{ ps/nm}^2 \tag{36}$$

where the transmission coefficient has an amplitude, $t(\Delta)$, given by^[1]

$$t(\Delta) = 4Z(0, \Delta) \left| \frac{Z(L, \Delta)}{Z(0, \Delta)} \right|^{1/2} \cdot \{1/[e^{i\varphi}(Z(0, \Delta)+1)(Z(L, \Delta)+1) - e^{-i\varphi}(Z(0, \Delta)-1)(Z(L, \Delta)-1)]\} \tag{37}$$

and a phase, $\varphi(z)$, given by^[2]

$$\varphi(z) = \int_0^L n_{eff}(z, \Delta) dz. \tag{38a}$$

When AFBGs are used, the grating profile, $h(z)$, grows and decays continuously from and to zero value; therefore $\kappa(0) = \kappa(L) = 0$. Hence,

$$Z(0, \Delta) = Z(L, \Delta) = 1 \tag{38b}$$

Therefore, the transmission coefficient can be rewritten in the form

$$t(\Delta) = e^{-i\varphi} \tag{39}$$

Well away from the reflection band edges, the argument of the transmission coefficient, $\arg\{t\}$, can be simplified to

$$\arg\{t(\Delta)\} = \text{sgn}(\Delta)\varphi(\Delta) \tag{40}$$

at its lowest order, because, n_{eff} , is real, where

$$\text{sgn}(\Delta) = +1 \text{ for } \Delta > 0 \tag{41-a}$$

$$\text{sgn}(\Delta) = -1 \text{ for } \Delta < 0 \tag{41-b}$$

This result is precise for AFBGs (i.e. $h(0) = h(L) = 0$ and is continuous for all z). The effect of apodization on the grating leads to express the dispersion and the dispersion slope of the compensator, respectively, as^[2]

$$d(k) = -\frac{2\pi n_o^2}{\lambda^2 c} \int_0^L \frac{\kappa^2(z) dz}{[\delta^2(z, \Delta) - \kappa^2(z)]^{3/2}} \tag{42}$$

and

$$d'(k) = \left(\frac{2\pi n_o}{\lambda^2}\right)^2 \frac{3n_o}{c} \int_0^L \frac{\kappa^2(z)\delta(z, \Delta) dz}{[\delta^2(z, \Delta) - \kappa^2(z)]^{5/2}} \tag{43}$$

We shall assume the grating is unchirped (i.e., $\varphi(z) = \text{constant}$) and $\sigma(z) = \text{constant}$ for the length of the grating, allowing to set $\sigma(z) = 0$ without loss of generality. These assumptions results in $\delta(z, \Delta) = \delta(k)$, i.e., δ is a function of k only. In the asymptotic region with large detuning away from Bragg wave number, we have $\delta(k) = k(z)$ which reduces (42) and (43) to the forms

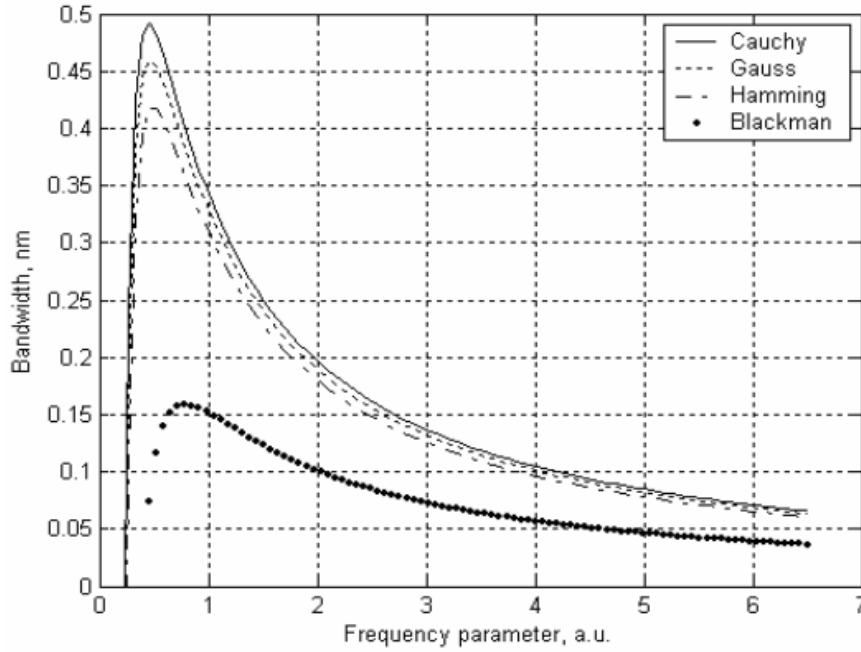


Fig. 7: Dispersion compensator bandwidth for off resonance grating (Apodization profiles group II).

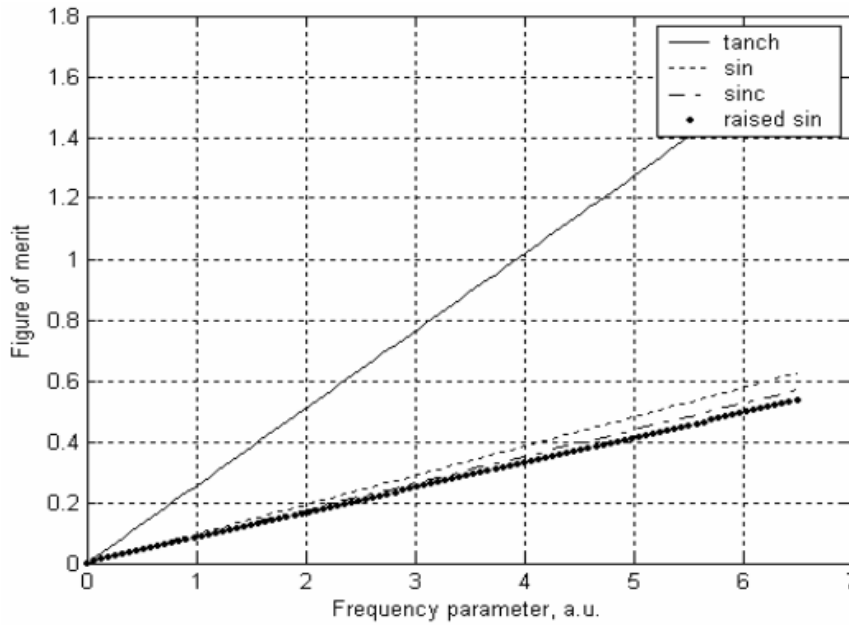


Fig. 8: Figure of merit against for off resonance grating (Apodization profiles group I).

$$d(k) \cong \frac{2\pi n_o^2}{\lambda^2 c \delta^3(k)} \int_0^L k^2(z) dz \quad (44)$$

and

$$d'(k) \cong \frac{12\pi^2 n_o^3}{\lambda^4 c \delta^4(k)} \int_0^L k^2(z) dz \quad (45)$$

For a dispersion compensator, the dispersion d of the compensator is set to negate the impact of fiber dispersion. That is $d(k) = -D/$. However, as seen from (44) and (45), if $d(k)$ is nonzero, then so is $d'(k)$. With the net dispersion zero, the impact of the dispersion slope d' must be considered. Then the compensator performance can be measured using its bandwidth. The bandwidth of a dispersion compensator, $\Delta\lambda$, is given by^[2]

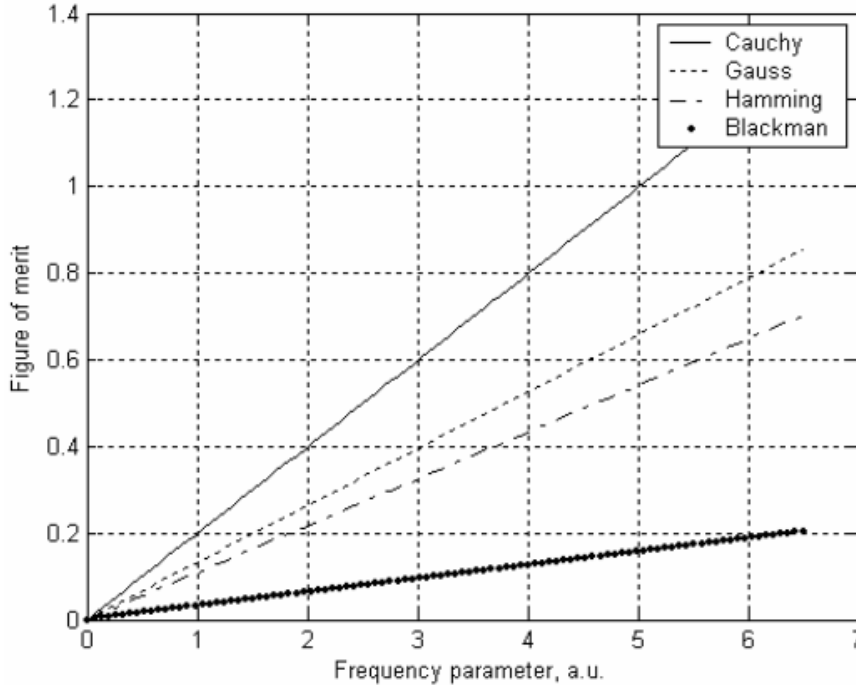


Fig. 9: Figure of merit against for off resonance grating (Apodization profiles group II).

$$\Delta\lambda = \frac{d(k)}{d'(k)} \tag{46}$$

This equation gives the maximum bandwidth of a signal which can be compensated without the AFBG dispersion slope affecting (degrading) the signal^[1]. For the compensator to properly compensate the fiber link dispersion, we require either $d' = -D'$ (i.e., the compensator negates both fiber dispersion and dispersion slope) or the signal bandwidth must be less than the quotient given by (46) for an ABFG as a compensator in transmission.

The asymptotic form for this bandwidth, $\Delta\lambda$, is given, through (44) and (45), by

$$\Delta\lambda = \frac{\lambda^2 \delta(k)}{6\pi n_o} \tag{47}$$

Figure of Merit (FOM): A variant ratio has been proposed as a “figure of merit (FOM)”^[4]. The FOM, for a given total fiber link dispersion, gives the maximum bandwidth of a signal which can be compensated, and is given by^[4]

$$F(\delta) = \sqrt{2} \left| \frac{\beta_{g2}}{\beta_{g3}} \sigma_0 \right| \tag{48}$$

where σ_0 is the transform limited rms pulse width, β_{g2} is quadratic dispersion of the off resonance grating (obtained by differentiating (18) off resonance grating wave number twice w.r.t. frequency), β_{g3} is the cubic dispersion (obtained by differentiating (18) three times w.r.t. frequency). Clearly to minimize the detrimental effect of third order dispersion, F should be as large as possible for a given set of design parameters.

RESULTS AND DISCUSSION

Using MATLAB ver. 6.5 with the described model, a computer simulation is performed for using the asymptotic form of the bandwidth as a measure for the performance of the dispersion compensator for different apodization profiles versus a combined parameter consisting of frequency with constant reference refractive index and Bragg period divided by speed of light of an off resonance Bragg grating. The same is parameter is used also for quadratic dispersion, cubic dispersion and figure of merit.

Different normalized apodization profiles is displayed in Fig. 1, against the grating length, as the following order tanh, Cauchy, sin, Gauss, Hamming, sinc, Raised sin, Blackman. The displayed results show that the positive-tanh profile is the most successful profile. It has $h(0) = h(L) = 0$, with $h(z)$ all continuous in a 5 cm grating. It also satisfies the previous terms for selection of apodization profile with a flat region at the grating center which is the biggest among the

different used profiles and a constant slope decaying characteristics towards the grating edges. For simplicity, in the following, the eight profiles are divided into two groups: Group I: tanh, sin, sinc and raised sin profiles, and Group II: Cauchy, Gauss, Hamming and Blackman profiles. Also, we define the quantity $\omega \bar{n} d / c$ as a frequency parameter.

The quadratic dispersion resulting from the grating is calculated by differentiating the wave number of the off resonance grating twice with respect to frequency. This quadratic dispersion is important for compensating dispersion resulting from the optical fiber and also from the fiber lasers. From Figs. 2 and 3, it is clear that the highest quadratic dispersion is the positive tanh profile and the lowest is in the Blackman profile.

Cubic dispersion is calculated by differentiating the wave number three times w.r.t. frequency as declared in Figs. 4 and 5, where the cubic dispersion is independent of the frequency (for each profile, it has a constant value). It is preferred to choose a profile with small cubic dispersion.

The dispersion compensator bandwidth, (47), is displayed against the frequency parameter in Fig. 6 and 7 for the two apodization profile groups. It is clearly seen that the best profile is the tanh one giving the greatest bandwidth.

The FOM is displayed in Figs. 8 and 9 for the considered profiles for the grating off resonance. It is found that the best profile that has the maximum value of figure of merit is the positive hyperbolic tangent profile. This profile was also chosen the best in the asymptotic form of the bandwidth method. Clearly to minimize the detrimental effect of third order dispersion, F should be as large as possible for a given set of design parameters.

Conclusion: The light wave number as a function of frequency, obtained by J. E. Sipe *et al.*^[1], is used to design a dispersion compensator using the off resonance grating operating in transmission. The asymptotic form for the compensator bandwidth using different apodization profiles which is demonstrated by Kerry Hinton^[2] is used to judge the compensator performance. The obtained results show that the positive hyperbolic tangent profile results in an overall superior performance, as it provides the highest value for asymptotic form for the compensator bandwidth. The obtained results are then verified by calculating the figure of merit, demonstrated by A. Othonos and K. Kalli^[4]. Again, the positive hyperbolic tangent profile is found to be the best profile that has the maximum value of figure of merit.

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