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THEORY AND ANALYSIS OF THREE-PHASE SERIES-CONNECTED PARAMETRIC MOTORS

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Abstract - **This paper presents the steady-state performance of a three phase wound-rotor parametric motor. This type of motor can be practically realized by series connection of stator and rotor phases of a conventional wound-rotor induction machine. The analysis is based on the d-q axes model, from which a phasor diagram is presented. The analysis is extended to include the magnetic saturation effect. Comparison between theoretical and experimental results showed a satisfactory agreement proving the validity of the mathematical model as well as magnetic saturation effect representation. Also the motor stability is investigated.**

Keywords: Parametric Machines, Induction motors, Wound Rotor, Synchronous Motors.

II-INTRODUCTION

The principle of operation of the parametric machine has been originally established for the generator mode when studying the behavior of periodically varying inductance *RLC* series circuits [1]. The Single phase parametric generator has been extensively studied theoretically and experimentally using different mathematical techniques based on the Floquet theory [2,3]. The three phase parametric generator has been analyzed using the d-q model [3,4], Floquet theory [3,5] and phasor diagrams [6,7].

It was found that such a machine is inherently of synchronous type allowing electromechanical energy conversion only if :

1. The rotor speed corresponds to an angular frequency of double the angular frequency of the stator mmf i.e.

$$\omega_r = 2\,\omega \qquad (1)$$

2. The series–connection of the stator and the rotor windings are such that the phase sequence of the rotor mmf is in reverse sense to that of the stator mmf as shown in Fig. 1.

Experimental and theoretical investigation for controlling the terminal voltage via the excitation capacitor has been presented [8] using a fixed thyristor controlled reactor.

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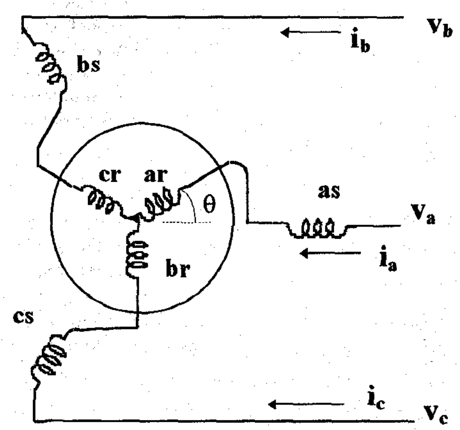

Fig. 1 ***Connection between stator and rotor windings of the parametric motor***

The parametric machine can be used as a motor or a generator depending on its terminal conditions [3]. The parametric motor has the advantage of operating at fixed speed of double the synchronous speed which depends only on the number of poles and supply frequency.

The previous work in the field of the parametric machines gave an essential attention to the generator mode of operation. Therefore, this paper is concerned with the motor mode. The aim of this paper is to :

1. propose a mathematical model for the parametric motor,
2. verify the experimental behavior to check the validity of the mathematical model and
3. investigate the limits for which the motor will be stable.

II- MATHEMATICAL MODEL

Based on the electrical connection between the balanced stator and rotor phases shown in fig. 1, the terminal voltages can be expressed as follows:

$$\left.\begin{aligned} V_a &= V_{as} + V_{ar} \\ V_b &= V_{bs} + V_{cr} \\ V_c &= V_{cs} + V_{br} \end{aligned}\right\} \qquad (2)$$

while the winding currents can be defined as follows:

$$\left.\begin{aligned} I_a &= I_{as} = I_{ar} \\ I_b &= I_{bs} = I_{cr} \\ I_c &= I_{cs} = I_{br} \end{aligned}\right\} \qquad (3)$$

The machine parameters are defined as follows:

$$\left.\begin{aligned} R_a &= R_s + R_r \\ L_a &= L_s + L_r \end{aligned}\right\} \qquad (4)$$

By writing the voltage balance equations of the six coils and substituting from equs. (2-4), the machine equations reduce to three equations describing the terminal conditions. These equations can be written in the following operational form :

$$\mathbf{V} = \mathbf{Z}(p)\,\mathbf{I} \qquad (5)$$

where $\mathbf{V} = (v_a \;\; v_b \;\; v_c)^T$ and $I = (i_a \;\; i_b \;\; i_c)^T$

where T stands for the transpose operation.

$\mathbf{Z}(p)$ is the transient impedance matrix =

$$\begin{pmatrix} R_a + p\left\{\begin{matrix} L_a + 2M \\ \cos(\theta) \end{matrix}\right\} & p\left\{\begin{matrix} -0.5L_a + 2M \\ \cos(\theta - 120^\circ) \end{matrix}\right\} & p\left\{\begin{matrix} -0.5L_a + 2M \\ \cos(\theta + 120^\circ) \end{matrix}\right\} \\ p\left\{\begin{matrix} -0.5L_a + 2M \\ \cos(\theta - 120^\circ) \end{matrix}\right\} & R_a + p\left\{\begin{matrix} L_a + 2M \\ \cos(\theta + 120^\circ) \end{matrix}\right\} & p\left\{\begin{matrix} -0.5L_a + 2M \\ \cos(\theta) \end{matrix}\right\} \\ p\left\{\begin{matrix} -0.5L_a + 2M \\ \cos(\theta + 120^\circ) \end{matrix}\right\} & p\left\{\begin{matrix} -0.5L_a + 2M \\ \cos(\theta) \end{matrix}\right\} & R_a + p\left\{\begin{matrix} L_a + 2M \\ \cos(\theta - 120^\circ) \end{matrix}\right\} \end{pmatrix}$$

The periodically varying coefficients in $\mathbf{Z}(p)$ can be changed into constant coefficients by applying a synchronously rotating reference frame transformation for voltages and currents. If zero sequence quantities do not exist, the transformation factor is given by:

$$\mathbf{K}^T = \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\omega t) & \cos(\omega t - 120^\circ) & \cos(\omega t + 120^\circ) \\ \sin(\omega t) & \sin(\omega t - 120^\circ) & \sin(\omega t + 120^\circ) \end{pmatrix} \qquad (6)$$

Applying the transformation (6) to (5) yields [9]:

$$\mathbf{V}' = \mathbf{Z}'(p)\,\mathbf{I}' \qquad (7)$$

$$\text{where} \quad \mathbf{V}' = \mathbf{K}^T\mathbf{V}, \qquad (8)$$

$$\mathbf{I}' = \mathbf{K}^T\mathbf{I}, \qquad (9)$$

$$\text{and} \quad \mathbf{Z}'(p) = \mathbf{K}^T\mathbf{Z}(p)\mathbf{K} \qquad (10)$$

The transformed impedance matrix $\mathbf{Z}(p)$ is given by:

$$\mathbf{Z}'(p) = \begin{pmatrix} R_a + L_d p & \omega L_q \\ -\omega L_d & R_a + L_q p \end{pmatrix} \qquad (11)$$

where

$L_d = 1.5\,\{L_a + 2M\}$; the direct axis inductance.

$L_q = 1.5\,\{L_a - 2M\}$; the quadrature axis inductance.

III- Steady-state Analysis

III-1 Voltage Balance equation

The mathematical model given by Equations (7-11) describes the dynamic behavior of the parametric motor. If the applied voltage is sinusoidal and balanced, the transformed voltages and currents are all constants. Therefore the operator p can be replaced by zero. The transformed voltage equation becomes:

$$\begin{pmatrix} V_d \\ V_q \end{pmatrix} = \begin{pmatrix} R_a & X_q \\ -X_d & R_a \end{pmatrix} \begin{pmatrix} I_d \\ I_q \end{pmatrix} \qquad (12)$$

where $X_d = \omega L_d$ is the direct axis reactance.

$X_q = \omega L_q$ is the quadrature axis reactance.

The voltage balance equation (12) suggests a phasor diagram for the parametric motor as shown in fig. 2, from which the following relations can be written:

$$\left.\begin{aligned} V_d &= V \sin\delta \\ V_q &= V \cos\delta \end{aligned}\right\} \qquad (13)$$

where δ is the load angle in electrical degrees.

Solving equation (12) for I_d and I_q one gets:

$$\left.\begin{aligned} I_d &= \frac{1}{\Delta}\left(R_a V_d - X_q V_q\right) \\ I_q &= \frac{1}{\Delta}\left(X_d V_d + R_a V_q\right) \end{aligned}\right\} \qquad 14)$$

where $\Delta = R_a^2 + X_d X_q$

Consequently the motor phase current is given by:

$$I = \sqrt{I_d^2 + I_q^2} \qquad (15)$$

The power factor can also be calculated from the relation:

$$\cos\phi = \sin(\psi - \delta) \qquad (16)$$

III-2 Torque Expression

The speed voltage coefficient matrix $\mathbf{G}$ can be written from either (11) or (12). The elements of $\mathbf{G}$ are the coefficients of the electrical angular speed ω_r. Considering equation (1), $\mathbf{G}$ matrix is given by:

$$\mathbf{G} = \begin{pmatrix} 0 & L_q/2 \\ -L_q/2 & 0 \end{pmatrix} \qquad (17)$$

The air-gap torque exerted by the motor is given by :

$$\begin{aligned} T_g &= \frac{3}{2} P\,\mathbf{I}'^T\mathbf{G}'\,\mathbf{I}' \\ &= -\frac{3}{4} P\left(L_d - L_q\right) I_d I_q \end{aligned} \qquad (18)$$

The torque can be expressed in terms of δ by using (13), (14) and (18) in the following form:

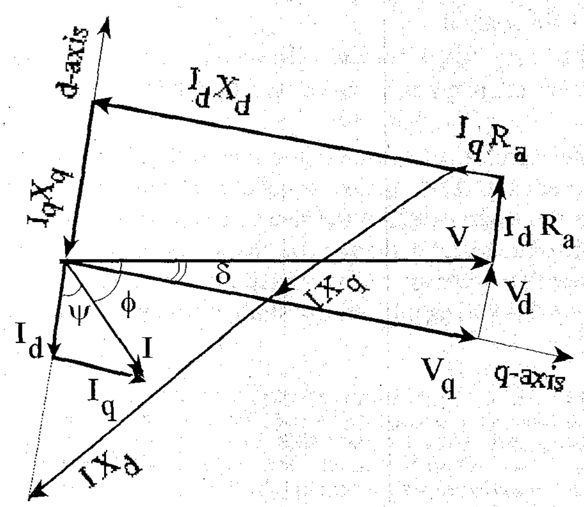

***Fig. 2** Phasor diagram of three phase parametric motor*

$$T_g(\delta) =$$

$$\frac{P}{8}\left(\frac{V}{\cos\phi_R}\right)^2 \frac{L_d - L_q}{Z_d Z_q}\left\{\sin\left(\phi_d + \phi_q - 2\delta\right) - \sin\phi_R\right\} \qquad (19)$$

where : $Z_d = \sqrt{R_a^2 + X_d^2}$, $Z_q = \sqrt{R_a^2 + X_q^2}$,

It was found that L_d is nearly constant in the operating range and equals 1.2 H while L_q is highly affected by I_q. Fig. 3 shows the experimental relation between L_q and I_q as well as a suitable curve fitting described by the following relations:

$$L_q = 0.034\left\{1 - e^{-2 I_q}\right\} \text{ H} \quad \textit{for} \quad I_q < 3\text{A} \qquad (25)$$

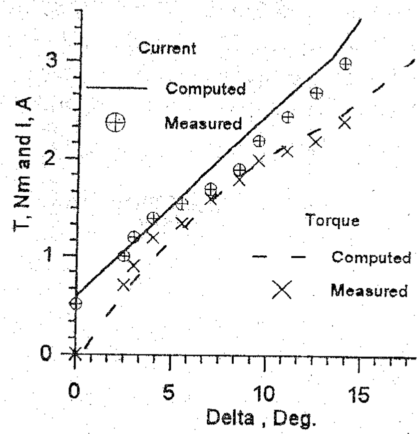

Fig. 4: *Variation of both of the output torque* T_o *and phase current* **I** *against load angle* δ

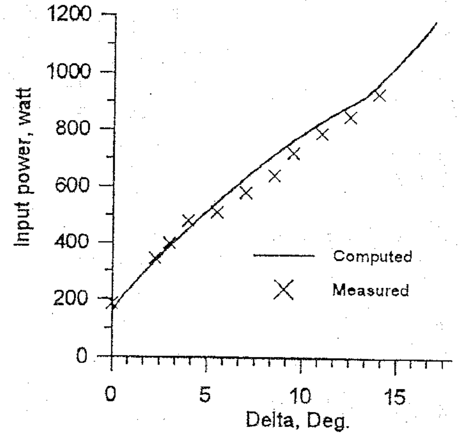

Fig. 5: *Variation of the input power* P_{in} *against load angle* δ

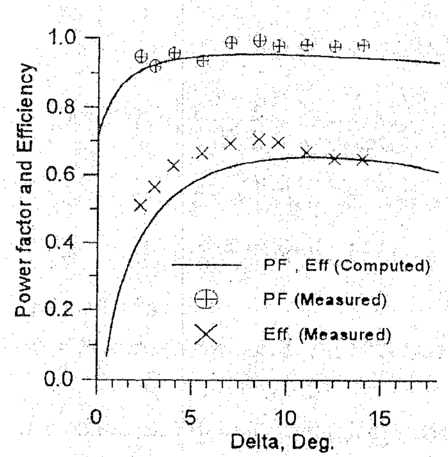

Fig. 6: *Variation of both of the power factor and efficiency against load angle* δ

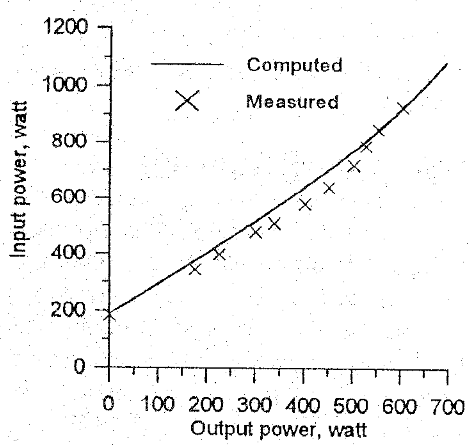

Fig. 7 : *Variation of the input power* P_{in} *against output power* P_{out}

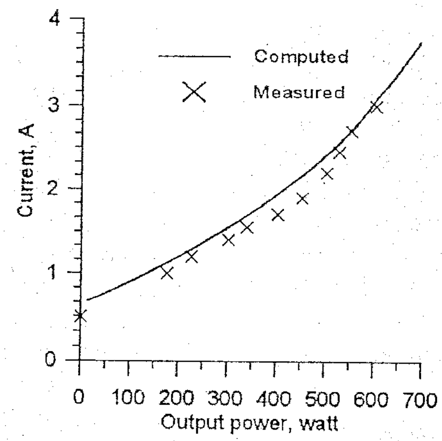

Fig. 8 : *Variation of the phase current* **I** *against output power* P_{out}

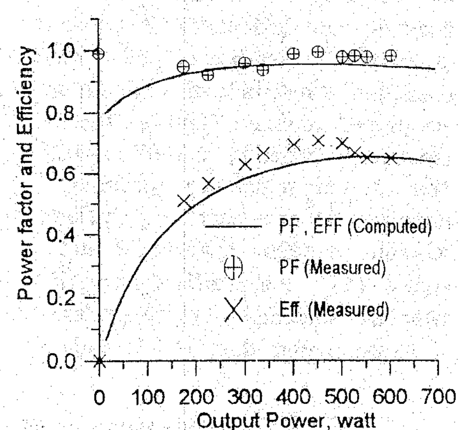

Fig. 9 : *Variation of both of the power factor and efficiency against output power* P_{out}

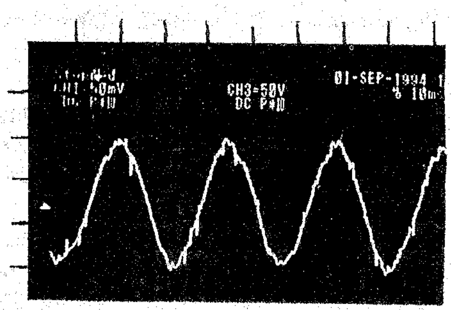

Fig. 10 : *Steady-state current waveform at* T_o*=1.6 Nm.*
Scale : 50 mV/Div.
1mV=0.03 A

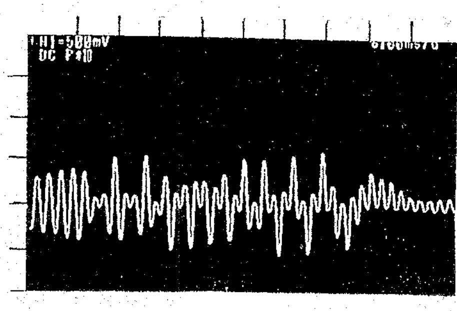

Fig. 11 : *Current waveform at pull-out of synchronism.*
Scale : 500 mV/Div.
1mV=0.03 A

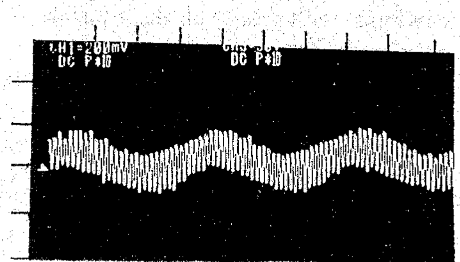

Fig. 12 : *Current waveform after pull-out of synchronism.*
Scale : 200 mV/Div.
1mV=0.03 A

Appendix

The data of the employed slip-ring induction motor are as follows:

Power : 2.2 KW, Frequency : 50 Hz, Speed : 1390 r/min
Stator 220/380 V , Δ/Y , 6.3/3.6 A
R_s=2.1 Ω/phase , X_s=5.28 Ω/phase
Rotor 328 V , Y-connected , 4.2 A
R_r=1.96 Ω/phase , X_r=3.92 Ω/phase
Rotor to stator turns ratio = 0.86

List of Symbols

v_d , v_q direct and quadrature axes voltages, V.
i_d , i_q direct and quadrature axes currents, A.
V terminal phase rms voltage, V.
V_L terminal line rms voltage, V.
I phase rms current, A.
R_s ,R_r stator and rotor phase resistances, Ω.
L_s ,L_r stator and rotor phase self inductances, H.
M max. mutual inductance between one stator phase and one rotor phase, H.
θ electrical angle between stator phase 'a' and rotor phase 'a'.
ω electrical angular frequency of the supply voltage, rad/s.
ω_r electrical angular rotor speed, rad/s.
δ electrical load angle, rad/s.
T_g air-gap torque, Nm.
T_o output torque, Nm.
P_{out} output power, W.
P_{in} input power, W.
P number of motor poles.
p differential operator d/dt.

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Biography

Dr. Essam El-Din M. Rashad (M94) was born in Shebin El-kom, Egypt on Sept. 1960. In 1983, he received B.Sc. degree in electrical power and machines engineering from Faculty of Engineering, Shebin El-Kom, Menoufia University, Egypt. He received M.Sc. and Ph.D. degrees both in electrical engineering from Alexandria University, Egypt in 1987 and 1992 respectively. From 1985-90 he was an offshore electrical engineer in Belayim Petroleum Company, Abo-Rudies fields, Sinai, Egypt. In 1990 he joined Ministry of Higher Education, Egypt as an engineering education lecturer. In 1992 he was appointed as an assistant lecturer in the electrical engineering department, Faculty of Engineering, Tanta University , Egypt. In 1993 he was promoted a lecturer in the same department

Dr. Mostafa E. Abdel-Karim was born in Kallin, Kafr El-Sheikh, Egypt on Dec. 1958. He received B.Sc., M.Sc. and Ph.D. degrees all in electrical engineering from faculty of engineering , Shebin El-Kom , Menoufia University, Egypt in 1981, 1985 and 1991 respectively. From 1987 to 1990 he was a channel system member in I.N.P.L. Nancy, France. In 1981 he was appointed as a demonstrator in the electrical engineering department, Faculty of Engineering, Shebin El-Kom, Egypt. From 1985 to 1991 he was an assistant lecturer in the same department where he is now a lecturer. His research interest includes power electronics, electrical machines and automatic control systems.

Mr. Yasser G. Desouky received B.Sc. and M.Sc. both in electrical engineering from Faculty of Engineering , Alexandria University, Egypt in 1991 and 1993 respectively. In 1991 he joined the Arab Academy for Science and Technology, Miami, Alexandria, Egypt as a demonstrator. Since 1994 he is working toward Ph.D. degree from Heriot-Watt University, Edinburgh, United Kingdom.