

# Transient and Steady State Analysis of High Efficiency Single-Phase Induction Motor

دراسة تحليلية للحالة الديناميكية والمستقرة لمحرك حتى على الكفاءة مغذى من مصدر كهربي احادى الوجه

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**Abstract:-** Operating a three-phase induction motor from a single-phase supply via three capacitors is studied. The capacitor values may be adjusted so as to get a motor balanced mode of operation. The model of the machine is derived using instantaneous symmetrical components. Equations describing the machine dynamics have been derived then used to simulate and analyze the machine performance for different modes of operation. The machine transient and steady state performance is predicted using Matlab-Simulink software. Experimental results have been obtained and compared with the simulated waveforms, showing the effectiveness of the proposed machine model.

ملخص:- يدرس هذا البحث عملية تشغيل محرك ثلاثى الأوجه من خلال مصدر كهربي احادى الوجه باستخدام ثلاثة مكثفات كهربية. بضبط قيم المكثفات لكل سرعة دوران يمكن تشغيل المحرك فى الحالة المتزنة للأوجه الثلاثة. تم خلال هذا البحث استنتاج نموذج للمحاكاة للمحرك قائم على المعادلات اللحظية لمركبات التيار والجهود المتماثلة وبأستخدام الحاسب الآلى تم عمل محاكاة للمحرك ودراسة الأداء الوظيفى للماكينة المستقرة وكذلك المتزنة والغير متزنة. وقد أثبتت فاعلية البرنامج بمقارنة النتائج النظرية مع نظيراتها العملية.

**Keywords:** Induction motor, Single-phase supply, Simulink, Symmetrical components

## 1. Nomenclature

$I_A, I_B, I_C$	RMS motor phase currents
$I_1, I_2, I_3$	RMS currents through capacitors
$I_0, I_p, I_n$	RMS zero, +ve and -ve phase currents
$I_{sr}, I_{rr}$	RMS of real part of stator and rotor currents
$I_{sg}, I_{rg}$	RMS of imaginary part of stator and rotor currents
$i_A, i_B, i_C$	Instantaneous phase currents
$i_{sr}, i_{rr}$	Real part of instantaneous stator and rotor currents
$i_{sg}, i_{rg}$	Imag. part of instantaneous stator and rotor currents
$i_{ps}, i_{ns}$	Instantaneous +ve and -ve stator currents
$i_{pr}, i_{nr}$	Instantaneous +ve and -ve referred rotor currents
$V_A, V_B, V_C$	RMS phase voltages
$V_1, V_2, V_3$	RMS voltages across capacitors
$V_S, I_S, \phi_S$	RMS supply voltage, current and phase shift
$V_0, V_p, V_n$	RMS zero, +ve and -ve phase voltages
$v_A, v_B, v_C$	Instantaneous phase voltages
$v_p, v_n$	Instantaneous +ve and -ve stator voltages
$v_S, i_S$	Instantaneous supply voltage and current
$R_s, R_r$	Stator and referred rotor resistances
$T_L, T_e$	Load torque and motor developed torque
$T_{av}, T_{OSC}$	Average motor torque and double supply frequency oscillatory torque component
$X_m, X_1, X_2$	Magnetizing, stator and referred rotor reactance
$C_1, C_2, C_3$	Balancing capacitors
$B_1, B_2, B_3$	Balancing susceptances
$J$	Inertia of the rotating parts
$\omega, n, s$	Electrical angular speed, per unit speed and slip
$P$	Number of poles
$a$	Unit complex operator ( $exp[j2\pi/3]$ )
$Y_p, Y_n$	Stator +ve and -ve admittance
$y_p, \phi_p$	Magnitude and angle of stator +ve admittance
$\phi_{sr}, \phi_{rr}, \phi_{sg}, \phi_{rg}$	Angle of $i_{sr}, i_{rr}, i_{sg}$ and $i_{rg}$

## 2. Introduction

The 3-ph induction motor (IM) can be operated from a 1-ph supply. This can be achieved by leaving one terminal unexcited and the machine is partially loaded [1]. Capacitor banks have been used to generate the split phase [2]. Autotransformer with a capacitor were also used to excite the motor third terminal for full load operation [3]. In [4], 3-ph IM is supplied by a 1-ph supply via three capacitors as shown in figure 1. This phase balancing scheme is called high efficiency single-phase induction motor [4], or SEMIHEX [5]. The machine can work at full load with the same high efficiency, which would have had from a 3-ph supply. With appropriate choice of the terminal capacitors, it is possible for the induction motor to operate with balanced winding voltages and currents. This configuration has lower cost and higher efficiency than a comparable 1-ph motor [5].

## 3. Performance Analysis

This machine performance is sensitive to the phase connection for a given direction of rotor rotation [5]. Referring to figure 1, the machine sequence is ACB. Therefore the vector diagram of

the machine can be depicted as shown in figure 2 [4]. Applying Kirchoffs' laws, the circuit equations at steady state can be written as follows:

$$V_S = V_A - V_C \quad (1)$$

$$I_B = I_1 + I_2 \quad (2)$$

$$I_A + I_C + I_1 + I_3 = 0 \quad (3)$$

$$I_1 = Y_1 V_1 = Y_1 (V_A - V_B) \quad (4)$$

$$I_2 = Y_2 V_2 = Y_2 (V_A - V_B - V_C) \quad (5)$$

$$I_3 = Y_3 V_A \quad (6)$$

$$I_S = I_2 - I_C \quad (7)$$

$$Y_1 = jB_1, Y_2 = jB_2 \text{ and } Y_3 = jB_3 \quad (8)$$

Where:  $B_1 = \omega C_1$ ,  $B_2 = \omega C_2$  and  $B_3 = \omega C_3$

#### 4. Balanced Operation

Under perfect phase balance condition, the sum of the machine phase currents must equal zero.

This requires the currents  $I_2$  and  $I_3$  to be equal (compare equations (2) and (3)), implying that  $C_3$  must be equal to twice  $C_2$ , as follows:

$$I_2 = I_3 \quad (9)$$

Substitute by (5) and (6) into (9) to get:

$$Y_3 V_A = Y_2 (V_A - V_B - V_C) \quad (10)$$

And for balanced operation, the sum of phase voltages is zero and from (5):

$$V_2 = (V_A - V_B - V_C) = 2V_A \quad (11)$$

Substitute by (8) and (11) into (10) to get:

$$C_3 = 2C_2 \quad (12)$$

Also, substituting by (4) and (5) into (2) results in:

$$I_B = (Y_1 + 2Y_2)V_A - Y_1 V_B \quad (13)$$

Note that  $I_A$ ,  $I_B$  and  $I_C$  are lagging behind  $V_A$ ,  $V_B$  and  $V_C$  respectively by angle  $(\phi_p)$ , which is the machine load angle. Also  $I_1$  and  $I_2$  lead  $V_1$  and  $V_2$  respectively by  $90^\circ$ . The voltages  $V_1$  and  $V_S$  are line voltages (i.e.  $\sqrt{3}$  phase voltage), where supply voltage leads  $V_A$  by  $30^\circ$  and  $V_1$  lags behind  $V_A$  by  $30^\circ$ . With balanced currents flowing in the stator phases, a perfect rotating magnetic field is produced and the machine runs as if it is supplied from a balanced 3-ph supply. Calculating the capacitors can be carried out using symmetrical components method. The symmetrical component equations for a Y-connected system may be written as follows:

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_0 \\ V_p \\ V_n \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_0 \\ I_p \\ I_n \end{bmatrix} \quad (15)$$

For balanced operation, the sum of phase voltages and currents are zeros, and therefore zero sequence current and voltage are absent. Since

$$I_p = Y_p V_p \text{ \& } I_n = Y_n V_n \quad (16)$$

Substitute by (14), (15) and (16) into (1) and (13) yields:

$$V_p = \frac{\sqrt{3}V_S(2Y_2 - a^2Y_n + (1 - a^2)Y_1)}{a(1 - a)(Y_p + 2Y_2 + 3Y_1 + Y_n)} \quad (17)$$

$$V_n = \frac{\sqrt{3}V_S(aY_p - 2Y_2 - (1 - a)Y_1)}{a(1 - a)(Y_p + 2Y_2 + 3Y_1 + Y_n)} \quad (18)$$

For balanced operation, the negative sequence voltage must be absent. It could be calculated from (18) that:

$$aY_p - 2Y_2 - (1 - a)Y_1 = 0 \quad (19)$$

Substitute by (8) into real and imaginary parts of (19), and rearranging, the phase capacitors can be written as follows:

$$\left. \begin{aligned} B_1 &= \frac{2}{\sqrt{3}}(y_p) \sin(\phi_p - 30^\circ) \\ B_2 &= (y_p) \sin(60^\circ - \phi_p) \\ B_3 &= 2B_2 = (2y_p) \sin(60^\circ - \phi_p) \end{aligned} \right\} \quad (20)$$

The values of phase converter capacitors depend on  $(y_p)$  and  $(\phi_p)$  which are both function of the rotor speed only. Therefore, each operating speed has certain susceptance values for the balanced mode. Table 1 shows the type of the balancing susceptances as the load angle  $(\phi_p)$  varies. Normally, the load angle of the machine within the torque-speed stable range varies between  $\pi/6$  to  $\pi/3$ . Therefore in this paper, the balancing susceptances are considered capacitive. A software program is written to predict the performance of the machine at balanced mode for each speed.

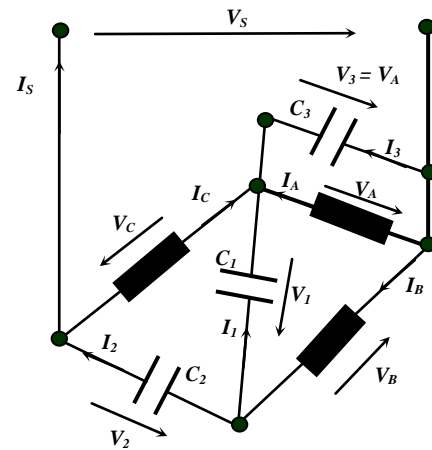


Fig. 1 Connection diagram of a 3-ph IM fed from a 1-ph supply using three capacitors

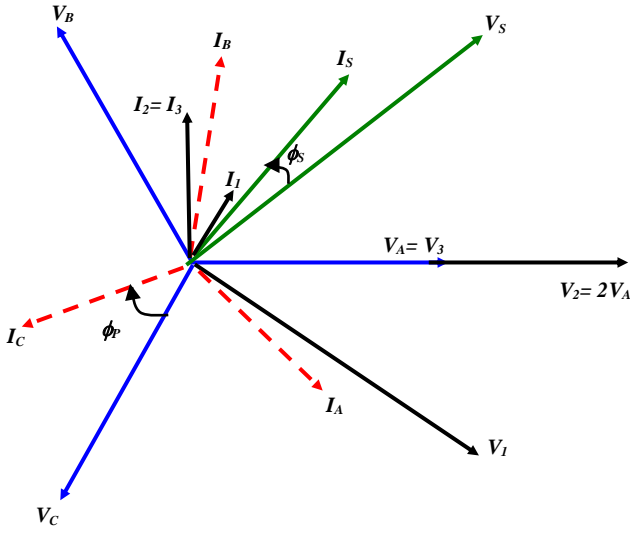


Fig. 2 Motor under perfect phase balance

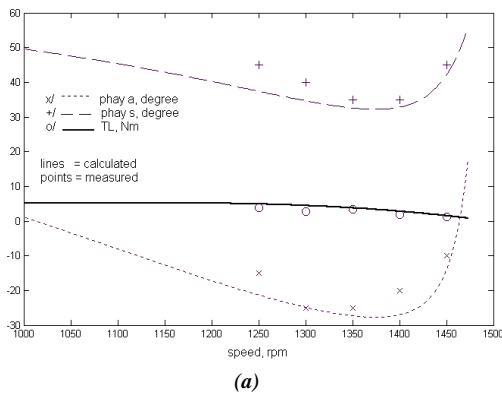
The values of  $(y_p)$  and  $(\phi_p)$  are calculated from the machine equivalent circuit, then  $C_1$ ,  $C_2$  and  $C_3$  are obtained from equation (20). For a given phase voltage, relationships between speed versus torque, admittance angle, power factor, balancing capacitors, current magnitudes and current phase angles are evaluated as depicted in figure 3. Experimental results, between 1250 rpm and 1450 rpm, are obtained and plotted as points in the same graph as shown in figure 3. A sort of correlation between simulated and experimental results can be seen from figure 3.

Table 1: Type of balancing susceptances with the load angle

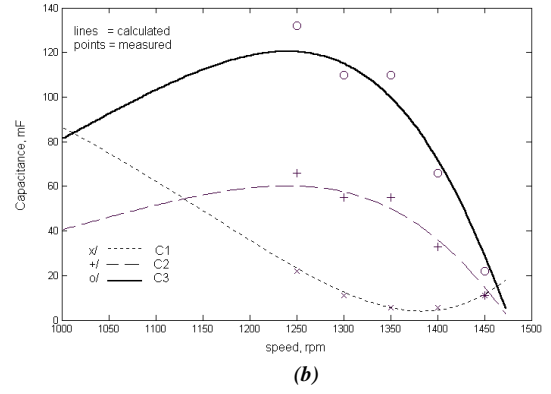
	B1	B2	B3
$\pi/6 > \phi_p$	inductor	capacitor	Capacitor
$\pi/6 < \phi_p < \pi/3$	capacitor	capacitor	Capacitor
$\phi_p > \pi/3$	capacitor	inductor	Inductor

## 5. Motor Dynamic Equations

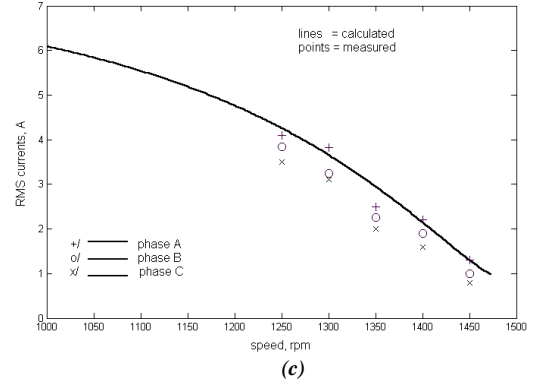
The equations describing motor transient performance can be derived using instantaneous symmetrical component (ISC) approaches [6-7].



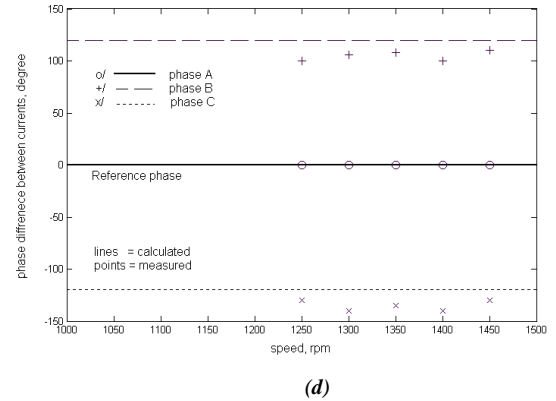
(a)



(b)



(c)



(d)

Fig. 3 Simulated (lines) and experimental (points) relationship at balanced operation between speed versus (a) torque, admittance angle and power factor (b) balancing capacitors (c) current magnitudes and (d) current phase angles

Equations (1) and (13) may be rewritten in an instantaneous form as:

$$v_s = v_a - v_c \quad (21)$$

$$i_b = B_1 p(v_a - v_b) + B_2 p(v_a - v_b - v_c) \quad (22)$$

$$\text{Where: } p = \frac{1}{\omega} \frac{d}{dt}$$

Equations (14) and (15) are also valid in the transient period, they represent the instantaneous symmetrical components.

Substitute by (14), (15) into (21) and (22) yields:

$$\sqrt{3}v_s = (1 - a^2)v_p + (1 - a)v_n \quad (23)$$

$$ai_{ps} + a^2i_{ns} = (B_1p(1-a) - 2B_2p)v_p + (B_1p(1-a^2) + 2B_2p)v_n \quad (24)$$

Voltage balance equations in terms of sequence current and impedance can be derived from the equivalent circuit shown in figure 4, where the rotational losses are neglected, as follows:

$$v_p = (R_s + X_s p)i_{ps} + X_m p i_{pr} \quad (25)$$

$$v_n = (R_s + X_s p)i_{ns} + X_m p i_{nr} \quad (26)$$

$$\left(\frac{R_r}{1-j\frac{n}{p}} + X_r p\right)i_{pr} + X_m p i_{ps} = 0 \quad (27)$$

$$\left(\frac{R_r}{1+j\frac{n}{p}} + X_r p\right)i_{nr} + X_m p i_{ns} = 0 \quad (28)$$

Where:  $X_s = X_1 + X_m$  and  $X_r = X_2 + X_m$   
The ISC currents ( $i_{ps}$ ,  $i_{ns}$ ,  $i_{pr}$  and  $i_{nr}$ ) can be written in terms of its real and imaginary parts as:

$$\left. \begin{aligned} i_{ps} &= i_{sr} + j i_{sg} \\ i_{ns} &= i_{sr} - j i_{sg} \\ i_{pr} &= i_{rr} + j i_{rg} \\ i_{nr} &= i_{rr} - j i_{rg} \end{aligned} \right\} \quad (29)$$

Equations (23) to (29) may be rearranged to conclude the following state space representation:

$$p[i]^{s,r} = [X]^{-1} \cdot [v] \quad (30)$$

Where:  $[i]^{s,r} = [i_{sr} \ i_{sg} \ i_{rr} \ i_{rg} \ q_{sr} \ q_{sg}]^T$

And  $q_{sg} = \frac{i_{sg}}{p}$  and  $q_{sr} = \frac{i_{sr}}{p}$

The matrices  $[X]$  and  $[v]$  are given by:

$$[X] = \begin{bmatrix} \sqrt{3}X_s & -X_s & \sqrt{3}X_m & X_m & 0 & 0 \\ X_m & 0 & X_r & 0 & 0 & 0 \\ 0 & X_m & 0 & X_r & 0 & 0 \\ (3B_1 + 4B_2)X_s & \sqrt{3}B_1X_s & (3B_1 + 4B_2)X_m & \sqrt{3}B_1X_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (31)$$

$$[v] = \begin{bmatrix} v_s - \sqrt{3}R_s i_{sr} + R_s i_{sg} \\ -nX_m i_{sg} - R_r i_{rr} - nX_r i_{rg} \\ nX_m i_{sr} + nX_r i_{rr} - R_r i_{rg} \\ -q_{sr} - \sqrt{3}q_{sg} - (3B_1 + 4B_2)R_s i_{sr} - \sqrt{3}B_1 R_s i_{sg} \\ i_{sr} \\ i_{sg} \end{bmatrix} \quad (32)$$

Moreover, the instantaneous electromagnetic torque can be expressed in terms of real and imaginary parts of motor currents as follows:

$$T_e = \frac{PX_m}{\omega} (i_{rr} i_{sg} - i_{sr} i_{rg}) \quad (33)$$

The mechanical equation of the machine is given by:

$$T_e - T_L = \frac{2}{P} J \omega \frac{dn}{dt} \quad (34)$$

## 6. Transient Analysis

Figure 5 shows a schematic representation of the machine model. A Matlab program is written to calculate the matrix  $[X]^{-1}$  for a given operating point correspondent to certain values of  $(B_1)$  and  $(B_2)$  which are in turn functions of speed. Then a Simulink program simulates the model given in figure 5 whose input is sinusoidal waveform having the supply frequency and maximum voltage. Each operating point involving certain supply voltage, load torque and capacitors will result in balanced operation of the machine at certain speed. Having get the currents  $[i]^{s,r}$ , the positive and negative currents can also be obtained from (29).

## 6. Steady State Analysis

Equations (26) to (30) may also predict the steady state analysis of the machine by substituting “ $j$ ” for “ $p$ ” and “ $1-s$ ” for “ $n$ ”. This can be written as:

$$[V] = [Z][I] \quad (35)$$

Where:

$$[I] = [I_{sr} \ I_{sg} \ I_{rr} \ I_{rg}]^T, \quad [V] = [V_s(RMS) \ 0 \ 0 \ 0]^T,$$

$$[Z] = \begin{bmatrix} \sqrt{3}Z_s & -Z_s & \sqrt{3}jX_m & -jX_m \\ jX_m & (1-s)X_m & Z_r & (1-s)X_r \\ -(1-s)X_m & jX_m & -(1-s)X_r & Z_r \\ 1 + Z_s(3Y_1 + 4Y_2) & \sqrt{3}(1 + Z_s Y_1) & -(3B_1 + 4B_2)X_m & -\sqrt{3}B_1 X_m \end{bmatrix},$$

$$Z_s = R_s + jX_s \text{ and } Z_r = R_r + jX_r$$

Solving (35) results in the vector of state currents  $[I]$  which when transferred to the time domain yields:

$$i_{(sr,sg,rr,rg)} = \sqrt{2} I_{(sr,sg,rr,rg)} \sin(\omega t - \phi_{(sr,sg,rr,rg)}) \quad (36)$$

Substitute by (36) into (33), the instantaneous torque ( $T_e$ ) at steady state then can be expressed as:

$$T_e = T_{av} + T_{osc} \quad (37)$$

where:

$$T_{av} = \frac{X_m P}{\sqrt{2}\omega} (I_{rr} I_{sg} \cos(\phi_{sg} - \phi_{rr}) - I_{sr} I_{rg} \cos(\phi_{rg} - \phi_{sr}))$$

$$T_{osc} = \frac{X_m P}{\sqrt{2}\omega} (I_{rr} I_{sg} \cos(2\omega t - \phi_{sr} - \phi_{rg}) - I_{rr} I_{sg} \cos(2\omega t - \phi_{sg} - \phi_{rr}))$$

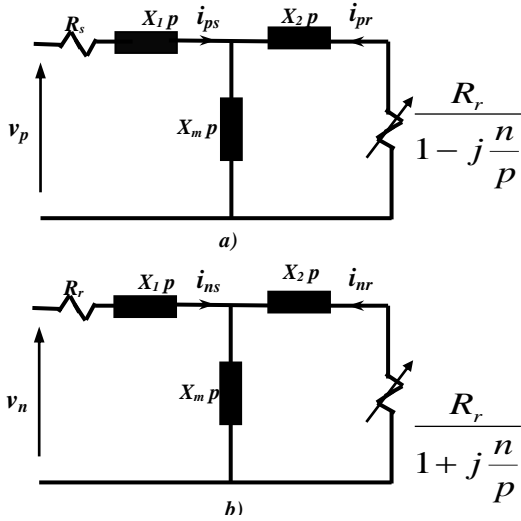


Fig. 4 Motor equivalent circuit (a) positive sequence (b) negative sequence

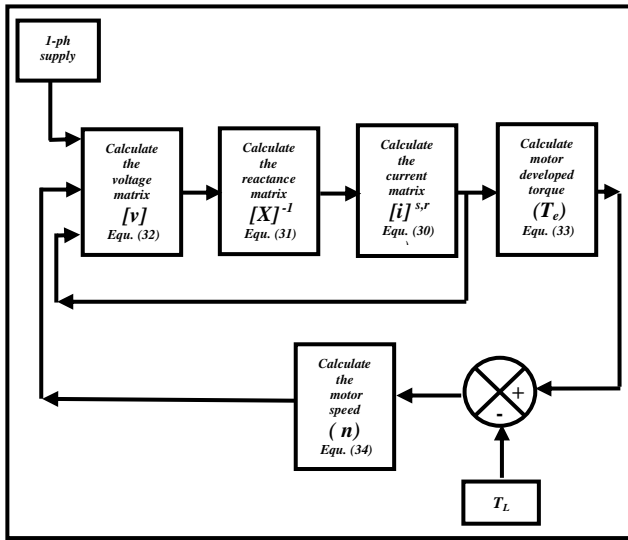


Fig. 5 SIMULINK model of the machine

It should be noted that the research of this paper is concentrated at steady state with balanced mode of operation. Also the steady state results for the unbalanced mode can be obtained using (35) as far as  $B3 = 2B2$ . Thus, phase currents, voltages and torque can be determined using (14), (15) and (29) for any values of  $C1, C2$ , speed and supply voltage.

## 7. Results

The simulation is carried out on a 3-ph 1.5kW 220V 1500rpm Y-connected induction motor whose data is given by:  $R_1=2.08\Omega$ ,  $R_2=2.64\Omega$ ,  $X_1=5.28\Omega$ ,  $X_2=5.28\Omega$ ,  $X_m=104\Omega$ ,  $P=4$ -pole and  $J = 0.01\text{kg}\cdot\text{m}^2$ . Figure 6a depicts the experimental waveform of the motor speed, scored by digital scope, when the 3-ph IM is fed from 1-ph supply and three capacitors are connected as shown in figure 1. Figure 6b depicts the corresponding simulated speed waveform using Simulink model based on figure 5. Figures 6c to 6h

illustrate different simulated current, torque and voltage waveforms. Figure 6 can be divided into the following four periods:-

**Period 1 ( $t_0 - t_1$ )** : The machine starts with a ramp increasing voltage from standstill up to the operating speed, during which the machine is expected to run at unbalanced mode, and reaches the balanced mode at the operating speed. The machine will start if the mean value of the torque at starting is higher than the load torque. Therefore, conventional methods to start the induction motor may be used or the machine is started at no-load then the load is introduced when the machine runs as shown in figures 6e and 6f.

**Period 2 ( $t_1 - t_2$ )** : At  $t_1$  the machine reaches the operating speed, whose capacitors are determined as given in (20), hence operates at a balanced mode. At steady state, the machine runs as if it is supplied from a 3-ph balanced supply whose phase voltage equals to  $(1/\sqrt{3})$  of the 1-ph supply voltage.

**Period 3 ( $t_2 - t_3$ )** : While the machine runs at the balanced mode for a certain speed and load torque, the load torque is changed suddenly as shown in figure 6e. Since the value of the capacitor results in a balanced mode of operation depends on the operating speed, and if the load is changed, the speed will change causing the machine to run in an unbalanced mode.

**Period 4 ( $t_3 - t_4$ )** : To bring the motor back to the balanced mode of operation, two methods may be used:

- The capacitors are changed to new values correspondent to the new speed. A smooth variation of capacitor can be obtained using a static exciter employing a fixed capacitor-thyristor controlled reactor (FC-TCR) [9].
- The voltage is changed to run the machine at the previous speed and the new load torque. The machine can resume the balanced mode by controlling the supply voltage, as shown in figure 6f, to keep the speed constant regardless the variation of the load torque.

From figures 6d and 6e it can be seen that during the balanced operation ( $t_1 - t_2$  or  $t_3 - t_4$ ), the negative sequence current  $I_n$  and the oscillatory torque  $T_{OSC}$  are zero while during the unbalanced operation ( $t_0 - t_1$  or  $t_2 - t_3$ )  $I_n$  and  $T_{OSC}$  are not zero. Also during the balanced operation the 3-ph currents are equal (see figures 6c and 6g), while these currents are not equal through the unbalanced operation (see figures 6c and 6h). It should be noted that the induction motor is seen by the supply as a lagging power factor load.

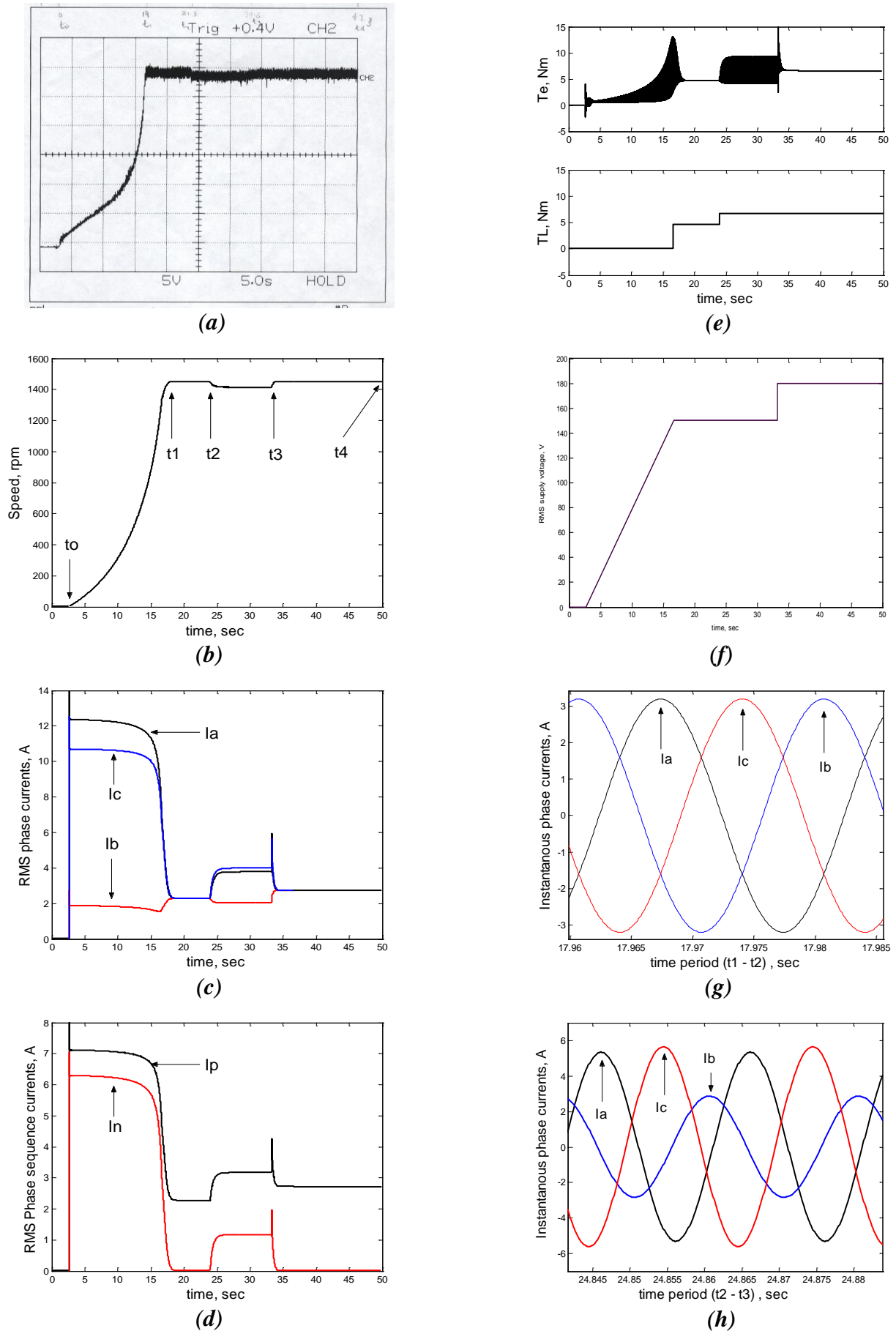


Fig. 6 Relationship between time versus (a) experimental speed, (b) simulated speed, (c) 3ph RMS phase currents, (d) RMS +ve and -ve phase sequence currents, (e) motor and load torques, (f) RMS of the 1-ph supply voltage, (g) closer view for the instantaneous 3-ph current during balanced operation [ $t_1 - t_2$ ] and (h) closer view for the instantaneous 3-ph current during unbalanced operation [ $t_2 - t_3$ ]

This configuration may operate at leading power factor due to the connection of the capacitors as seen from figure 2.

## 8. Conclusions

The three-phase induction motor can be supplied from a single-phase supply in a balanced mode of operation if three capacitors are used whose values are function of operating motor speed. The machine can work at full load with the same high efficiency that it would have had on a 3-ph supply. The machine performance is sensitive to the phase rotation. The steady state transient response of the machine is predicted using instantaneous symmetrical components. The Simulink model has been implemented and used to analyze the machine performance for different operating conditions. The simulated speed is verified and matched with the corresponding experimental waveform. If the load is changed, the speed will correspondingly change and the machine will continue to run but at unbalanced mode. The supply voltage can be then controlled to bring the machine to the balanced mode.

## 9. References

- [1]T. C. McFarland, 'Alternating current machines', D. Van Nostrand Company, 1948, pp 464.
- [2]G.M.Hertz, 'Current techniques in phase conversion systems', IEEE Rural electric power conference, Minneapolis, Minnesota, May, 1978 pp 78-83.
- [3]K.Jon,' An up to date look at conversion of single phase to three phase power', contactor electrical equipment (CEE), Jan., 1981.
- [4]O. J. M. Smith, 'High efficiency single phase motor', IEEE Trans. on EC, vol.7, No.3, Sept. 1992, pp 560-569.
- [5]O. J. M. Smith, 'Large low cost single phase semihex<sup>TM</sup> motors', IEEE Trans. on EC, vol.14, No.4, Dec. 1999, pp 1353-1358.
- [6]T. F. Chan, 'Single phase operation of a three phase induction generator with the Smith connection', IEEE Trans. on EC, vol.17, No.1, March. 2002, pp 47-54.
- [7]S. S. Murthy et al, 'Transient analysis of a three phase induction motor with single phase supply', IEEE Trans. on PAS, PAS-102, 1983, pp 28-37.
- [8]A. Al-Ohaly, A.L.Mohamadein and A.H.Al-Bahrani, 'Effect of phase balancer capacitance on the dynamic behavior of a three phase induction motor operated from a single phase supply', J. King Saud Univ., Riyadh, Eng. Sci.(1,2), vol.1, 1989, pp 123-146.
- [9]M.B.Brennen and A.A.Abbaondanti, 'Static exciter for induction generators', IEEE Trans. on IA, vol. IA-13, No.5, 1977, pp 422-428.