Simulation of Ship Maneuvering Behavior Based on The Modular Mathematical Model

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Abstract --- With the rapid development of the computer technology and its successful application in ship engineering, the method of computer simulation based on the mathematical models became more and more popular; it provides a convenient tool for predicting ship maneuverability. One of the preconditions for applying that is the modeling of the dynamic differential equations that represent the ship dynamics in three degrees of freedom. The effectiveness of simulation is guaranteed by how accurate the model is. There are different types of ship mathematical modeling. In this work, the ship modular mathematical model was investigated. Simulink software was utilized to develop the ship subsystems as individual modules. Modules hydrodynamic forces, and moments were implemented in simulating the ship maneuvering behaviors of the ESSO OSAKA tanker class ship. Moreover, different types of maneuvering are tested in particular, turning and zigzag motion.

I. INTRODUCTION

Studying the maneuverability of a ship has a great importance in order to avoid collusion with unpredictable objects. Moreover, it helps in determining the ship constraints either in its dynamics or control signal commands. Various types of mathematical model for maneuvering have been developed by different institutions and they are based on either theoretical approaches or experimental techniques [6,7]. However, generally, the mathematical model could be classified into two types; the whole ship model and the mathematical modeling group (MMG) model.

The whole ship model also called "Abkowitz" model [8] is named so after Prof. Abkowitz,1964 who proposed a method for expressing the hydrodynamic forces and moments by implementing a regression model that treats the maneuvering ship as a complete entity and the forces acting upon it are represented as a Taylor Series in kinematics and geometrical variables.

The other method (MMG) is proposed by the Japanese Mathematical Modeling Group, (JMMG) in 1970 [11,12], In such modular mathematical model; each of the ship individual elements (such as the hull, rudder, propeller, and engines) is considered as a separate module and contribute to the total hydrodynamic forces acting on the ship.

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The advantage of using the modular type model is that changing the parameters of an individual module does not altering other modules, e.g. a change in rudder size or propeller geometry could be done without having to change other modules.

The MMG modeling method is addressed here and applied in Simulink software to model, simulate, and analyze the ship dynamic system. In the proposed Simulink program, each ship subsystem was built as an individual model, and all hydrodynamics nonlinear properties of the ESSO OSAKA tanker class ship [14] were added and modeled in continuous time simulation to predict the ship maneuvering behavior. Two types of maneuvering tests; turning test and zigzag test were performed on the simulated model and the ship behavior results were analyzed

The paper is organized as follow: section II introduces the ship dynamic and kinematic and the ship maneuvering prediction; the mathematical model based on MMG is deduced in section III; the implementation of the MMG model and the maneuver simulation is explained in section IV; the result analysis is discussed in section V; and finally the conclusion of the work is summarized.

II. SHIP DYNAMICS AND KINEMATICS

In essence a set of equations of motion are constructed based on rigid body dynamics to describe the ship motion in three degree of freedom. These three motions are; the longitudinal translational motion 'surge motion' produced by the longitudinal force 'X', the lateral translational motion 'sway motion' produced by the lateral force 'Y', and the rotational motion around the *z*-axis 'yaw motion' produced by the moment around the *z*-axis 'N', these Forces and moment are driven according to the Newtonian law of motion[1].

A.Coordinate systems

To investigate the ship maneuverability by means of mathematical tool, two right-handed coordinate systems are adopted: the earth-fixed (Inertial) coordinate system " O_0 - x_0 y_0 z_0 " and the body-fixed coordinate system "O-x y z" which moves together with the ship as shown in Fig.1, the " O_0 - x_0 y_0 " plane and the "O-x y" plane lie on the undisturbed free surface, with the x_0 axis pointing to the direction of the original course of the ship, whereas the z_0 axis and the z axis point downwards vertically [2].

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Fig 1 Coordinate systems

The angle between the directions of x_0 axis and x axis is defined as the heading angle " ψ ", at the moment the maneuvering motion starts, the two coordinate systems coincide with each other, at any later moment, the position of the ship is determined by the coordinates x_{0G} and y_{0G} of the ship center of gravity in the earth-fixed coordinate system, and the orientation of the ship is determined by the heading angle " ψ ". The maneuvering motion of the ship in the horizontal plane is described by "V", the speed of translational motion, and "r", the yaw rate of rotational motion about the z axis. The components of the speed "V" in the directions of x axis and y axis are "u" and "v" respectively. The angle between the directions of speed "V" and x axis is defined as the drift angle " β ", the equations of motion in the earth-fixed coordinate system are:

$$X_o = m \, \ddot{x}_{OG}$$

$$Y_o = m \, \ddot{y}_{OG} \qquad (1)$$

$$N_o = I_a \, \ddot{\psi}_{OC}$$

where;

 X_o , Y_o and N_o are the components of external force in the directions of x_0 , y_0 and z_0 respectively.

m is the mass of the ship.

 I_z is the moment of inertia about z axis.

 ψ is the yaw acceleration.

 \ddot{x}_{OG} , \ddot{y}_{OG} are the components of acceleration in the direction of $x_{0, y_{0}}$ respectively.

B. Axis transform

For purpose of ship maneuver prediction, it is more convenient to use the equations of motion in the body fixed system instead of the earth-fixed inertial system. The relations between the two coordinate systems are given by :

$$x_0 = x_{0G} + x \cos \psi - x \sin \psi$$

$$y_0 = y_{0G} + x \sin \psi + y \cos \psi$$

$$z_0 = z$$

(2)

assume that $x_0' = x_0 - x_{0G}$, $y_0' = y_0 - y_{0G}$, and $z_0' = z_0$ Then;

$$\begin{aligned} x'_0 &= x \cos \psi - y \sin \psi \\ y'_0 &= x \sin \psi + y \cos \psi \\ z'_0 &= z \end{aligned}$$
 (3)

The components of the body system and the components of the inertial system can be rewritten as element pairs of the vectors \underline{X} and \underline{X}_0

where $\underline{X}^T = [x \ y] \text{ or } [u \ v] \text{ or } [X \ Y]$, and $\underline{X}_0^T = [x_0 \ y_0] \text{ or} [u_0 \ v_0] \text{ or } [X_0 \ Y_0]$ respectively.

 $\underline{X} = [R] \underline{X}_0$, and $\underline{X}_0 = [R]^T X$ Where ;

 $[R] \text{ is the Axis transform matrix} \\ [R] = \begin{pmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{pmatrix}$

The forces in the directions of x axis and y axis by X and Y, respectively, are expressed by:

$$X = X_0 \cos \psi + Y_0 \sin \psi$$

$$Y = -X_0 \sin \psi + Y_0 \cos \psi$$
(4)

Similar the components of ship speed and accelerations in the directions of x axis and y axis can be obtained, and by substitution into equation (4), we obtain the equations of motion in the body-fixed coordinate system in the form

$$X = m (\dot{u}_G - v_G \dot{\psi})$$
(5)
$$Y = m (\dot{v}_G - u_G \dot{\psi})$$

The moment about the *z* axis through the center of gravity is same in the body-fixed coordinate system as in the earth-fixed coordinate system [3,4,5], the origin of the body-fixed coordinate system is lying on the mid-ship point, the center of gravity has the coordinates (x_G , θ , *z*_G). In such a coordinate system the components of ship speed at the center of gravity are u_G and v_G , and at the original are *u* and *v*, therefore, the equations of motion in the body-fixed coordinate system also called the Euler equation are expressed as;

$$X = m (\dot{\boldsymbol{u}} - \boldsymbol{v} \ \boldsymbol{r} - \boldsymbol{x}_G \ \boldsymbol{r}^2)$$

$$Y = m (\dot{\boldsymbol{v}} + \boldsymbol{u} \ \boldsymbol{r} + \boldsymbol{x}_G \ \dot{\boldsymbol{r}})$$

$$N = I_Z \ \dot{\boldsymbol{\psi}} + m \ \boldsymbol{x}_G (\dot{\boldsymbol{v}} + \boldsymbol{u} \ \boldsymbol{r})$$
(6)

C. Maneuver Prediction

Ship motion differential equations, can be solved by numerical methods to determine the components of maneuvering motion u(t), v(t) and r(t) at any time [8], the proposed maneuvering prediction method is based on utilizing the parameters and the hydrodynamic coefficients of the ship in three degrees of freedom to evaluate the forces and moment acting on the ship, these forces are implemented in the Euler's equations of motion where the accelerations and velocities are obtained and fed into the axis transform module where the position and the orientation of the ship can be obtained by numerical integrations.

The Euler equations (6), is rearranged in the acceleration form, the acceleration due to surge force can be obtained by:

$$\dot{u} = (X/m) + v r + x_G r^2$$
 (7)

Solving equation (7) by numerical integration in time domain the velocity, and the displacement of the surge motion (x_{OG}) can be obtained by integration. Similarly, the sway force and yaw moment, the ship displacement in the y axis (y_{OG}) and the turning angle (ψ_s) for the ship heading.

$$x_{OG} = \int u(t) dt = \iint [(X/m) + v r + x_G r^2] dt$$

$$y_{OG} = \int v(t) dt = \int \int [(Y/m - ur - x_G r)] dt$$
(8)
$$\psi_s = \int r(t) dt = \iint [(N/Iz - mx_G/Iz (v + ur))] dt$$

III. SHIP MATHEMATICAL MODEL

To simplify the scope of the work, the MMG mathematical model has been adopted ; the MMG type model separates the forces experienced by the hull, propeller, and rudder and also includes the interaction effects between the components of model that could be developed and tested separately [11,12]. The model individual elements hydrodynamic forces and moments acting on the ship can be divided into the following components;

$$X = X_H + X_P + X_R$$

$$Y = Y_H + Y_P + Y_R$$

$$N = N_H + N_P + N_R$$
(9)

Where ; The subscripts H, P, and R denote the hull, the propeller, and the rudder respectively,

A. Hydrodynamic forces acting on the hull

The hull forces could be represented by the hydrodynamic derivatives model that represents the hull characteristic such as hull-form geometry, the hydrodynamic forces and moment acting on the hull are expressed by the components X_H , Y_H , and N_H as:

$$\begin{split} X_{H} &= -m_{x} u + R_{T}(u) + X_{vv} v^{2} + (X_{vv} - m_{y}) v_{r} \\ &+ X_{rr} r^{2} + X_{vvvv} v^{4} \\ Y_{H} &= -m_{y} v + Y_{v} v + (Y_{r} - m_{x}) r + Y_{vvv} v^{3} + Y_{vvr} \\ &+ v^{2} r + Y_{vrr} v r^{2} + Y_{rrr} r^{3} \\ N_{H} &= -I_{z} r + N_{v} v + N_{r} r + N_{vvv} v^{3} + N_{vvr} v^{2} r \\ &+ N_{vrr} v r^{2} + N_{rrr} r^{3} \end{split}$$
(10)

Where:

 m_x , m_y are the added masses in the x, y direction

is the resistance force = $a_0 + a_1 u + a_2 u^2$ $R_T(u)$

 I_z is the moment of inertia about the z axis

- X_{vv} , X_{vr} , X_{rr} , X_{rrvvv} are Non-linear and coupling components of surge force due to sway ,yaw motion for surge force
- Y_{v} , Y_{vvv} , Y_{vvr} , Y_{vrr} , Y_{rrr} are the Non-linear and coupling component of sway force due to sway and yaw motion for sway force
- N_v , N_{vv} , N_{vvr} , N_{vrr} , are Non-linear and coupling components of the yaw moment due to sway motion and yaw for yaw moment.

The proposed ship model in this study used the specifications, characteristics, as well as the hydrodynamic coefficients of commercial Esso tanker class ship [14], Table 1 illustrates the parameters, and hydrodynamic coefficients used in this study.

Table 1 Parameters and hydrodynamic coefficients

т	3500000	N_v	98631.9
M_x	523500	N_r	-1293619
m_y	533700	Y_v	-159684
I_z	56470	Y_r	-119712
N_vvr	86003	N_vrr	27866
X	20567	X	94225
X_{vr}	79987	Y_vvr	47866
$Y_{_rrr}$	-41239	N_rrr	2667498
Y	79871	$N_{\nu\nu\nu}$	-6423869

B. Hydrodynamic forces induced by propeller

The calculation of propulsive forces is dependent upon accurate representation of thrust geometry, wake, lifting and drag or more generally in the context of propeller-hull-rudder interaction. In the proposed model, propulsive forces are calculated using formulations in Inoue et al. (1981) and Spyrou (1990) for the propeller system [13].

$$X_{P} = (1-t) \rho K_{T} D_{P}^{4} n^{2}$$

$$Y_{P} = \rho n^{2} D^{4} Y_{P}^{*}$$

$$N_{P} = \rho n^{2} D^{5} N_{P}$$
(11)

Where:

 K_T is the thrust coefficient of the propeller $D_{P_{n}} n$ are the propeller diameter and propeller turn rate Y_P^*, N_P^* are generally dependent on propeller pitch Y_P, N_P have small effect on the ship motion thus they can be neglected

C. Hydrodynamic forces induced by the rudder

Rudder system is an important element of the excitation for ship motions in following and quartering seas, the performance of rudder is greatly influenced by the interactions between rudder to hull and rudder to propeller, due to the change of the lift [10]. In 1981, Inoue et al. adopted a formula for calculating the rudder forces including the aforementioned interactions that later was used in the MMG model, the rudder forces and moments including rudder to-hull interaction is as follows:

$$X_{R} = -(1-t_{R}) F_{N} \sin \delta$$

$$Y_{R} = -(1+a_{H}) F_{N} \cos \delta$$

$$N_{R} = -(x_{R}+a_{H} x_{H}) F_{N} \cos \delta$$
(12)

where;

are the interactive force coefficients t_R , a_H are the coordinate of interaction force $x_{H_i} x_R$

$$F_N$$
 is the rudder normal force given by;
 $F_{\nu r} = \alpha/2 A_{\nu r} f_{\nu} u_{\nu}^2 \sin(\alpha_{\nu})$

$$K_R$$
 are the rudder area and aspect rati

 A_R is the gradient of the lift coefficient of the fα rudder and is given by the formula;

$$f_{\alpha} = 6.13 K_R / (K_R + 2.25)$$

are the longitudinal inflow velocity and $u_{R_i} \alpha_R$ the effective rudder inflow angle

IV. MODEL IMLEMENTATION

Simulation of the ship maneuver behavior by changing the ship control parameter is achieved by using Simulink software program. As a consequence, three modules were built to express the ship subsystems representing the ship hull, propeller, rudder modules. The outputs of all modules are summed and fed back to the ship motion equation module that represents the Euler's equations of motion (6), and by using maneuvering prediction method, the accelerations and velocities are obtained and fed back at the next computation time step.

The simulated control parameter representing the rudder and the propeller rotational rate are presented to simulate the navigator maneuvering commands, and the output of the ship model are the position of the ship in the earth fixed coordinate (x_{OG}, y_{OG}) which read out by a

digital displays and plotted by a X Y plot to indicate the ship motion ,and the ship heading angle (ψ_s) which is shown as a direction of an analog compass. Fig .2 illustrates an overview of the simulated modular program that represents the ship MMG model.

A. Ship motion module

The ship motion module represents the Euler's equations of motion, the inputs to the module are the 3DOF hydrodynamic forces and moment (surge, sway, and yaw), and by using maneuvering prediction method, the accelerations and velocities are obtained by applying numerical integrations blocks and fed into the axis transform module and output the ship's current position (x_s, y_s) and the heading angle (ψ_s) as shown in Fig.3. The ship characteristics mass (m), moment of inertia (I_z) , the distance to the center of gravity (x_G) , and the hydrodynamic coefficients of the case studied ESSO OSAKA tanker class ship were used as input values, the outputs of the module include the surge, sway, and the yaw angular velocity and accelerations, they all are fed back to the three modules representing the Hull, Propeller, and rudder subsystem.



Fig 3 Ship motion module

B. Hull subsystem

The hull module represents all the interacting hydrodynamic forces and moment acting on the hull that are explained in the equation (10) which expresses the forces X_{H} , Y_{H} , and N_{H} with regard to the hydrodynamic coefficients of the ESSO OSAKA tanker class ship [14], Fig.4 represents the Hull subsystem model.



Fig.4 Hull subsystem module

C. Propeller subsystem

The propeller subsystem module represents the ship thrust based on the calculation of the propulsive forces, the most influenced force produced by the propeller and cause the ship translational motion is the surge force X_P , the sway force Y_P and the yaw moments N_P can be neglected. As shown in Fig.5.





Fig. 5 Propeller Subsystem module

D. Rudder subsystem module

The forces acting on a rudder, when considered as a separate individual element, can be obtained with the derivation of the lift and drag generated from the rudder as shown in Fig.6, the rudder control command is expressed as the rudder angle (δ),



Fig 6 Rudder Subsystem module

V. MODEL SIMULATION RESULTS

The accuracy of ship maneuvering behavior depends mainly on the predefined hydrodynamic coefficients and the precision of the ship technical data. The ship control command signals are the rudder angle denoted as (δ) and the propeller turn rate denoted as (n), the pilot action to change the rudder angle could take different shape like actions depending on the required maneuver; therefore the following test maneuvers were implemented.

A. Turning Maneuver Test

The turning test is performed to evaluate the ship's turning ability, a turning maneuver is to be performed to both starboard and port sides with the maximum design rudder angle permissible at the test speed, the rudder angle is executed following a steady approach with zero yaw rates, the actual turning test maneuver is shown in Fig 7.



Fig 7 Turning test maneuver

The turning test is applied by the simulating the turning maneuver for a ship running with a propeller turn rate equals to (11rps) revolution per second, and applying a rudder angle in a step like action of 20 degree to the starboard side and keep the applied angle to the end of simulation time as shown in Fig.8, the ship behavior is very much close to the actual turning maneuver [1]



Fig.8 Model Turning maneuver

B. Zigzag test

The zigzag test is performed to evaluate the initial turning, the yaw-checking and the course-keeping abilities [1]. There are two kinds of zigzag tests, the $10^{\circ}/10^{\circ}$ and $20^{\circ}/20^{\circ}$ tests, as shown in Fig.9, the model test is simulated by applying a generated signal similar to the pulse shown in Fig.10 that reflects the rudder action of $20^{\circ}/20^{\circ}$ zigzag maneuver.



Fig.9 Zigzag test maneuver

The accumulated errors of the empirical formulas used in the calculations along with the inaccuracy of the parameters, and hydrodynamic coefficients of ship model causes a dynamically unstable ship model or a model with poor dynamic stability. This instability can be noticed while performing the zigzag maneuvering test.



Fig.10 Model Zigzag maneuver

CONCLUSION

One of specific characteristics of MMG model is to express the hydrodynamic forces caused by propeller and rudder and interaction among hull, propeller and rudder correctly, the ship modular mathematical model proposed in the study used to simulate the ship behavior versus different maneuvers that can be used to investigate the different effects caused by applying hydrodynamic forces on the turning and zigzag motion, measuring the ship turning ability by obtaining the ship tactical parameters, time histories of the rudder angle, turning angle, overshoot angles due to zigzag maneuvering can also be investigated.

The ship model represented in the study is under condition of calm and deep water conditions, in the actual navigational situation, there are many kinds of environmental forces such as wind, wave and current. It will be proper in the future to investigate the mathematical formulas of such forces and add them as additional modules.

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